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Introduce and Compare Two Approaches for Monitoring a Two-Stage Process by Profile Quality Characteristic in the Second Stage

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Abstract

The quality of products is monitored according to the different control charts for quality characteristics of them. These quality characteristics are either variable or attribute can be a single variable, or a vector of variables or a profile relationship. However, because the quality of the product is the result of the performance of different procedures on the product, and usually these stages are not independent of each other, therefore, the assumption of independent process affecting error on the quality of the output. Up to now the effects of these situations on monitoring the multi-stage processes with single or multi variables were being examined. Though, multi-stage profile processes were less appealing for researchers. This paper introduces a model for a two-stage profile in addition to two different approaches that have been proposed for monitoring process.And the variation of the coefficients of the profile, as well as changes in the quality characteristics of the first stage, in a two-stage process, on the second phase control charts were reviewed.

Keywords: Average run length (ARL), exponentially weighted moving average (EWMA) control chart, multistage processes, T^2 multivariate control chart, profile monitoring.

Introduction

Nowadays, quality of many productions[1] and service environment[2] is monitored by statistical quality control based on statistical methods[3]. Statistical process control[4], design of experiment[5] and process capability are three major issues in statisticalquality control. Control charts are powerful tools in statistical process control. Research activities undertaken in the field of control charts, have great emphasis on the proper use of control charts in the proper position and number of investigation has been done on the error resulting from improper use of them.Two of these studies are the major source of this article, which is trying to consider the both together. The first group believed that because many of the manufacturing processes are complex systems and this process is often not a single stage, hence, the output quality should be evaluated by monitoring several interdependent processes that take place. This type of control is called multistage processes monitoring[6]. Multistage processes have cascade properties. This means that at each stage of the process, quality is dependent on two parameters. One is particular quality, which is the quality of operations in the current period. And the other is the overall quality, which is defined as the quality of pre-and current stages, The secondgroup had tried to describe the quality of the product and the process performance by monitoring the relationship between a response variable and one or more independent variables. They have named this equation (relationship) as profile[7].

Zheng[8] first carried out monitoring a multistage processes. The foundation of these efforts were based on the cascade property, then Hawkins[9-10] provided similar charts regardless of the cascade property. This new control chart created new horizons in the analysis and improvement of a multistage processes, and then Wade and Woodall[11] and Yang and Yang[12] began to develop, expand, and emphasize the use of the charts. Several examples of multistage processes in the semiconductor industry by Skinner et al[13] and Jearkpaporn et al[14-16] have been raised, assuming that the data is not normalized. Loredo et al[17], Shu and Tsung[18] and Yang and Yang[19] conducted their research with premise of data correlation. Also using neural network by Niaki and Davoodi[20] was studied.

In the first study in profile monitoring, Kang and Albine[21] proposed two approaches including T^2 and EWMA-R control charts for monitoring simple linear profile in Phase I and II. Then, profile monitoring has been investigated by many authors. For example, in simple linear profile, Kim et al[22] coded the explanatory variables to achieve uncorrelated parameters. Then, they proposed using 3 EWMA control charts to monitor intercept, slope and variance of errors, separately. Afterward, Gupta et al[23] proposed a method in which the performance of the EWMA-3 method is justified by replacing Shewhart control charts. Zouet al[24] proposed a method based on generalized likelihood ratio (GLR) for monitoring simple linear profile in Phase II.The effects of non-normality residual on simple linear profile monitoring are investigated by Noorossana et al[25] and the effects of non-independent data on profiles monitoring are studied by Jensen et al[26]. The major achievement in profile monitoring can be founded in Noorossana et al[27].

In the research that has been cited, few studies have been carried on these two topics; profile monitoring and controlling multistage processes, together. In this paper we will examine the monitoring of a two-stage process in a manner that there is a simple linear profile in one of the stages. One of the researchers that has been done in this field can be Niakiet al[28] research that investigates the methods of monitoring the linear profiles in a two-stage process where at any stage, presents a profile instead of a quality characteristics.

In this paper we have considered a two-stage process. In the first stage, there is a quality characteristic as a random variable and in the second stage, there is a profile.Also the quality characteristic of the first stage is one of the profile independent variables in the second stage. We decided to monitor the changes in the profile coefficients of the second stage and also the changes in first stage quality characteristic in the second stage of the process by the two approaches: $\bar{X}-R, T^2, \chi^2$ and $EWMA-R, T^2, \chi^2$ and then compare the results.

In the approach $\bar{X}-R, T^2, \chi^2$ for monitoring the quality characteristics of the first phase, diagram $\bar{X}-R$, for monitoring the profile parameters of the second phase, chart T^2, and for monitoring the amount of residuals, χ^2 control chart are used. In the approach *EWMA – R*, T^2, χ^2 for monitoring the quality characteristics of the first phase, graph *EWMA – R*, for monitoring the profile parameters of the second phase, chart T^2, and for monitoring the amount of residuals, chart χ^2 are used. The paper is structured as follows:

In the second part, we analyze the problem and model assumptions and then in the third section, we introduce charts and statistics we use. In the fourth and fifth parts, respectively, method of data collection for monitoring profiles in the two-stage process and sensitivity analysis of the "Average Run Length" related to the changes of model parameters in the second phase are assessed. Finally, conclusions are presented.

Methodology

Defining the Problem and Model Assumptions: In many situations, the quality of a process or a product is characterized by the relationship between a response variable and one independent variable. Thus at each stage of sampling, a set of data is collected which can be shown by using a profile. But sometimes it is necessary that monitoring takes place at different stages of processes. This type of monitoring is named multistage processes monitoring. In fact, in this case the steps are not independent of each other. And based on the cascade property, former stages have their impact on the latter stages. Figure 1 shows the first stage of a two-stage process in which there is a quality characteristic x_1. According to equation 1 the quality characteristic x_1 has normal distribution.

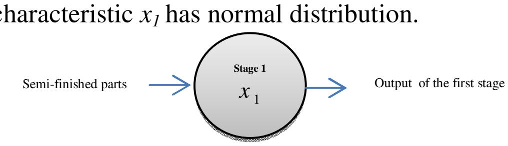

Figure-1
The first stage of a two-stage process with a qualitative characteristic x_1

$$x_1 \sim N(\mu_{x_1}, \sigma^2_{x_1}) \tag{1}$$

Also as it is shown in Figure 2 in the second stage profile y_2 is available, as in equation 2. In the second phase profile, besides x_2 as an independent variable of the profile that gets constant values, the quality characteristic of the first stage (x_1) as another independent variable of the profile is considered to be constant like the assumptions.

