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Nonlinear Analysis and Mechanical Characterization of a Micro-Switch under Piezoelectric Actuation

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Abstract

In this article, a comprehensive model of a micro-switch subjected to piezoelectric excitation, which accounts for the nonlinearities due to inertia and curvature, is presented. Dynamic equations of this model is derived by the Lagrange method and solved by the Galerkin method using five modes. Micro-switch movable part is assumed as an elastic Euler-Bernoulli beam with clamped-free end conditions. Whereas the major drawback of electrostatically actuated micro-switches is the high driving voltage, using the piezoelectric actuator in these systems can provide less driving voltage. The effect of variation in geometry of piezoelectric layer such as length and thickness as well as piezoelectric voltage on mechanical characterizations is discussed. The aim of this work is to design and control of a micro-switch with the change in length and thickness of the piezoelectric actuator.

Keywords: MEMS, Micro-Switch, Piezoelectric actuator, Curvature and inertia nonlinearities.

Introduction

Micro and Nano electro-mechanical systems (MEMS and NEMS) have attracted a lot of attention in many applications such as sensors and actuators, and also in energy conversion mechanisms in different engineering applications such as micro-switches[1-6]. A typical electrostatic micro-switch includes of a conductive movable micro-beam and a fixed electrod placed on a substrate front the micro-beam. This micro beam can deflected through electrostatic or piezoelectric actuation. When the micro beam is deflected and pulled into the front fixed plate; this puts the micro-switch in the ON position[7] (figure 1(a)). The major drawbacks of electrostatically actuated micro-switches are high driving voltage and low reliability[8].

Due to their low weight, fast response, low hysteresis, low energy consumption and high bandwidth performance, piezoelectric materials have been extensively used in Microsystems as actuators and sensors[9]. Zhou et al. suggested piezoelectric excitation for increasing the sensitivity and selectivity of a gas sensor micro cantilever[10]. Cattan et al. experimentally investigated the piezoelectric attributes of PZT film for use in micro system[11]. In 2008 Li et al showed that a piezoelectric micro-resonator with clamped-clamped condition demonstrated a hardening behavior[12]. In 2007, Mahmoodi and Jalili experimentally and analytically evaluated the nonlinear analysis of a micro system under piezoelectric actuation experimentally and analytically. They assumed that no extension of the beam mid-plane when the beam deflection occurred. Their results indicated that the nonlinear models are much more accurate than their linear counterparts. Also they concluded that due to nature of piezoelectric excitation the vibration response demonstrated a hardening behavior[13].

In this article, a micro-switch model subjected to the piezoelectric actuation was developed. Nonlinear terms due to inertia and curvature were considered since they play important role in the micro-scale response. The movable part of micro-switch is assumed as an elastic Euler-Bernoulli beam with clamped-free end conditions. The piezoelectric actuator is bonded onto a portion of the micro-beam's surface. Since there is no external axial force acting on the beam, we were able to use the shortening effect assumption (no extension of the beam mid-plane after deformation) in deriving the dynamic equations. The objectives of this model are reducing the driving voltage and tune deflection and natural frequency in micro-switches.

Methodology

A flexible slender micro-beam with uniform rectangular cross section and Euler-Bernoulli beam theory assumption is considered as a movable micro-switch. A clamped-free boundary condition exists and therefore, the micro-beam is called micro-cantilever beam. As shown in figure 1 we cover the upper surface of this micro-switch with a piezoelectric actuator with the same width as that of the micro-switch and length of $l_2 - l_1$. Now we applied the DC voltage v_e to piezoelectric actuator.

