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A solution to determining the reliability of products "Using Generalized Lambda Distribution"

M.M. Movahedi[1], M.R. Lotfi[2], M. Nayyeri[1]
[1]Management Department, Firoozkooh Branch, Islamic Azad University, Firoozkooh, IRAN
[2]Industrial Engineering Dep., Firoozkooh Branch, Islamic Azad University, Firoozkooh, IRAN

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Abstract

At the end of the manufacturing cycle, performance tests are often carried out to ensure that the product meets or exceeds all specified performance parameters. In addition to initial performance, customers are interested in knowing how long the product will last, how many products will fail per year, and how many will last more than some number of years. One method for determining the reliability is the application of statistical distributions. Of the most significant and common distributions currently utilized are normal, weibull, exponential, and lognormal distributions, which are used to study most of the products' and systems' reliability. However, there are products that do not follow a specified lifetime distribution and cannot be investigated by these distributions. Instead, Generalized Lambda Distribution (GLD) can be deployed to investigate the identified and unidentified distributions, so it can resolve the problem. In this research, we introduce a method for determining the reliability, using GLD in a practical and operational way.

Keywords: Reliability, generalized lambda distribution (GLD), life cycle.

Introduction

Simply stated, reliability is quality over the long run. In other words, it is the ability of the product to perform its intended function over a period of time. A product that "works" for a long period of time is a reliable one. Since all units of a product will fail at different times, reliability is a probability. So, a more precise definition is: *Reliability is the probability that a product will perform its intended function satisfactorily for a prescribed life under certain stated environmental conditions*[1].

Reliability of a product is considered as one of its most significant quality characteristics that both the producer and the customer tend to recognize and determine. To measure reliability, it is initially necessary to figure out the life distribution of the product and then using the reliability definition, its reliability can be determined. However, there are cases in which the life distribution of a product is not specified or cannot be figured out. What can be done in such circumstances?

A key part of any quality program is to ensure that the products are produced according to performance objectives. It does not suffice just to predict the reliability; solid evidence is required of what the reliability really is. Many different methods are used to estimate product reliability, including formal life tests, field tracking studies, analyses of repair data, and accelerated life testing.

As for reliability prediction, in reliability estimation the exponential distribution is the most widely used model. This distribution assumes a constant failure rate and may not be an appropriate model for more products. The distribution that engineers use most frequently, other than exponential, is the two-parameter weibull. Weibull distribution has two parameters β and λ. Exponential and normal distributions are special cases of weibull distribution[2]. The normal distribution is often utilized for devices or systems in the wear out phase, when the failure rate of their life rises. Another distribution that is sometimes used in reliability is the lognormal although it is a subject of some controversy. Many engineers think that there are few realistic models that lead naturally to the characteristic failure rate exhibited by the lognormal distribution[3].

The reliability of a system or product may be determined by operating a large number of systems or products for the necessary length of time and observing the failures. These procedures are called life tests. Life tests are effective but expensive. They are often used for components and small parts when the cost of the items and the test facilities is not prohibitively expensive and it is practical to simulate the actual operating environment. For complex products or systems, it is usually more practical to develop procedures for mathematically determining the reliability by studying the system architecture and component reliability[4].

In this article, it is attempted to employ the generalized lambda distribution to determine the life distribution parameters and compute reliability. The article begins with the statement of the problem and purpose of the study and also refers to the significance and necessity of the research. Then it goes on to describe how this research was conducted and what theoretical framework informs this study. There follows a consideration of the statistical population and methods of sampling. The article concludes with a brief presentation of the theoretical framework which serves as the basis for this research and introducing and defining the terminology used in the article. The next section addresses reliability, its measurement in varied distributions, and the review of literature on the topic.

In the remainder of the article, the investigation procedure is mentioned. First off, it examines how the data was obtained and the table of frequency of the life distribution of the product is drawn. Then, there is a description of how experience moments were calculated, followed by how equations were attained. It goes to elaborate on the procedures for solving the equations through MATLAB software and the parameters gained from equipments were analyzed and the concerning distribution was extracted. Finally, using the obtained distribution, reliability of the product was computed and manifested in the table.

Reliability: Given the fact that the rapid growth of industry at present illustrates the importance of the efficiency of production systems more than it did in the past; therefore, it is imperative to predict the reliability of production systems so as to impede the probable failures. The reliability of a system refers to the probability of its properly working for a certain time and under specified conditions planned in advance. In other words, *reliability* is theoretically defined as the *probability* that a product might work at least for a given period of *time specified by the designer. That is:*

$$R(t) \equiv P(x>t) \equiv 1-F(t) \equiv \overline{F}(t) \tag{1}$$

The random variable *X* represents the life cycle of the product and *t* refers to the time in question, which is measured using an appropriate unit. *F(t)* is the cumulative distribution function and $R(t) \equiv 1-F(t) \equiv \overline{F}(t)$ is the function of the reliability or the survival or the cumulative distribution function of risk.