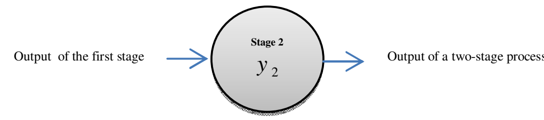

Figure-2
The second stage of a two-step process with profile y_2

$$y_2 = \beta_0 + \gamma_1 x_1 + \beta_2 x_2 + \varepsilon \tag{2}$$

In equation 2 β_i's and γ_1 are coefficients of second stage profile so that the expected change in the y_2 per unit change in x_1 alone or x_2 in isolation with all other variables being constant shows that x_2 is the effective qualitative characteristics on the profile with constant values in the second stage and x_1 is the qualitative characteristics of the first step and ε is the error term. Model assumptions are: i. The profiles are intended to be linear. ii. due to regression, the values of x_2 are constant (not random variables), iii. There is no autocorrelation within the profiles. iv. ε has a normal distribution.

$$\varepsilon \sim N(0, \sigma^2_\varepsilon) \tag{3}$$

Control Charts and Statistics Used in the Phase II: According to the article, which is performed in the second phase of the control chart, the main objective of this phase is to explore changes in the process once possible. So, we review and explain each chart in both approaches $\bar{X}-R, T^2, \chi^2$ and $EWMA-R, T^2, \chi^2$.

$\bar{X}-R$ Control chart: $\bar{X}-R$ control chart can be one of the primary charts for monitoring a characteristic feature when the sample size in each sample is between 2 and 9. In this paper, we use this chart for monitoring the mean and distribution of the first stage quality characteristics x_1 of the $\bar{X}-R, T^2, \chi^2$ approach. Statistic of graphs $\bar{X}$ and R are expressed in equations4 and 5, i is the counter of the sample size n_1 of the quality characteristics of the first stage (x_1). $i=1,\dots,n_1$. j is the counter of the number of samples $(j=1,\dots,m)$. Also limits of the control chart $\bar{X}$ and R are expressed in equations 6 and 7. d_2, d_3 are constants that are dependent on sample size.

$$\bar{x}_j = \frac{\sum_{i=1}^{n_1} x_i}{n_1} \quad ; j = 1,\ldots,m \tag{4}$$

$$R_j = \max(x_i) - \min(x_i) \quad ; i = 1,\ldots,n_1 \quad ; j = 1,\ldots,m \tag{5}$$

$$\begin{cases} UCL_{\bar{X}} = \mu_{x_1} + L\dfrac{\sigma_{x_1}}{\sqrt{n_1}} \\ LCL_{\bar{X}} = \mu_{x_1} - L\dfrac{\sigma_{x_1}}{\sqrt{n_1}} \end{cases} \tag{6}$$

$$\begin{cases} UCL_R = \sigma_{x_1}(d_2 + Ld_3) \\ LCL_R = \max\{0, \sigma_{x_1}(d_2 + Ld_3)\} \end{cases} \tag{7}$$

$EWMA-R$ Control chart: As mentioned in the introduction, one of the methods used for monitoring simple linear profiles is $EWMA-R$ approach[36], usually in profile monitoring $EWMA$ control chart for monitoring the mean residuals and R control chart for monitoring the diagram distribution are used. In this paper according to approach $EWMA-R, T^2, \chi^2$ the $EWMA$ and R control charts are used to monitor the mean and distribution of the first stage qualitycharacteristics x_1. The equations 8 and 9 show statistic and limits of $EWMA$ control chart respectively. It should be noted that $\bar{x}_j$, the average j^{th} sample of quality characteristic x_1 for stage 1 and λ has a constant value between zero and one, and L is the controlling factor that is calculated according to the type one error.

$$EWMA_j = \lambda\bar{x}_j + (1-\lambda)EWMA_{j-1} \quad ; EWMA_o = \mu_{x_1} \tag{8}$$

$$\begin{cases} UCL_{EWMA} = \mu_{x_1} + L\sigma\sqrt{\dfrac{\lambda}{(2-\lambda)n_1}} \\ LCL_{EWMA} = \mu_{x_1} - L\sigma\sqrt{\dfrac{\lambda}{(2-\lambda)n_1}} \end{cases} \tag{9}$$

The statistic and limits of R control chart are the same as equations 5 and 7.

T^2 Multivariate control chart: In quality control for monitoring a process that has more than one quality characteristic and quality characteristics are interdependent, T^2 multivariate control chart is used. That, when estimating the parameters of a simple linear regression according to equation 2, $(\hat{\beta}_{0j}, \hat{\gamma}_1, \hat{\beta}_{2j})$ are dependent with the method of least squares error. So they can be monitored simultaneously by a T^2 multivariate control chart[36] Hence the statistic used in this diagram, is obtained from equation 10.

$$T_j^2 = (z_j - \mu)^T \sigma^{-1}(z_j - \mu) \tag{10}$$

Where:

$$z_j = (\hat{\beta}_{0j}, \hat{\gamma}_1, \hat{\beta}_{2j})$$

$$\hat{\beta}_{2j} = \frac{r_{y_2x_2} - r_{y_2x_1}r_{x_1x_2}}{1 - r_{x_1x_2}^2} \times \frac{S_{y_2}}{S_{x_2}} \tag{11}$$

$$\hat{\gamma}_1 = \frac{r_{y_2x_1} - r_{y_2x_2}r_{x_1x_2}}{1 - r_{x_1x_2}^2} \times \frac{S_{y_2}}{S_{x_1}}$$

$$\hat{\beta}_{0j} = \bar{y}_j - \hat{\beta}_{2j}\bar{x}_2 - \hat{\gamma}_1\bar{x}_1$$

$$\mu = (\beta_{0j}, \gamma_1, \beta_{2j}) \tag{12}$$

$$\sigma = \begin{pmatrix} \sigma_0^2 & \sigma_{01}^2 & \sigma_{02}^2 \\ \sigma_{10}^2 & \sigma_1^2 & \sigma_{12}^2 \\ \sigma_{20}^2 & \sigma_{21}^2 & \sigma_2^2 \end{pmatrix} \tag{13}$$

Care must be taken that if the process is controlled, T_j^2 has a chi-square distribution. The upper control limit for this control chart is in accordance with the following formula.

$$UCL_{T^2} = \chi_{\alpha,\upsilon}^2 \tag{14}$$

In this paper, that we tend to monitor the three profile parameters by T^2 multivariate control chart the upper limit of the control chart has a chi-square distribution with $\upsilon = 3$ degrees of freedom.

χ^2 Control chart: Usually χ^2 control chart is used to monitor the distribution, so in this article we use it to monitor residuals. If the chart warns it is because it has only upper limit according to equation 15, It means at least of the residuals rises above the limit, and this means that the difference between the actual and predicted values of the profile has increased. In equation 16 the graph relation is expressed with regards to the residuals that have a normal distribution with zero mean and variance σ_e^2.

$$\chi^2 = \sum_{i=1}^{n_1}\sum_{k=1}^{n_2}\left(\frac{e_{ik}}{\sigma_e}\right)^2 \tag{15}$$

$$UCL_{\chi^2} = \chi_{\alpha,n_1 \times n_2}^2 \tag{16}$$

In equation 15, k is the counter of the sample size of the effective qualitative characteristics of profile in the second phase ($k = 1,\ldots,n_2$).

