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Heat and Mass Transfer Effects on Flow past an Oscillating Infinite Vertical Plate with Variable Temperature through Porous Media

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Abstract

An exact solution of heat and mass transform on flow past an oscillating infinite vertical plate with variable temperature through porous media has been presented. The dimensionless governing equations are solved by using Laplace transform technique. The velocity and temperature profiles are studied for different physical parameters like phase angle (ωt), thermal Grashof number (Gr), mass Grashof number (Gc), permeability parameter (K), Prandtl number (Pr), Schmidt number (Sc) and time t. It is observed that the velocity increases with decrease in ωt and increase in Gc, Gr, Pr, Sc, K and t.

Keywords: Porous medium, oscillating infinite, vertical plate, heat and mass transfer, variable temperature.

Introduction

Effect of heat and mass transfer plays vital role, in space craft design, in nuclear reactors, pollution of environment etc. Flow through porous media have numerous engineering problems, for example, in the study of underground water resources ,the movement of oil and natural gas through oil reservoirs, purification of crude oil, pulp. The purpose of present study is to study the heat and mass transfer effects on flow past an oscillating infinite vertical plate with variable temperature through porous media.

Soundalgekar[1] presented an exact solution to the flow of a viscous fluid past an impulsively started infinite isothermal vertical plate. The solution was derived by the Laplace-transform technique. Heat transfer effects on flow past an impulsively started infinite vertical plate in the presence of constant heat flux and variable temperature are studied[2,3]. Muthucumaraswamy et al.[4] studied the unsteady flow past an accelerated infinite vertical plate with variable temperature and uniform mass diffusion.

Muthucumaraswamy and Valliamal[5] considered first order chemical reaction on exponentially accelerated isothermal vertical plate with mass diffusion. Muthucumaraswamy et al.[6] studied heat transfer effects on flow past an exponentially accelerated vertical plate with variable temperature. Recently R. Muthucumaraswamy and A. Vijayalakshmi[7] studied the effects of heat and mass transfer on flow past an oscillating vertical plate with variable temperature.

Formulation of Problem: The unsteady flow of an incompressible viscous fluid which is initially at rest past an infinite vertical plate with variable temperature through a porous medium has been considered. The flow is assumed to be in x-direction which is taken along the vertical plate in the upward direction. The y-axis is taken to be normal to the plate. Initially the plate and the fluid are at the same temperature T' with same concentration level C' at all points. At time $t'>0$, the plate starts oscillating in its own plane with a velocity $u = u_0 \cos\omega' t'$. The plate temperature and the level of concentration near the plate are raised linearly with time t. Then by usual Boussinesq's approximation, the unsteady flow is governed by the following equations:

$$\frac{\partial u}{\partial t'} = g\beta(T - T_\infty) + g\beta^*(C' - C'_\infty) + \nu\frac{\partial^2 u}{\partial y^2} - \nu\left(\frac{u}{K'}\right) \tag{1}$$

$$\rho C_p \frac{\partial T}{\partial t'} = \kappa \frac{\partial^2 T}{\partial y^2} \tag{2}$$

$$\frac{\partial C'}{\partial t'} = D\frac{\partial^2 C'}{\partial y^2} \tag{3}$$

With the following initial and boundary conditions:

$t' \le 0$, $u = 0$, $T = T_\infty$, $C' = C'_\infty$ for all y

$t' > 0$, $u = u_0 \cos\omega' t'$, $T = T_\infty + (T_W - T_\infty)At'$, $C' = C'_\infty + (C'_W - C'_\infty)At'$, at $y = 0$