The governing equations of motion: Since in the present model there is no external axial load along the beam and the boundary condition are clamped-free the assumption of no extension in the beam mid-plane after deformation (or in other words, the 'shortening effect') is corrected and the following relationship is established between longitudinal displacement 'u' and derivation of transverse displacement 'v'':

$$\left(1+u'\right)^2 + v'^2 = 1 \Rightarrow u = -\frac{1}{2}\int_0^l v'^2 ds \qquad (1)$$

Nonlinear equation of motion in transverse direction for a micro-cantilever with piezoelectric actuator deposited on a section of it is as below:

$$m(s)\ddot{v} + \left(C_\eta(s)v''\right)'' + \left(v'\left(C_\eta(s)v'v''\right)'\right)' + \left(v'\int_l^s m(s)\left(\int_0^s \frac{\partial^2}{\partial t^2}\left(\frac{1}{2}v'^2\right)ds\right)ds\right)' = -V_e\left(C_\gamma(s)\right)'' \qquad (2a)$$

where:

$$m(s) = w_b\left(\rho_b t_b + \left(H_{l_1} - H_{l_2}\right)\rho_p t_p\right)$$

$$C_\eta(s) = \left(1-H_{l_1}\right)E_b I_b + \left(H_{l_1}-H_{l_2}\right)E_b \bar{I}_b + H_{l_2}E_b I_b + \left(H_{l_1}-H_{l_2}\right)E_p I_P$$

$$C_\alpha(s) = \left(H_{l_1}-H_{l_2}\right)\frac{E_p d_{31} A_p}{t_p}, \; C_\gamma(s) = \left(H_{l_1}-H_{l_2}\right)\frac{E_p d_{31}\bar{A}_p}{t_p} \qquad (2b)$$

$$I_b = \frac{w_b t_b^3}{12}, \; \bar{I}_b = \frac{w_b t_b^3}{12} + w_b t_b \bar{z}_n^2, \; I_p = w_p\left(t_p \bar{z}_n^2 - \left(t_p^2 + t_b t_p\right)\bar{z}_n + \frac{1}{3}\left(t_p^3 + \frac{3}{2}t_b t_p + \frac{3}{4}t_b^2 t_p\right)\right)$$

$$A_p = w_p t_p, \; \bar{A}_p = \frac{w_p}{2}\left(t_b t_p + t_p^2 - 2t_p \bar{z}_n\right), H_{l_i} = Heaviside\ function\ (s-l_i) = \begin{cases} 1 & s \ge l_i \\ 0 & s < l_i \end{cases} \quad i = 1,2$$

The boundary conditions for above equation are: $v\big|_{s=0} = 0, \;\; v'\big|_{s=0} = 0, \;\; v''\big|_{s=l} = 0, \;\; v'''\big|_{s=l} = 0 \qquad (2c)$