How to Collect Data for Monitoring Profile in a Two-Stage Process: Since this model is different from the other models of multi-step processes, samples are obtained from relation (17).

$$\begin{aligned} &(y_{111}, x_{111}, x_{12}), (y_{112}, x_{111}, x_{22}), \ldots, (y_{11n_2}, x_{111}, x_{n_22}) \\ &,(y_{121}, x_{121}, x_{12}), (y_{122}, x_{121}, x_{22}), \ldots, (y_{12n_2}, x_{121}, x_{n_22}) \\ &,\ldots,(y_{1n_11}, x_{1n_11}, x_{12}), (y_{1n_12}, x_{1n_11}, x_{22}), \ldots, (y_{1n_1n_2}, x_{1n_11}, x_{n_22}) \\ &,\ldots,(y_{m11}, x_{m11}, x_{12}), (y_{m12}, x_{m11}, x_{22}), \ldots, (y_{m1n_2}, x_{m11}, x_{n_22}) \\ &,\ldots,(y_{mn_11}, x_{mn_11}, x_{12}), (y_{mn_12}, x_{mn_11}, x_{22}), \ldots, (y_{mn_1n_2}, x_{mn_11}, x_{n_22}) \end{aligned} \tag{17}$$

According to equation 17, x_{ji1} is the i^{th} amount of stage onequality characteristic of the n_1 sample on the j^{th} sampling, and x_{k2} is the k^{th} amount of stage twoquality characteristic of the n_2 sample, and y_{jik} is the profile amount of the j^{th} sampling for the i^{th} amount of stage onequality characteristic of the n_1 sample and the k^{th} amount of stage two qualitative characteristic of the n_2 sample.

The $\mathbf{X}$ vector and the $\mathbf{Y}$ matrix are according to the equation 18.

$$\mathbf{X}=\begin{bmatrix} 1 & x_{111} & x_{12} \\ 1 & x_{111} & x_{22} \\ 1 & x_{111} & x_{32} \\ . & . & . \\ 1 & x_{111} & x_{n_2 2} \\ 1 & x_{121} & x_{12} \\ 1 & x_{121} & x_{22} \\ . & . & . \\ 1 & x_{121} & x_{n_2 2} \\ . & . & . \\ 1 & x_{1n_1 1} & x_{12} \\ . & . & . \\ 1 & x_{1n_1 1} & x_{n_2 2} \\ 1 & x_{211} & x_{12} \\ . & . & . \\ 1 & x_{2n_1 1} & x_{n_2 2} \\ . & . & . \\ 1 & x_{mn_1 1} & x_{n_2 2} \end{bmatrix}_{(m\times n_1\times n_2)\times 3}, \mathbf{Y}=\begin{bmatrix} y_{111} \\ y_{112} \\ y_{113} \\ . \\ y_{11n_2} \\ y_{121} \\ y_{122} \\ . \\ y_{12n_2} \\ . \\ y_{1n_1 1} \\ . \\ y_{1n_1 n_2} \\ y_{211} \\ . \\ y_{2n_1 n_2} \\ . \\ y_{mn_1 n_2} \end{bmatrix}_{(m\times n_1\times n_2)\times 1} \quad (18)$$

Then, using the least squares error, we have equation 19 for our model.

$$\hat{\beta}=\left(\mathbf{X}'\mathbf{X}\right)^{-1}\mathbf{X}'\mathbf{Y} \quad (19)$$

In this paper, according to the information of sensitivity analysis of average run length towards changes in parameters of the model and according to the above relations, we have obtained the profile's intercept and slope coefficients. In fact, because our research is in the phase II of the control chart, at first we assume real profiles with default coefficients in the control mode. Then with simulation, we will estimate them. Moreover, according to the comparison of two monitoring approach proposed in this paper, $\bar{X}-R, T^2, \chi^2$ and $EWMA-R, T^2, \chi^2$ we are trying to compare these two approaches to the positive performance of the phase II.

Results and Discussion

Sensitivity Analysis of the Average Run Length to the Changes in Model Parameters: The structure of the sensitivity analysis: As it was the case, since the research has been done in phase II of the control chart, therefore at first according to the equation 1 and 2, we consider the model coefficients as given. Because the goal of the Phase II control chart is monitoring process. This is why we want to achieve: First, sensitivity of each of the proposed approaches for variation in the profile coefficients, and the changes in average of the first step quality characteristics. Second, among the proposed approaches, which one is the most sensitive to the variation in profile coefficients and changes in qualitative parameters of phase I. And finally, the best approach according to the model presented in equation 1 and 2.

Hence, in the first approach $\bar{X}-R, T^2, \chi^2$ for monitoring the phase one quality characteristics, a graph $\bar{X}-R$, is used. Secondly profile parameters are monitored by T^2 control chart. And plot χ^2 is the residuals monitoring chart. In the approach $EWMA-R, T^2, \chi^2$ for monitoring the quality characteristics of the first stage, a graph $EWMA-R$, is used. Secondly, profile parameters are monitored by T^2 control chart. Also, a chart χ^2 is used to monitor the residuals. Decision criterion is the average run length. Because one of the achievements of this paper is to compare the two approaches $\bar{X}-R, T^2, \chi^2$ and $EWMA-R, T^2, \chi^2$, both of these approaches must have the same values of ARL_0 and it should be considered at least equal to 200. Hence in this paper $\alpha=0.005$ is equivalent to $ARL_0=200$. According to the number of charts of each approach we achieved this objective. On the other hand, according to equation 1 the phase one quality characteristic has a standard normal distribution $x_1 \sim N(\mu_{x_1}=0, \sigma^2_{x_1}=1)$ and in the simulation we assume $n_1=5$. The effective quality characteristics of the second stage on profile are constant numbers $X_2=[2\ 4\ 6\ 8]$, and $n_2=4$. These numbers are considered according to the second assumption of the model. The number of sampled loads is 100 $(m=100)$.

Because this research has been done on phase II of control chart, at first to control the diagram, the values of the coefficients are considered as given. The values $\beta_0=1$ and $\beta_2=0.5$ and $\gamma_1=1$ are considered for the state of control in a two-stage process as in equation 1 and 2. MATLAB software is used for simulation with 10,000 repetitions for each (ARL) output. At first we obtain control limits for the control charts with respect to the coefficients in the profiles in each proposed approach. Then with changes in each of these coefficients, it is possible that the charts detect the change and warn. From the time of change to the time that at least one of the charts identifies this change, it is called the run length. Then we repeat this activity as many as 10,000 times to obtain an average run length. Then we repeat the same operation for other profile changes for all parameters and qualitative characteristics of phase one.