$u = 0, \quad T \to T_\infty, \; C' \to C'_\infty \qquad \text{as } y \to \infty$ (4)

where $A = \left(\dfrac{u_0^2}{\nu} \right)$

On introducing the following dimensionless quantities:

$$U = \left(\frac{u}{u_0} \right), t = \left(\frac{t' u_0^2}{\nu} \right), Y = \left(\frac{y u_0}{\nu} \right), \theta = \left(\frac{T - T_\infty}{T_w - T_\infty} \right), \omega = \left(\frac{\nu \omega'}{u_0^2} \right), Sc = \frac{\nu}{D}\; Gr = \left(\frac{g\beta\nu(T_w - T_\infty)}{u_0^3} \right), Gc = \left(\frac{g\beta^* \nu(C'_w - C'_\infty)}{u_0^3} \right),$$

$$C = \left(\frac{C' - T_\infty}{T_w - T_\infty} \right), \; Pr = \frac{\mu C_p}{k} \quad \frac{1}{K} = \frac{u_0^2 K'}{\nu^2} \tag{5}$$

in equations (1) to (4) , leads to

$$\frac{\partial U}{\partial t} = Gr\theta + GcC + \frac{\partial^2 U}{\partial Y^2} - KU$$ 6)

$$\frac{\partial \theta}{\partial t} = \frac{1}{Pr} \frac{\partial^2 \theta\theta}{\partial Y^2} \tag{7}$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc} \frac{\partial^2 C}{\partial Y^2} \tag{8}$$

Initial and boundary conditions in non dimensional form are

$U= 0, \quad \theta = 0, \quad C = 0 , \qquad$ for all $Y \le 0, t \le 0$

$U = \cos\omega t, \; \theta = t, \; C = t, \qquad$ at $Y = 0, t > 0$

$U = 0, \; \theta \to 0, \; C \to 0 \qquad$ as $Y \to \infty$ (9)

All the physical variables are defined in the nomenclature.

Method of Solution

The governing equations in exact form are solved by laplace transform technique. On taking laplace transform of the equations (6), (7), (8) and (9),we get

$$\frac{d^2\bar{\theta}}{dY^2} - s\,Pr\,\bar{\theta} = 0 \tag{10}$$

$$\frac{d^2\bar{C}}{dY^2} - sSc\bar{C} = 0 \tag{11}$$

$$\frac{d^2\bar{U}}{dY^2} - (s + K)\bar{U} = -Gr\bar{\theta} - Gc\bar{C} \tag{12}$$

$\bar{U} = 0, \bar{\theta} = 0, \bar{C} = 0 \qquad$ for all Y, $t \le 0$

$\bar{U} = \dfrac{s}{s^2 + \omega^2}, \bar{\theta} = \dfrac{1}{s^2}, \bar{C} = \dfrac{1}{s^2} \qquad$ at $Y = 0, t > 0$

$\bar{U} = 0, \bar{\theta} \to 0, \bar{C} \to 0 \qquad$ as $Y \to \infty$, $t > 0$ (13)

On solving the equations (10), (11), (12) with the help of equation (13),we get

$$\bar{\theta} = \frac{e^{-Y\sqrt{s\,Pr}}}{s^2} \tag{14}$$

$$\bar{C} = \frac{e^{-Y\sqrt{sSc}}}{s^2} \tag{15}$$

$$\bar{U} = \left[\frac{s}{s^2+\omega^2} + \frac{1}{s^2}\left\{ \frac{Gr}{s(Pr-1)-K} + \frac{Gc}{s(Sc-1)-K} \right\} \right] e^{\sqrt{s+K}} - \frac{1}{s^2}\left\{ \frac{Gr e^{-Y\sqrt{s\,Pr}}}{s(Pr-1)-K} + \frac{Gc e^{-Y\sqrt{sSc}}}{s(Sc-1)-K} \right\} \quad (16)$$

Where s is the laplace transform parameter.

On taking inverse laplace transform of of equations (14), (15) and (16) we get

$$\theta = t\left[(1+2\eta^2\,Pr)\operatorname{erfc}(\eta\sqrt{Pr}) - \frac{2}{\sqrt{\pi}} e^{-\eta^2 Pr}\eta\sqrt{Pr} \right] \quad (17)$$

$$C = t\left[(1+2\eta^2 Sc)\operatorname{erfc}(\eta\sqrt{Sc}) - \frac{2}{\sqrt{\pi}} e^{-\eta^2 Sc}\eta\sqrt{Sc} \right] \quad (18)$$