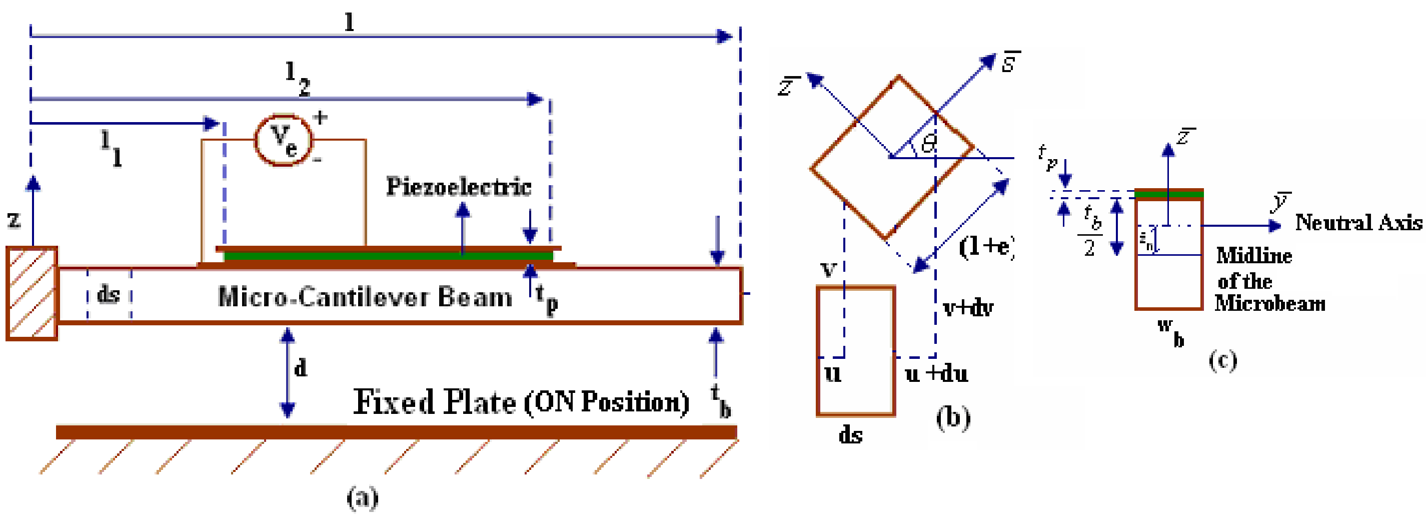

Figure-1
(a) The micro-switch model, (b) Deflected element of micro-switch beam, and (c) The cross-section of the micro-switch with piezoelectric layer

In equation (2a), the third and fourth terms are considered due to the bending-bending curvature (geometry) and inertia nonlinearities. These nonlinear terms are cubic. The impact of these terms in the frequency response curve is softening and hardening respectively[14]. The right hand side term in equation (2a) denotes bending effect associated with the piezoelectric actuator. This term produces a hardening effect on the presented micro system.

Now, by using the dimensionless quantities of equation (3), equations (2a) and (2c) are rewritten as dimensionless equations (4a) and (4b):

$$v^{*} = -\frac{v}{l},\ x = \frac{s}{l},\ \tau = \frac{t}{T}, T = \sqrt{\frac{\rho_b w_b t_b l^4}{E_b I_b}} \quad (3)$$

$$m(x)\ddot{v} + \left(H(x)v''\right)'' + \left(v'\left(H(x)v'v''\right)'\right)' + \left(v'\int_l^x m(s)\left(\int_0^x \frac{\partial^2}{\partial t^2}\left(\frac{1}{2}v'^2\right)dx\right)dx\right)' =$$
$$-\alpha_1 V_e \left(H_{l_1/l} - H_{l_2/l}\right)''$$

$$v\big|_{x=0} = 0,\quad v'\big|_{x=0} = 0,\quad v''\big|_{x=1} = 0,\quad v'''\big|_{x=1} = 0 \quad (4b)$$

In equations (4a) and (4b), the (*) symbol is omitted for simplification, and the parameters of equation (4a) are defined as follows:

$H_{l_i/l}$ = Heaviside function

$$\left(x - \frac{l_i}{l}\right) = \begin{cases} 1 & x \geq \frac{l_i}{l} \\ 0 & x < \frac{l_i}{l} \end{cases},\qquad m(x) = \left(1 + \left(H_{l_1/l} - H_{l_2/l}\right)\frac{\rho_p t_p}{\rho_b t_b}\right)$$

$$H(x) = \left(1 - H_{l_1/l}\right) + \frac{\bar{I}_b}{I_b}\left(H_{l_1/l} - H_{l_2/l}\right) + H_{l_2/l} + \frac{E_p I_p}{E_b I_b}\left(H_{l_1/l} - H_{l_2/l}\right) \quad (4c)$$

$$\alpha_1 = \frac{E_p d_{31}\bar{A}_p l^2}{E_b I_b d t_p}$$

Parameter $\alpha_1 V_e$ is dimensionless parameters for designing the micro-system. $\alpha_1 V_e$ Express the impacts of piezoelectric excitations on the present micro-system.

Static deflection of micro-switch: To obtain the static deflection of the micro-switch, we set time derivatives in equation (6a) equal to zero:

$$\left(H(x)v_s''\right)'' + \left(v_s'\left(H(x)v_s'v_s''\right)'\right)' = \alpha_1 V_e\left(H_{l_1/l} - H_{l_2/l}\right)'' \quad (5)$$

And the boundary conditions are according to equation (4c).