In addition to the description above, in each step of the simulation, other outputs can be obtained. For example, the diagram $EWMA-R$ in figure 3 and the T^2 control chart in Figure 4 and the χ^2 control chart in Figure 5 are given for the approach $EWMA-R$, T^2, χ^2.

As you can see the $EWMA-R$ chart is drawn for the 570 samples. These two charts are used for monitoring the quality characteristics of the first stage. In figure 3 both diagrams show out-of-control state. At first figure 3a, the $EWMA$ control chart for monitoring the quality characteristics of the first stage, is focused on the first 100 samples in control mode and after simulation, as soon as the average quality characteristics of stage one changes, it immediately shows sensitivity to this shift. And by repeating the outside the control sequence, we can obtain a measure for average run length. Besides this, in figure 3b, R control chart for monitoring the distribution of the quality characteristics of stage one has the same function.

Diagram shown in figure 4 is T^2 multivariate control chart which has been applied for the monitoring profile coefficients ($\beta_0, \gamma_1, \beta_2$) for both proposed approaches in this paper. As can be seen there has been a change in one of the profile coefficients in four hundred and sixtieth sample and the chart identifies this change. With performing more simulations and repetition, we can get the number of samples between the two out of control samples. And then their average is the average run length in T^2 control chart.

Diagram shown in figure 5 is χ^2 control chart which has been applied for the monitoring residuals for both proposed approaches in this paper. As can be seen all of the residuals in 570 samples were drawn under control. It means between the actual values and the predicted values no significant difference exists. If this chart warns, at least one of the residuals shows out-of-control state, this means that it is out of range data and there is a difference between actual and predicted values.

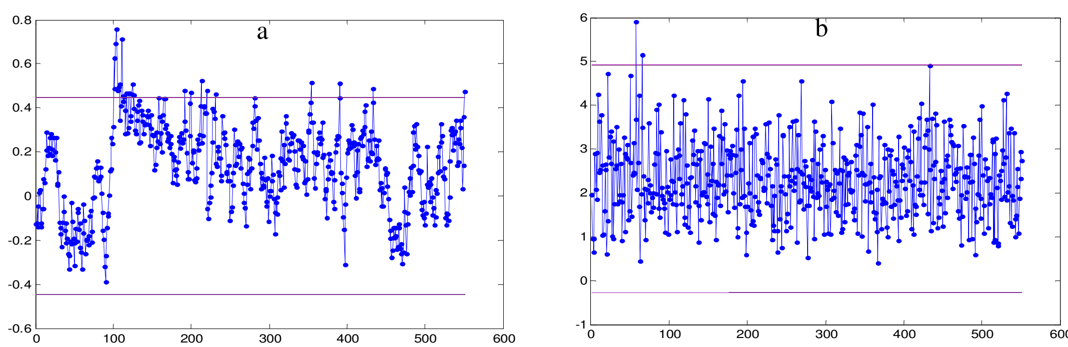

Figure-3
part of the $EWMA-R$ chart for monitoring the quality characteristics of stage one

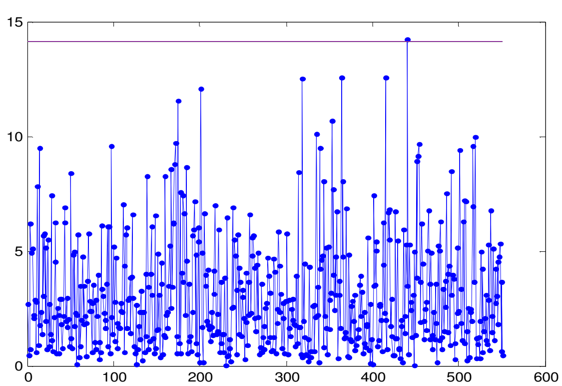

Figure-4
Part of the T^2 control chart for monitoring profiles coefficients

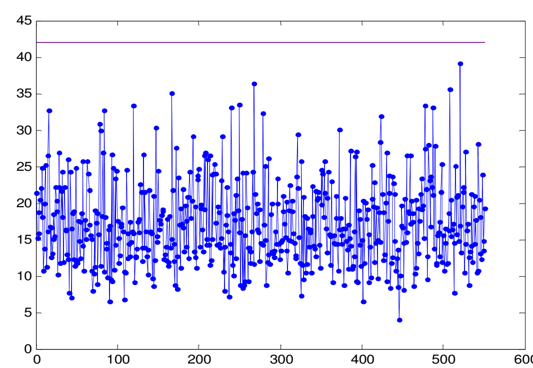

Figure-5
Part of the Graph χ^2 for monitoring residuals

Sensitivity analysis on the coefficients Profile: 1- Sensitivity Analysis on γ_1 coefficient: At this step, the changes on the coefficients $\beta_0 = 1$ and $\beta_2 = 0.5$ and γ_1 have been done from 1 to 3 to the amount of 0.05 for each parameter. Table 1 shows the simulated output for ARL calculation according to the rate of change γ_1 for both proposed approaches. As it is apparent, the best ARL is for $\gamma_1 = 1$. This is shown in figure 6. However, as stated the total ARL of both approaches has been fixed at 200.Column γ_1 shows the change in terms of γ_1. It is important to note that in this paper, the control mode has been considered for $\gamma_1 = 1$.SDRL column is the standard deviation run length.

For this reason, in figure 6 we have the highest ARL for $\gamma_1 = 1$. The ARL column states that per each γ_1, on average, how many samples have been plotted in control charts to show a warning. Calculated ARL for each of the approaches $\bar{X} - R, T^2, \chi^2$ and $EWMA - R, T^2, \chi^2$ is the total ARL. For example, in column ARL for approach $\bar{X} - R, T^2, \chi^2$ and for $\gamma_1 = 1.05$, amount 198.3621 is calculated. This number means that a positive change of 0.05 in γ_1, in the long run, shows at least one time out-of-the-control state in each 198 samples in at least one of the four graphs used in this approach.

As it is shown in figure 6, by an increase in γ_1 from value one, the ARL decreases. This is the same for both approaches. Both approaches $\bar{X} - R, T^2, \chi^2$ and $EWMA - R, T^2, \chi^2$ have same reaction to changes in γ_1, because the changes applied to γ_1 have sensitivity affects on T^2 control chart, and the graph has the same performance in both approaches. Also for positive changes in γ_1 by more than 2, which means when $\gamma_1 \geq 3$, both approaches show the change in the first sample.

2-The Sensitivity Analysis on β_0 Coefficient: In the next step we perform changes on β_0 coefficient, which means $\beta_2 = 0.5$ and $\gamma_1 = 1$ and β_0 have changed from 1 to 3 to the amount of 0.05. In table 2 as is defined the best ARL associated with the $\beta_0 = 1$ condition. This case is clear in figure 7. In this part, again, the total ARL for both approaches is the same, and it is fixed at least on 200.