$$\begin{aligned}
U &= e^{-i\omega t}\left[e^{Y\sqrt{K-i\omega}}\operatorname{erfc}\left\{\eta+\sqrt{(K-i\omega)t}\right\} + e^{-Y\sqrt{K-i\omega}}\operatorname{erfc}\left\{\eta-\sqrt{(K-i\omega)t}\right\} \right] \\
&+ e^{i\omega t}\left[e^{Y\sqrt{K+i\omega}}\operatorname{erfc}\left\{\eta+\sqrt{(K+i\omega)t}\right\} + e^{-Y\sqrt{K+i\omega}}\operatorname{erfc}\left\{\eta-\sqrt{(K+i\omega)t}\right\} \right] \\
&+ e^{Y\sqrt{K}}\operatorname{erfc}(\eta+\sqrt{Kt})\left[\frac{Gr}{Pr-1}\left(\frac{-1}{2c^2} + \frac{t}{2c} + \frac{Y}{4c\sqrt{K}} \right) + \frac{Gc}{Sc-1}\left(\frac{-1}{2b^2} + \frac{t}{2b} + \frac{Y}{4b\sqrt{K}} \right) \right] \\
&+ e^{-Y\sqrt{K}}\operatorname{erfc}(\eta-\sqrt{Kt})\left[\frac{Gr}{Pr-1}\left(\frac{-1}{2c^2} + \frac{t}{2c} - \frac{Y}{4c\sqrt{K}} \right) + \frac{Gc}{Sc-1}\left(\frac{-1}{2b^2} + \frac{t}{2b} - \frac{Y}{4b\sqrt{K}} \right) \right] \\
&+ \frac{Gr e^{-ct}}{2(Pr-1)c^2}\left[\begin{array}{l} e^{y\sqrt{-c\,Pr}}\left\{ \operatorname{erfc}\left(\eta+\sqrt{-c\,Pr\,t}\right) - \operatorname{erfc}\left(\eta\sqrt{Pr}+\sqrt{-ct}\right) \right\} \\ + e^{-y\sqrt{-c\,Pr}}\left\{ \operatorname{erfc}\left(\eta-\sqrt{-c\,Pr\,t}\right) - \operatorname{erfc}\left(\eta\sqrt{Pr}-\sqrt{-ct}\right) \right\} \end{array} \right] \\
&+ \frac{Gc e^{-bt}}{2(Sc-1)b^2}\left[\begin{array}{l} e^{y\sqrt{-bSc}}\left\{ \operatorname{erfc}\left(\eta+\sqrt{-bSct}\right) - \operatorname{erfc}\left(\eta\sqrt{Sc}+\sqrt{-bt}\right) \right\} \\ + e^{-y\sqrt{-bSc}}\left\{ \operatorname{erfc}\left(\eta-\sqrt{-bSct}\right) - \operatorname{erfc}\left(\eta\sqrt{Sc}-\sqrt{-bt}\right) \right\} \end{array} \right] \\
&- \frac{Gr}{Pr-1}\left[\operatorname{erfc}(\eta\sqrt{Pr})\left\{ \frac{-1}{c^2} + \frac{1}{c}\left(t + \frac{Y^2\,Pr}{2} \right) \right\} - \frac{Y}{c} e^{-\frac{Y^2\,Pr}{4t}} \sqrt{\frac{Pr\,t}{\pi}} \right] \\
&- \frac{Gc}{Sc-1}\left[\operatorname{erfc}(\eta\sqrt{Sc})\left\{ \frac{-1}{b^2} + \frac{1}{b}\left(t + \frac{Y^2 Sc}{2} \right) \right\} - \frac{Y}{b} e^{-\frac{Y^2 Sc}{4t}} \sqrt{\frac{Sct}{\pi}} \right] \quad (19)
\end{aligned}$$

Where $c = \frac{-K}{Pr-1}$, $b = -\frac{K}{Sc-1}, \eta = \frac{Y}{2\sqrt{t}}$

Results and Discussion

The problem of heat and mass transfer on flow past an oscillating infinite vertical plate with variable temperature through porous media has been formulated and solved analytically. The values of velocity, temperature and concentration are obtained for the physical parameters such as thermal Grashof number Gr, mass Grashof number Gc, Prandtl number Pr, Schmidt number Sc, time t and permeability parameter K. on the flow patterns, the computation of the flow fields are carried out. The value of the Prandtl number Pr is chosen to represent air (Pr = 0.71). The value of Schmidt number is chosen to represent water vapour (Sc = 0.6).