Now we solve equation (5) by using the Galerkin method to find the static deflection of the micro-switch. It can be showed that the solution of this equation is in the form of following equation:

$$v_s = \sum_{i=1}^{M} a[i] v_s[i] \quad (6)$$

Where M indicates number of modes and $a[i]$ are constant coefficients obtained by using the Galerkin method. Also, $v_s[i]$ is the ith mode shape, which should only satisfy the boundary conditions. For compression functions, the following linear un-damped mode shapes of a straight cantilever can be considered[13]:

$$v_s[i] = \sinh(\gamma_i x) - \sin(\gamma_i x) + \left(\frac{\sinh(\gamma_i l) + \sin(\gamma_i l)}{\cosh(\gamma_i l) + \cos(\gamma_i l)}\right)\left(\cos(\gamma_i x) - \cosh(\gamma_i x)\right)\ \ i = 1..M \quad (7)$$

where γ_i are associated with natural frequencies and can be get in the form of following equation:

$$1 + \cos(\gamma_i)\cosh(\gamma_i) = 0 \quad (8)$$

By inserting equation (6) into equation (5), multiplying by mode shapes, $v_s[n]\ \ n = 1..M$, and integrating the result from $x = 0$ to 1, we get M algebraic equations of the micro-system The constant coefficients $a[i]$ will be obtained by solving this equations, and then by substituting the obtained $a[i]$ coefficients into equation (6), we will get the static deflection of micro-system.

Corresponding natural frequency: The solution of equation (4a) can be written in the form of a summation of dynamic deflections $v_d(x,\tau)$ and static deflection v_s [14]:

$$v(x,\tau) = v_d(x,\tau) + v_s(x) \quad (9)$$

By inserting equation (9) into equation (4a) and by considering equation (5), we get:

$$m(x)\ddot{v}_d + \left(H(x)v_d''\right)'' + +\alpha_1\left(v_d'\begin{pmatrix} H(x)v_d'v_d'' + H(x) \\ v_s'v_d'' + H(x)v_d'v_s'' + H(x)v_s'v_s'' \end{pmatrix}'\right)' +$$

$$+\alpha_1\left(v_s'\left(H(x)v_d'v_d'' + H(x)v_s'v_d'' + H(x)v_s''v_d'\right)'\right)'$$

$$+\alpha_1\left(v_d'\int_l^x m(s)\left(\int_0^x \frac{\partial^2}{\partial t^2}\left(\frac{1}{2}v_d'^2\right)dx\right)dx\right)'$$

$$+\alpha_1\left(v_s'\int_l^x m(s)\left(\int_0^x \frac{\partial^2}{\partial t^2}(v_s'v_d')\,dx\right)dx\right)' = 0 \qquad (10a)$$

$$v_d\big|_{x=0} = 0,\quad v_d'\big|_{x=0} = 0,\quad v_d''\big|_{x=1} = 0,\quad v_d'''\big|_{x=1} = 0 \qquad (10b)$$

Static deflection $v_s(x)$ can be calculated from last section. By considering only the linear terms in equation (10a), we can assume a harmonic response in the form:

$$v_d(x,\tau) = \varphi(x)e^{i\omega\tau} \qquad (11)$$

Where $\varphi(x)$ are the linear mode shapes and ω are dimensionless natural frequencies. Now by inserting equation (11) into (10a), the differential equation for linear mode shapes and the boundary conditions will be obtained as below:

$$\left(H(x)\varphi''\right)'' - m(x)\omega^2\varphi + \alpha_1\left(\varphi'\left(H(x)v_s'v_s''\right)'\right)' + \alpha_1\left(v_s'\left(H(x)v_s'\varphi'' + H(x)v_s''\varphi'\right)'\right)''$$

$$-\alpha_1\omega^2\left(v_s'\int_1^x m(x)\left(\int_0^x (v_s'\varphi' dx)\right)dx\right)' dx = 0 \qquad (12a)$$

$$\varphi\big|_{x=0} = 0,\quad \varphi'\big|_{x=0} = 0,\quad \varphi''\big|_{x=1} = 0,\quad \varphi'''\big|_{x=1} = 0. \quad (12b)$$