Column β_0 shows the change in terms of β_0. It is important to note that $\beta_0 = 1$ is the control mode in this paper. For this reason, in figure 7 we have the highest ARL for $\beta_0 = 1$. ARL column shows each value of β_0 on average, how many samples within the control charts were plotted for each of the proposed approaches so that it is viewed as a warning. SDRL column is the standard deviation of the run length.

Table-1

Output of simulation to compute the ARL with change for γ_1 for both approaches

$\beta_0 = 1 \& \beta_2 = 0.5$										
$\bar{X} - R, T^2, \chi^2$		$EWMA - R, T^2, \chi^2$		γ_1	$\bar{X} - R, T^2, \chi^2$		$EWMA - R, T^2, \chi^2$		γ_1
SDRL	ARL	*SDRL*	ARL		*SDRL*	ARL	*SDRL*	ARL	
0.890625	1.5258	0.8831	1.5276	2.05	207.7296	206.6287	211.4463	213.0227	1
0.787216	1.4421	0.8202	1.4501	2.1	201.8411	198.3621	204.9991	205.569	1.05
0.707173	1.3675	0.72	1.3804	2.15	175.9962	177.564	182.9721	184.364	1.1
0.633212	1.3111	0.6539	1.2716	2.2	144.9438	146.896	148.4641	148.6024	1.15
0.605381	1.2774	0.5775	1.2309	2.25	108.5703	110.4098	113.0406	113.524	1.2
0.534096	1.2321	0.531	1.212	2.3	73.86522	75.4851	75.76684	76.6993	1.25
0.494816	1.1987	0.5131	1.1797	2.35	48.71745	48.945	48.25706	49.0913	1.3
0.456592	1.1804	0.4587	1.1525	2.4	29.96688	30.7556	30.83073	31.7261	1.35
0.42584	1.1529	0.4131	1.1361	2.45	19.50562	20.1034	19.11001	19.8737	1.4
0.387006	1.129	0.3982	1.118	2.5	12.84504	13.4967	13.07149	13.498	1.45
0.366824	1.1231	0.3665	1.1042	2.55	8.834107	9.3088	8.852786	9.275	1.5
0.338307	1.1003	0.3393	1.1003	2.6	6.150389	6.7621	6.364552	6.8885	1.55
0.333542	1.1008	0.333542	1.1008	2.65	4.623046	5.2269	4.628095	5.1469	1.6
0.284214	1.0757	0.284214	1.0757	2.7	3.568873	4.0743	3.602793	4.1099	1.65
0.283625	1.0746	0.283625	1.0746	2.75	2.799374	3.3324	2.777203	3.3384	1.7
0.2636	1.0665	0.2636	1.0665	2.8	2.218206	2.7868	2.283987	2.8081	1.75
0.251758	1.0602	0.251758	1.0602	2.85	1.817829	2.4029	1.837065	2.4217	1.8
0.241022	1.0549	0.241022	1.0549	2.9	1.539275	2.1295	1.566143	2.1358	1.85
0.226153	1.0496	0.226153	1.0496	2.95	1.349574	1.9217	1.357844	1.9548	1.9
0.210254	1.0424	0.210254	1.0424	3	1.156462	1.7411	1.159989	1.7645	1.95
					1.023277	1.6342	1.061161	1.657	2

As in figure 7 is determined, by increasing the value β_0 from one, the ARL is reduced. This is the same for both approaches.The two approaches $\bar{X}-R, T^2, \chi^2$ and $EWMA-R, T^2, \chi^2$ have the same functions to changes in β_0, because the changes applied to β_0 have sensitivity affects on T^2 control chart, and the graph has the same performance in both approaches. Also for positive changes in β_0 by more than 1.8, which means when $\beta_0 \geq 2.8$, both approaches show the change in the first sample.

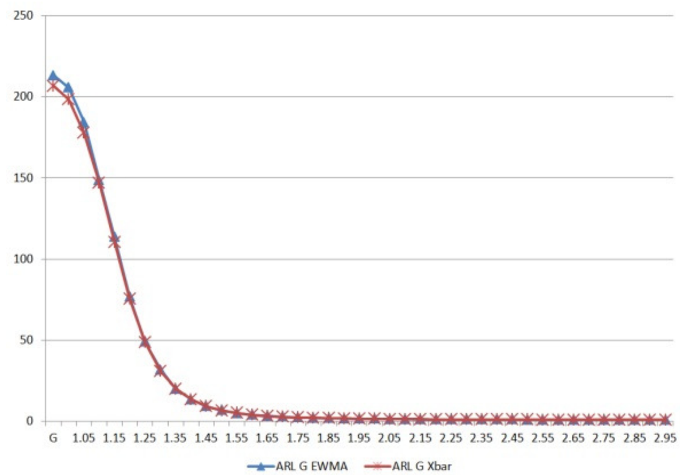

Figure-6
Average run length curve for both proposed approaches for changes in γ_1

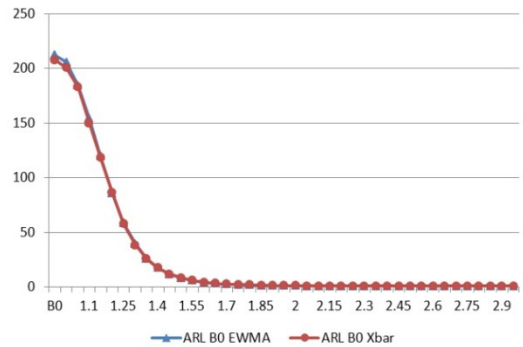

Figure-7
average run length curve for both proposed approaches for changes in β_0

3-The sensitivity analysis on β_2 coefficient: In the next step we perform changes on β_2 coefficient, which means $\beta_0 = 1$ and $\gamma_1 = 1$ and β_2 have changed from 0.5 to 2.5 to the amount of 0.05. In table 3 as is defined the best ARL associated with the $\beta_2 = 0.5$ condition. This case is clear in figure 7. In this part, again, the total ARL for both approaches is the same, and it is fixed at least on 200. Column β_2 shows the change in terms of β_2. It is important to note that $\beta_2 = 0.5$ is the control mode in this paper. For this reason, in figure 8 we have the highest ARL for $\beta_2 = 0.5$. ARL column shows each value of β_2 on average, how many samples within the control charts were plotted for each of the proposed approaches so that it is viewed as a warning. SDRL column is the standard deviation of the run length. Because of the likeliness of simulated data, part of the curve is summarized.