The velocity profiles has been studied and presented in figure 1 to 5. The effect of velocity for different values of phase angle ωt is presented in figure 1. It is observed that the velocity increases with decreasing phase angle. The velocity profiles for different values of time (t=0.2,0.4,0.6,0.8) is presented in figure 2. It is observed that velocity increases with increasing time. The velocity profiles for different values of mass Grashof number (Gc =-10,-5, 3, 5,) is presented in figure 3. It is observed that velocity increases with increasing Gc. The velocity profiles for different values of thermal Grashof number (Gr =-5,-2, 2, 5, 10) is seen in figure 4 It is observed that velocity increases with increasing Gr. The effect of velocity for different values of permeability (K=0.25, 0.5, 0.75,1) is seen in figure 5. It is observed that the velocity increases with increasing permeability.

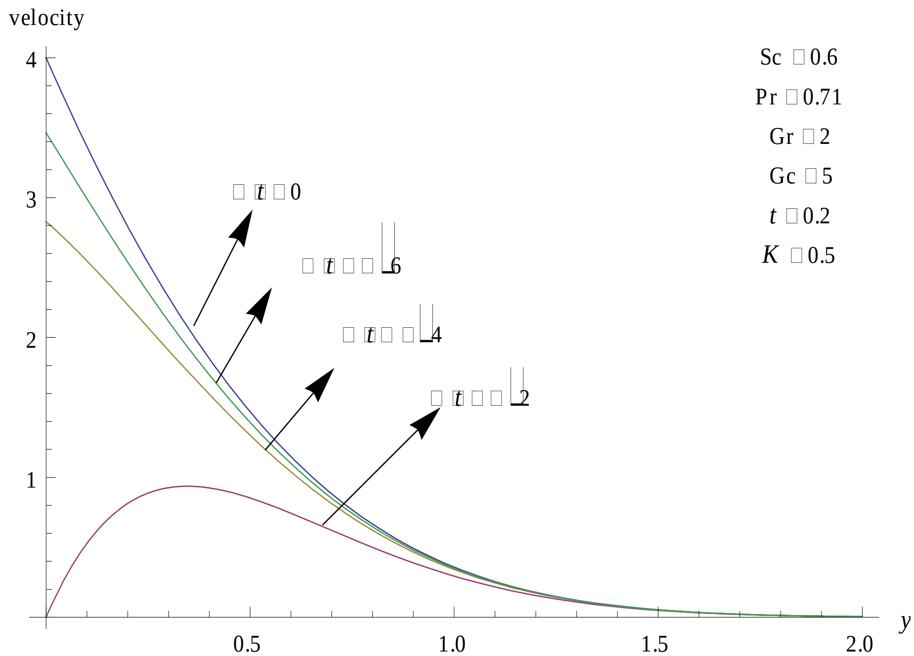

Figure-1
Velocity profiles for different Phase angles (ωt)