To solve equation (12a), we can write $\varphi(x)$ in the form of:

$$\varphi = \sum_{j=1}^{M} b[j]\varphi_a[j] \qquad (13)$$

Where $\varphi_a[i]$ is the ith linear mode shape, defined similar to $v_s[i]$ in equation (7), of a micro-cantilever and $b[i]$ are the constant coefficients and can be found using the Galerkin method. Now, we insert the above equation into (12a), multiply by $\varphi_a[n], n = 1..M$ and integrate resulting expression from $x = 0$ to 1:

$$\int_0^1 \sum_{i=1}^{M} b[i]\frac{d^2}{dx^2}\left(H(x)\frac{d^2\varphi_a[i]}{dx^2}\right)\varphi_a[n]dx + \alpha_1\int_0^1\sum_{i=1}^{M} b[i]$$

$$\left(\frac{d\varphi_a[i]}{dx}\left(H(x)v_s'v_s''\right)'\right)'\varphi_a[n]dx$$

$$+\alpha_1\int_0^1\sum_{i=1}^{M} b[i]\left(v_s'\left(H(x)v_s'\frac{d^2\varphi_a[i]}{dx^2} + H(x)v_s''\frac{d\varphi_a[i]}{dx}\right)'\right)''$$

$$\varphi_a[n]dx - \omega^2\int_0^1\sum_{i=1}^{M} b[i]m(x)\varphi_a[i]\varphi_a[n]dx$$

$$-\alpha_1\omega^2\int_0^l\sum_{i=1}^{M} b[i]\varphi_a[i]\varphi_a[n]\left(v_s'\int_1^x m(x)\left(\int_0^x (v_s'\varphi' dx)\right)dx\right)' dx = 0$$

Equation (14) has a nonzero solution when we set determinant of coefficient $b[i]$ to zero. So by solving this equation, we can find the dimensionless natural frequency ω.

Results and Discussion

In this section, in order to investigate the proposed model, the solutions of equations (5) and 12a will be explained. Properties and dimensions of the micro-switch and piezoelectric actuator are listed in table 1. We assume that the length of piezoelectric actuator is 0.8 of the beam length, and it is bonded to the fixed side of the micro-switch beam. Now we solve equation 5 by using 1 to 5 shape modes. Figure 2 shows the solution of this equation. In this graph, x axis indicates length of micro-switch and y direction indicates static deflection of the beam. From this graph we can realize that by using 5 mode shapes, the solution converges in the Galerkin method; so, 5 mode shapes were considered for all the investigations.

Table-1
Material and geometrical properties of the micro-switch beam and piezoelectric actuator

	Micro-Switch Beam	**Piezoelectric Actuator (PZT-4)**
Young's Modulus	$155.8\times10^9\ Pa$	$78.6\times10^9\ Pa$
Possion's Ratio	0.06	0.3
Mass Density	$2330\ \frac{kg}{m^3}$	$7600\ \frac{kg}{m^3}$
Length	$20\times10^3\ \mu m$	Variable
Width	$5\times10^3\ \mu m$	$5\times10^3\ \mu m$
Height	$57\,\mu m$	$0.57\,\mu m$
d_{31}	-	$-123\times10^{-12}\ \frac{m}{V}$

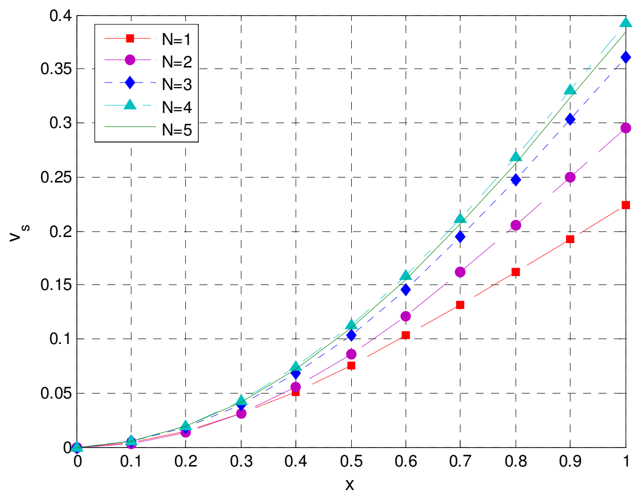

Figure-2

Dimensionless static deflection of micro-switch (v_s) versus micro-switch length for, $l_1 = 0, l_2 = 0.8l$