Table-2
Output of simulation to compute the ARL with changes of β_0 for both approaches

$\gamma_1 = 1 \& \beta_2 = 0.5$									
$\bar{X}-R, T^2, \chi^2$		$EWMA-R, T^2, \chi^2$		β_0	$\bar{X}-R, T^2, \chi^2$		$EWMA-R, T^2, \chi^2$		β_0
SDRL	ARL	*SDRL*	ARL		*SDRL*	ARL	*SDRL*	ARL	
1.264955	1.8609	1.279586	1.868	1.85	205.2427	207.9379	207.6309	212.362	1
1.000389	1.6016	1.005124	1.6069	1.9	202.1148	200.6812	207.0338	205.7173	1.05
0.789545	1.441	0.768539	1.4225	1.95	182.0202	183.0582	184.1808	184.4719	1.1
0.626239	1.3049	0.62132	1.2995	2	148.9851	150.1219	154.5815	154.3579	1.15
0.48368	1.1959	0.500394	1.2084	2.05	117.9695	118.3407	117.6674	119.6498	1.2
0.397985	1.1372	0.395124	1.1421	2.1	85.55559	86.2295	84.37637	85.6926	1.25
0.29986	1.0854	0.310605	1.0868	2.15	56.53753	57.7364	58.3152	58.5483	1.3
0.247875	1.058	0.254632	1.0589	2.2	38.24206	38.5194	39.36857	39.9413	1.35
0.185888	1.0339	0.19243	1.0357	2.25	25.8414	26.2504	25.56957	25.9534	1.4
0.147725	1.0219	0.140091	1.0194	2.3	17.22152	17.7311	17.02432	17.5283	1.45
0.110668	1.0124	0.105726	1.0111	2.35	11.3962	11.876	11.29075	11.7813	1.5
0.087977	1.0078	0.075302	1.0055	2.4	8.107003	8.5589	7.742323	8.3766	1.55
0.062343	1.0037	0.062331	1.0039	2.45	5.369095	5.9987	5.568809	6.0572	1.6
0.046855	1.0022	0.041198	1.0017	2.5	3.933316	4.4556	3.899855	4.4841	1.65
0.024489	1.0006	0.031609	1.001	2.55	2.869813	3.4117	2.880952	3.412	1.7
0.017319	1.0003	0.022356	1.0005	2.6	2.180582	2.7382	2.098724	2.6732	1.75
0	1	0	1	2.65	1.633299	2.2119	1.669979	2.2327	1.8

Table-3
Output of simulation to compute the ARL with changes for β_2 in both approaches

$\gamma_1 = 1 \& \beta_0 = 1$									
$\bar{X} - R, T^2, \chi^2$		$EWMA - R, T^2, \chi^2$		β_2	$\bar{X} - R, T^2, \chi^2$		$EWMA - R, T^2, \chi^2$		β_2
SDRL	ARL	*SDRL*	ARL		*SDRL*	ARL	*SDRL*	ARL	
0.394727	1.1375	0.407106	1.141	0.7	205.2427	207.9379	209.9821	207.2517	0.5
0.090186	1.0082	0.097028	1.0093	0.75	69.60755	71.6243	71.72321	71.9864	0.55
0.017319	1.0003	0.019997	1.0004	0.8	7.963178	8.5114	8.077916	8.5529	0.6
0	1	0	1	0.85	1.483909	2.0586	1.520199	2.0541	0.65

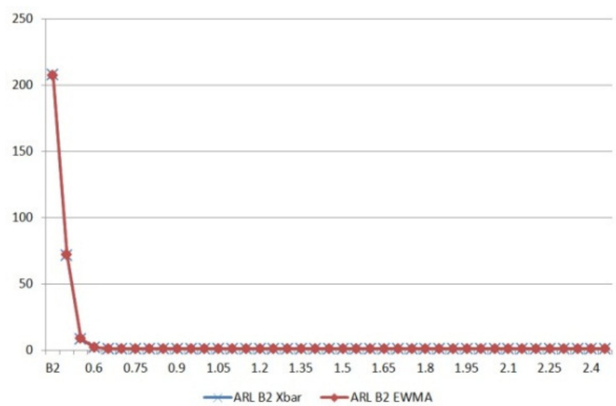

Figure-8
average run length curve for both proposed approaches for changes in β_2

As in figure 8 is determined, ARL value decreases with the increase in β_2 amount. However, comparing figure 8 with figure 7 is observed for the reduction in ARL is faster with the changes in β_2 than with the changes in β_0. Both approaches are more sensitive to such changes in β_2 than in β_0. On the other hand, both approaches $\bar{X} - R, T^2, \chi^2$ and $EWMA - R, T^2, \chi^2$ have the same reaction to changes in β_2 . The acquisition was obvious; since these changes applied to β_2 have sensitivity affects on T^2 control chart, and T^2 control chart works the same in both approaches. On the other hand, for each positive change in β_2 by more than 0.35, which means if $\beta_2 \geq 0.85$ both approaches show this change in the first sample.

The sensitivity analysis on the average quality characteristics of the stage one: In this step, the changes have applied on the average quality characteristics of the phase I, thus, the coefficients of the profile $\beta_0 = 1$ and $\gamma = 1$ and $\beta_2 = 0.5$ are in the controlled state. Table 4 shows the ARL values for changes in average qualitative characteristic of the first stage in both approaches. In this part, also, the total ARL for both approaches is the same, and is considered at least 200.Column μ_{x_1} shows changes based on the average qualitative characteristics of the first stage. It is important to note that in this article, $\mu_{x_1} = 0$ has been the control mode. For this reason, in figure 9 for $\mu_{x_1} = 0$ we have the highest ARL. ARL column states that per each value of μ_{x_1}, on average, how many samples of the control charts were plotted for each approach so that an alert has been observed.SDRL column is the standard deviation of the run length.

As in figure 9 is shown, ARL value decreases with increasing μ_{x_1}. Both approaches $\bar{X} - R, T^2, \chi^2$ and $EWMA - R, T^2, \chi^2$ adjust differently to changes in μ_{x_1}. The changes are not the same and as can be seen in the approach $EWMA - R, T^2, \chi^2$ for incremental changes in the average quality characteristics of the first stage μ_{x_1}, better adjustment is seen in compare to approach $\bar{X} - R, T^2, \chi^2$. The changes applied are on μ_{x_1} and the $EWMA$ control char is better than the $\bar{X}$ control chart for small changes in the mean.