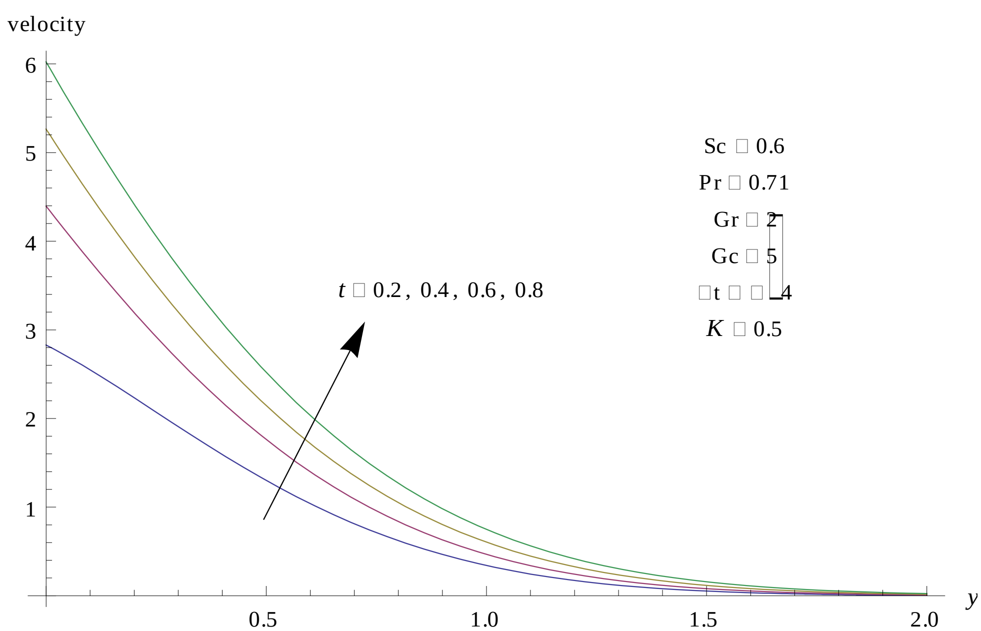

Figure-2
Velocity profiles for different values of t

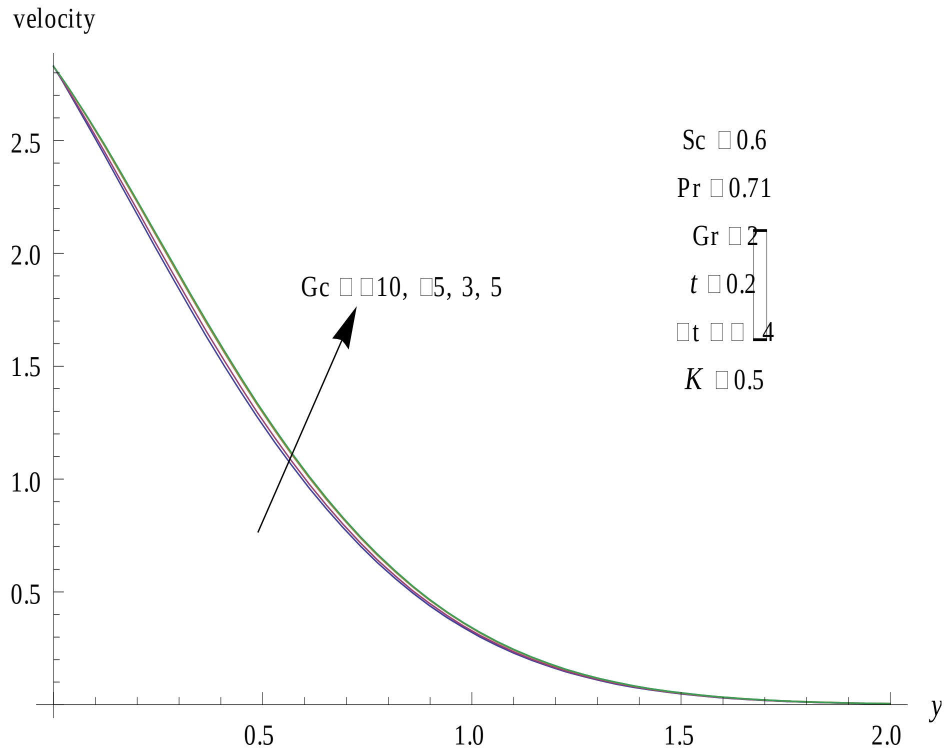

Figure-3
Velocity profiles for different values of Gc

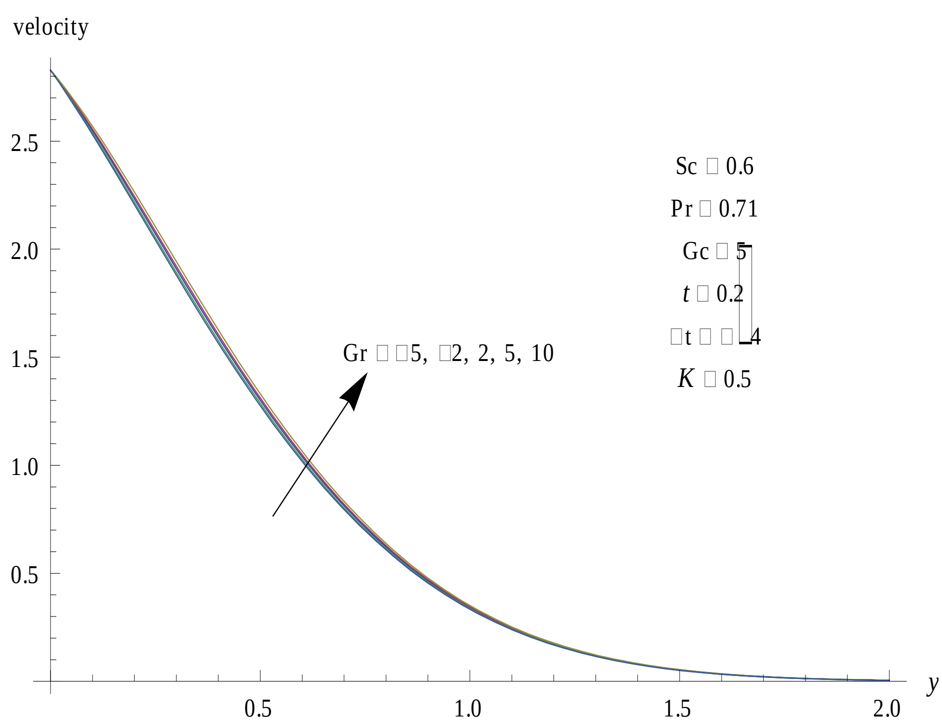