Effect of Piezoelectric actuator length on the mechanical behaviors*:* We show the dimensionless maximum static deflection of micro-switch at different piezoelectric voltages in figure 3. It is found from figure 3 that, by putting a piezoelectric actuator on the micro-switch beam, the maximum static deflection of the micro-switch increases under a constant piezoelectric voltage $V_e(v)$, as compared to no piezoelectric actuator ($l_1 = 0, l_2 = 0$). So, although adding a piezoelectric actuator increases the production cost, but in that way, we can have a micro-switch that operates with less driving voltage. This figure also indicates that by increasing length of piezoelectric actuator, the maximum deflection of beam under a constant piezoelectric voltage $V_e(v)$ increases. So, by increasing the piezoelectric length, the micro-switch goes on the ON state with less driving voltage (about 55 (v)). To explain this fact, we should mention that the impact of piezoelectric voltage is expressed in equation (5) by linear term ($\alpha_1 V_e (H_{l_1/l} - H_{l_2/l})''$). This term determine the bending effect. With solving equation (5) by Galerkin method, the linear bending term appears as the product of $\alpha_1 V_e$ multiplied by the first derivative of the comparison function at the beginning and end points of the piezoelectric actuator where it is deposited. So when the piezoelectric length increased, the bending effect increases, and more deflection occurs. Figure 4 illustrates the dimensionless first linear natural frequency of the micro-switch for different piezoelectric lengths. This figure shows that by increasing the piezoelectric length, the first linear natural frequency under constant $V_e(v)$ decreases. We should note that although by increasing the piezoelectric length, bending stiffness of the system increases, but the other terms in equation 12a, i.e., mass per unit length, are more effective on the variation of natural frequency.

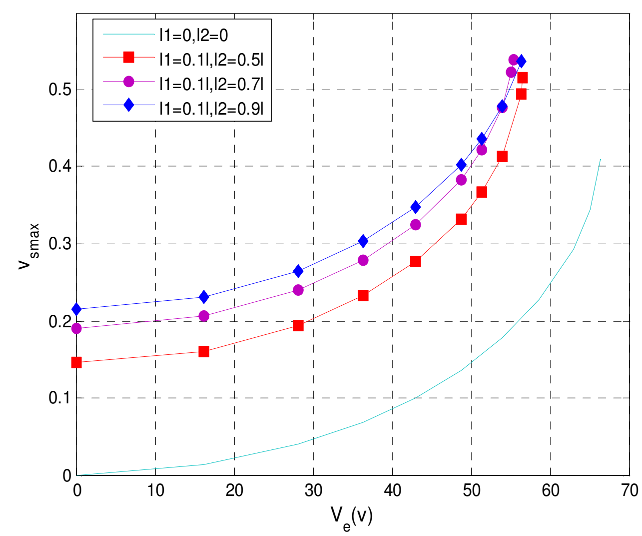

Figure-3
Dimensionless maximum static deflection of the micro-switch $v_{s\max}$ versus applied piezoelectric voltage $V_e(v)$ at different piezoelectric lengths