Figure 10 is a general diagram to compare the average run length for changes in all parameters $\beta_0, \gamma_1, \beta_2$ and the average quality characteristic of stage one μ_{x_1}

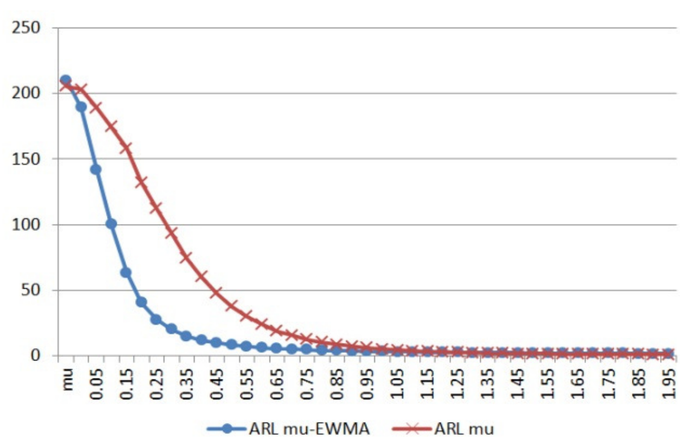

Figure-9
Average run length curve for both proposed approaches for changes in average quality characteristic of stage one μ_{x_1}

Thus, R is the rate of change in any of the cases mentioned. If R = 0, ie, the parameter is not changed, and if $R = 0.1$ for example for γ_1 it means that the parameter value is $\gamma_1 = 1.1$.

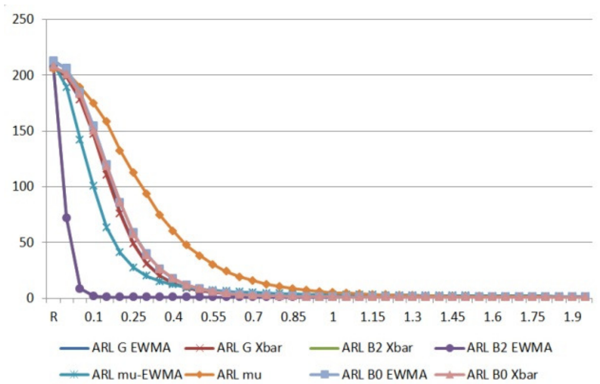

Figure-10
The curve of comparing all average run length in both approaches to the change in R

According to the simulation and the two monitoring approaches presented: $\bar{X} - R, T^2, \chi^2$ and $EWMA - R, T^2, \chi^2$ it can be stated that, in general, the most sensitivity to changes is to changes in parameter β_2. Both approaches have the same performance to the changes. However, the worst-case detection is related to changes in the average quality characteristic of the first stage by the approach $\bar{X} - R, T^2, \chi^2$. In general it can be stated that the approach $EWMA - R, T^2, \chi^2$ for monitoring a two-stage process, as in figures 1 and 2 mentioned, is better than approach $\bar{X} - R, T^2, \chi^2$.

The results of this paper can have special importance in many industries wherequality product is a function of more than one stage and profile monitoring is done in one of the stages. For example, products such as parts manufacturing, production of metals such as copper and loom, etc. thatquality product is not formed at a particular stage and pre-processing steps which have impacts on the nature of the profile, is of the utmost importance. For example of the case in the textile industry. The first phase is the spinning part and the quality characteristics of it, is the thickness of the thread, which as one of the independent variables in the second phase profile has an effective role in resistance of the fabric. On the other hand x_2 can also be a place on coils where the fabric resistance is measured. This measurement happens in certain areas of Coils. This means that x_2 gets constant values.

Table-4
The simulated output to compute ARL with changes of μ_{x_1} in qualitycharacteristicof stage one for both approaches

$\gamma_1 = 1 \& \beta_0 = 1 \& \beta_2 = 0.5$										
$\bar{X} - R, T^2, \chi^2$		$EWMA - R, T^2, \chi^2$			$\bar{X} - R, T^2, \chi^2$		$EWMA - R, T^2, \chi^2$		μ_{x_1}
SDRL	ARL	*SDRL*	ARL	μ_{x_1}	*SDRL*	ARL	*SDRL*	ARL	
4.627502	5.1033	1.213284	3.3324	1.05	207.1675	205.5633	210.0561	210.0388	0
3.972613	4.4821	1.164866	3.1659	1.1	201.04	203.0946	187.0086	189.1841	0.05
3.426977	3.9015	1.087632	3.0087	1.15	186.9102	189.003	140.4241	142.1297	0.1
2.857253	3.3639	1.004862	2.8823	1.2	170.6085	174.67	94.71455	100.3104	0.15
2.417764	3.0012	0.939428	2.742	1.25	156.5327	158.1093	58.44721	63.3811	0.2
2.094937	2.6424	0.889863	2.6415	1.3	129.4541	132.0665	35.19054	40.7941	0.25
1.797567	2.3822	0.847004	2.5256	1.35	111.1552	112.1983	22.57736	27.4731	0.3
1.603382	2.1674	0.801869	2.4503	1.4	92.97603	93.5414	15.60135	20.1508	0.35
1.388211	1.9535	0.761559	2.3716	1.45	73.44434	74.3279	10.86843	15.136	0.4
1.205851	1.8054	0.722412	2.2778	1.5	60.44918	60.2653	7.787788	12.025	0.45
1.038501	1.668	0.699564	2.2118	1.55	47.25281	47.4693	5.956902	9.9394	0.5
0.931477	1.5569	0.677978	2.1428	1.6	36.75006	37.9038	4.71898	8.3823	0.55
0.833903	1.474	0.646871	2.0755	1.65	30.02235	30.2055	3.775463	7.2234	0.6
0.726781	1.3815	0.628015	2.0117	1.7	23.61157	24.0671	3.211226	6.3337	0.65
0.645068	1.323	0.609955	1.9737	1.75	18.11843	19.0408	2.692741	5.6835	0.7
0.584423	1.2653	0.594424	1.9337	1.8	15.18259	15.6336	2.368705	5.186	0.75
0.514475	1.2237	0.5737	1.8759	1.85	12.06661	12.5541	2.049138	4.7183	0.8
0.461048	1.184	0.560719	1.8353	1.9	10.12139	10.496	1.801797	4.3278	0.85
0.412418	1.1457	0.547494	1.8008	1.95	8.241278	8.6809	1.620903	4.0289	0.9
0.363896	1.1208	0.543131	1.7684	2	6.745102	7.3052	1.499855	3.7991	0.95
					5.593926	6.0907	1.334781	3.5232	1

Conclusion

This paper introduces and compares two approaches for monitoring a two-stage process with profile quality characteristics in the second stage. And given that the quality of products are monitored by a variety of control charts on their quality characteristics. These quality characteristics that are either variable or attribute can be, a single variable, a vector of variables, or a profile equation. However, because the quality of the performance of different procedures is the quality of the product and usually these steps are not independent of each other, therefore, the assumption of independent process affecting the quality of the output with error. So far, the impact of such conditions on multi-stage monitoring processes of univariate and multivariate have been studied. However, a profile has been less studied in multi-stage monitoring process.

This paper introduces a model for a two-step profile in addition to two different approaches that have been proposed for monitoring process.