Figure-4
Velocity profiles for different values of Gr

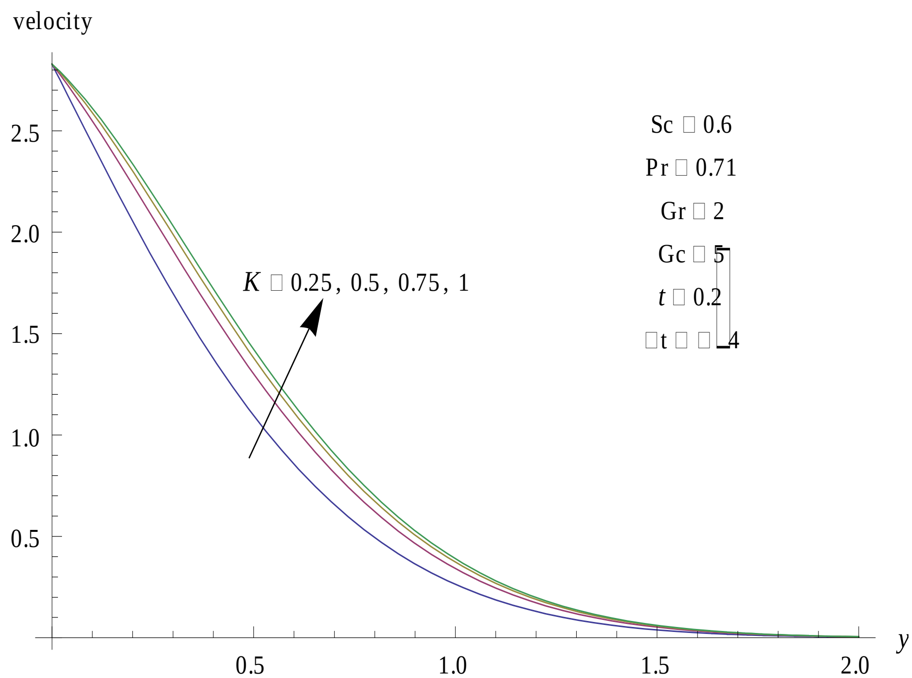

Figure-5
Velocity profiles for different values of K

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