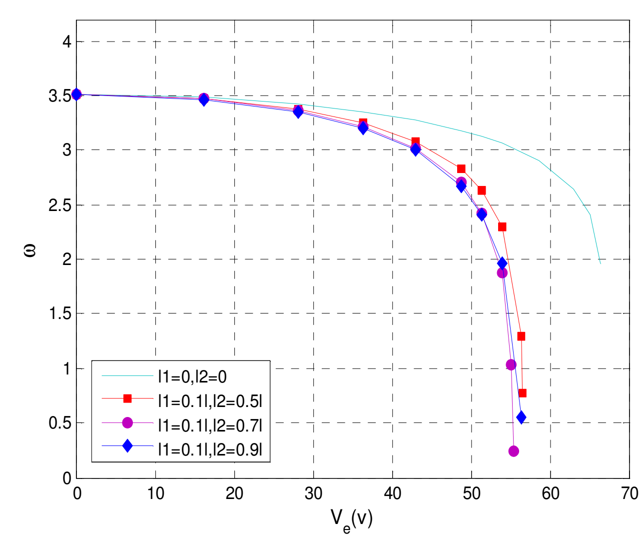

Figure-4
Dimensionless first linear natural frequency of the micro-switch ω versus applied piezoelectric voltage $V_e(v)$ at different piezoelectric lengths

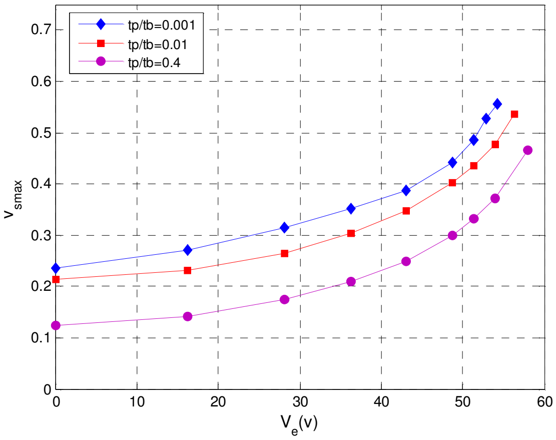

Figure-5
Dimensionless maximum static deflection of the micro-switch $v_{s\,\max}$ versus applied piezoelectric voltage $V_e\,(v)$ at different piezoelectric thicknesses for , $l_2 - l_1 = 0.8l$

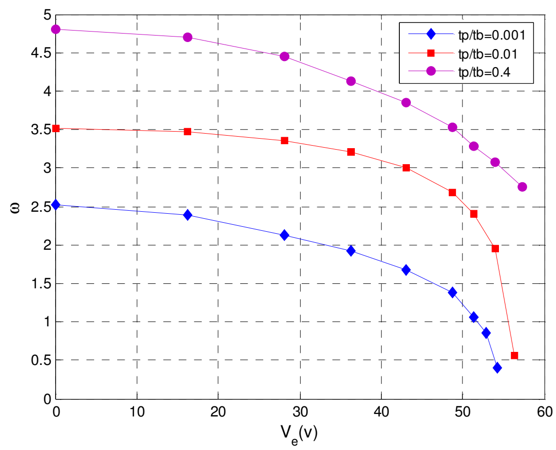

Figure-6
Dimensionless first natural frequency of the micro-switch ω versus applied piezoelectric voltage $V_e\,(v)$ at different piezoelectric thicknesses for, $l_2 - l_1 = 0.8l$

Conclusion

In this paper, a comprehensive model of a micro-switch with piezoelectric actuator is developed to determine the mechanical characterization of this micro-system. Due to clamped-free boundary conditions and no external axial force acting on the beam, the shortening effect has been used in the governing equations, which means that the neutral bending axis does not elongate when the micro-switch beam is deflected. Nonlinear terms due to inertia and curvature were considered in the dynamic equations. Nonlinearities due to inertia and curvature appear as cubic nonlinear terms. The Galerkin method was applied to solve static equation with five modes. Inertia term has no effect on static solution because it has a derivative with respect to time. This study indicates that piezoelectric actuation term lead to the bending and stiffening effects in the micro-system. It was demonstrated that static deflection and natural frequency are associated with piezoelectric layer geometry and piezoelectric voltage. Results showed that although adding the piezoelectric actuator increases the production cost of micro-switches, it reduces the driving voltage needed for the micro-switches. The results presented in this article can be effectively used to increase and adjust the accuracy and sensitivity of MEMS devices and to design a micro-switch with two goals: reducing the driving voltage, tuning deflection and natural frequencies

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