And the variation of the coefficients of the profile, as well as changes in the qualitative characteristics of the first stage, in a two-stage process, on the second phase control charts were reviewed. And observed changes in the coefficients γ_1, β_0 have almost the same effect on a two-step monitoring process. While the rate of change in β_2 influences a two-stage process in a narrower range. This paper also proposes two approaches for monitoring such processes and the final analysis was carried out related to these comparisons.

This new topic of research activities can be considered in the following cases: i. EWMA chart to monitor the residuals of the profile and compare two approaches of this article. ii. Analysis of the interaction between Independent quality characteristics in the profile equation. iii. Using other multivariate charts such as MEWMA and MCUSUM instead of T^2 control chart and compare the outputs with each other. iv. Examining the performance of the simple linear profile coefficients in monitoring processes with more than two stages. v. Evaluation of the effect of the first stage on the profile slope. vi. Examining the performance of the simple linear profile coefficients in monitoring processes with more than two steps that have a profile in each stages.

References

1. Rodrigues L.L.R., Kantharaj A.N., Kantharaj B., Freitas W.R.C. and Murthy B.R.N., Effect of Cutting Parameters on Surface Roughness and Cutting Force in Turning Mild Steel, *Res. J. Recent Sci.,*1(**10**), 19-26 **(2012)**
2. Sharma Kalpa, Health IT in Indian Healthcare System: A New Initiative, *Res.J.Recent Sci.,* **1**(**6**), 83-86(**2012**)
3. Agbo G.A., Ibeh G.F. and Ekpe J.E., Estimation of Global Solar Radiation at Onitsha with Regression Analysis and Artificial Neural Network Models, *Res.J.Recent Sci.,***1**(**6**), 27-31(**2012**)
4. Behmaneshfar Ali, Shahbazi S. and Vaezi S., Analysis of the Sampling in Quality Control Charts in non uniform Process by using a New Statistical Algorithm, *Res.J.Recent Sci.,***1**(**8**), 36-41(**2012**)
5. Murthy B.R.N., Lewlyn L.R. Rodrigues and AnjaiahDevineni, Process Parameters Optimization in GFRP Drilling through Integration of Taguchi and Response Surface Methodology, *Res.J.Recent Sci.,***1**(**6**), 7-15 (**2012**)
6. Zhang G.X., A New Type of Quality Control Chart Allowing the Presence of Assignable Causes the Cause-Selecting Control Chart, *Acta ElectronicaSanica*, **2**, 1-10 **(1980)**
7. Mahmoud M.A. and Woodall W.H., Phase I Analysis of Linear Profiles with Calibration Applications, *Technometrics*, **46**, 377-391 **(2004)**
8. Zhang G.X., Multiple Cause-Selecting Control Charts, *Acta Electronic Sin*, **3**, 31-36 **(1982)**
9. Hawkins D.M., Multivariate Quality Control Based on Regression Adjusted Variables, *Technometrics,* **33**(**1**), 61-75 **(1991)**
10. Hawkins D.M., Regression Adjustment for Variables in Multivariate Quality Control, *Journal of Quality Technology,* **25**(**3**), 170-182 **(1993)**
11. Wade M.R. and Woodall W.H., A Review and Analysis of Cause-Selecting Control Charts, *Journal of Quality Technology*, **25**(**3**), 161-169 **(1993)**
12. Yang S.F. and Yang C.M., An Approach to Controlling Two Dependent Process Steps with Auto correlated Observations, *International Journal of Advanced Manufacturing Technology,* **29**(**1-2**), 170-177 **(2006)**
13. Skinner K.R., Montgomery D.C. and Runger G.C., Generalized Linear Model-Based Control Charts for Discrete Semiconductor Process Data. *Quality and Reliability Engineering International*, **20**(**8**), pp. 777 786 (**2004**)
14. Jearkpaporn, D., Montgomery, D.C., Runger, G.C. and Borror, C.M. Process Monitoring for Correlated Gamma-Distributed Data Using Generalized-Linear-Model-Based Control Charts, *Quality and Reliability Engineering International,* **19**(**6**), 477-491 **(2003)**
15. Jearkpaporn D., Montgomery D.C., Runger G.C. and Borror C.M., Model-based process monitoring using robust generalized linear models, *International Journal of Production Research*, **43**, 1337-1354 **(2005)**

16. Jearkpaporn D., Borror C.M., Runger G.C. and Montgomery D.C., Process Monitoring for Mean Shifts for Multiple Stage Processes, *International Journal of Production Research*, **45**(**23**), 5547-5570 **(2007)**

17. Loredo E.N, Jearkpaporn D. and Borror C.M., Model-based control chart for autoregressive and correlated data, *Quality and Reliability Engineering International*, **18**, 489-496 **(2002)**

18. Shu L. and Tsung F., On multistage statistical process control, *Journal of the Chinese Institute of Industrial Engineers,* **20**, 1-8 **(2003)**

19. Yang S.F. and Yang C.M., Effects of Imprecise Measurement on the Two Dependent Processes Control for the Autocorrelated Observations, *International Journal of Advanced Manufacturing Technology,* **26**, 623-630 **(2005)**

20. Niaki S.T.A. and Davoodi M., Designing a multivariate-multistage quality control system using artificial neural networks, *International Journal of Production Research*, **47(1),** 251-271 **(2009)**

21. Kang L. and Albin S.L., On-Line Monitoring When the Process Yields a Linear Profile, *Journal of Quality Technology,* **32**, 418-426 **(2000)**

22. Kim K., Mahmoud M.A. and Woodall W.H., On the Monitoring of Linear Profiles, *Journal of Quality Technology,* **35**, 317-328 **(2003)**

23. Gupta, S., Montgomery D.C. and Woodall W.H., Performance Evaluation of Two Methods for Online Monitoring of Linear Calibration Profiles, *International journal of Production Research*, **44**, 1927-1942 (**2006**)

24. Zou C., Zhang Y. and Wang Z., Control Chart Based on Change-Point Model for Monitoring Linear Profiles, *IIE Transactions,* **38**, 1093-1103 **(2006)**

25. Noorossana R., Vaghefi A. and Dorri M., Effect of Non-Normality on the Monitoring of Simple Linear Profiles, *Quality and Reliability Engineering International,* Wiley Inter science, **27**, 1015-1021 (**2010**)

26. Jensen W.A., Birch J.B. and Woodall W.H., Monitoring correlation within linear profiles using mixed models, *Journal of Quality Technology*, **40**, 167–183 **(2008)**

27. Noorossana R., Saghaei A., Amiri A., *Statistical Analysis of Profile Monitoring,* John Wiley & Sons, **(2011)**

28. Niaki S.T.A., Soleimani P. and Eghbali M., Performance Evaluation of methods for monitoring simple linear profiles in Multi stage Process, *Journal of Engineering and Quality Management,* **1(1),** 1-14 **(2012)**