Table 1 depicts the specifications of the distributions and the computations of the reliability[4].

Table-1
The reliability of various statistical distributions

Distribution	Distribution Function	Cumulative Distribution Function	Reliability
Exponential	$f(t)=\lambda e^{-\lambda t}$	$F(t)=1-e^{-\lambda t}$	$\overline{F}(t)=e^{-\lambda t}$
Weibull	$f(t)=\lambda\beta(\lambda t)^{\beta-1}e^{-(\lambda t)^{\beta}}$	$F(t)=1-e^{-(\lambda t)^{\beta}}$	$\overline{F}=e^{-(\lambda t)^{\beta}}$
Lognormal	$f(t)=\frac{0.4343}{\sqrt{2\pi}t\sigma}\exp\left[-\frac{1}{2\sigma^2}(Log\,t-\mu)^2\right]$	$F(t)=\phi(\frac{Log(t)-\mu}{\sigma}$	$\overline{F}(t)=1-F(t)$

The failure or conditional risk rate constitutes an appropriate value of reliability:

$$\lambda(t)=\lim_{t\to 0}\frac{p(t\le x\le t+\Delta t/x>t)}{\Delta t}=\frac{f(t)}{\overline{F}(t)} \tag{2}$$

Where $f(t)$ is the probability distribution function. The failure rate takes different forms in varied distributions. Figure 1 illustrates the failure rate for human beings.

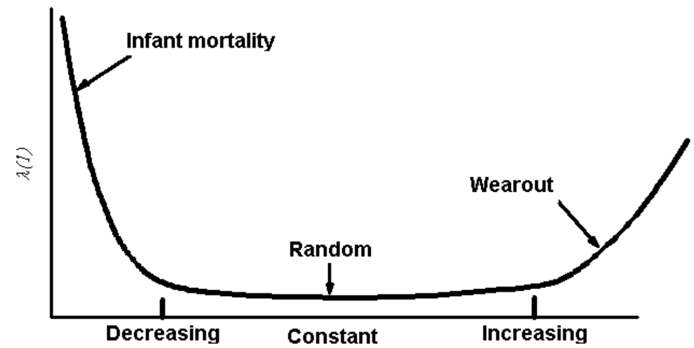

Figure-1
The failure rate for human beings

Function of life distribution differs in industrial products (such as normal, weibull, etc.). Moreover, identifying the function of life distribution needs test and hypothesis; it is not feasible to recognize all distributions. Even when a test is done for the normality of the distribution in question, the following problems might occur:

In the test above, when H_0 (the hypothesis of the normality of the distribution) is rejected, it reflects the fact that the concerned distribution is not normal. But when H_0 is not rejected, it does not necessarily mean that H_0 is accurate. Besides, according to goodness of fit test, numerous data is required to make a reliable judgment concerning H_0. Due to cost or time limitations or lack of enough data, the tests in questions are run with less data[5]. Therefore, if the life distribution is not properly recognized, the computed reliability is not credible valid. In such cases, the generalized lambda distribution can serve as an alternative of varied distributions. By altering its parameters, such a distribution can simulate and estimate most of the identified and even unidentified distributions[6]. In fact, the main problem of the research is the computation of the parameters of GLD, and the reliability of the product is then, extracted by using those parameters.

Generalized Lambda Distribution: This distribution was first advanced by Tukey[7], and later on was developed by Junior and Rosenblatt[8]. This distribution can precisely fit the ordinary distribution like normal, lognormal, Weibull, etc. The flexibility of this distribution exerts influences on estimating continual distributions and matching on histogram data and estimating the distribution type. As a matter of fact, it serves as a powerful device for research in different areas like estimating parameters, adjusting distributions on data and simulating research based on data production. For example, it is deployed in operational[9], psychology[10], meteorology[11], process statistic control[12], safety and fault tolerance[13], and queue systems[14]. Zaven and et al[15] studied about generalized lambda family of distributions, generalized bootstrap and Monte Carlo, and fitting distributions and data with.

In literature, several imputation techniques are described. Thakur and et al[16] present the estimation of mean in presence of missing data under two-phase sampling scheme while the numbers of available observations are considered as random variable. Rekha R. C. and Vikas S[17], have formulated an Inventory model for deteriorating items with Weibull distribution deterioration rate with two parameters. Roman, and et al[18], have used Goodness-of-Fittest such as Anderson-Darling, Chi-square and Kolmogorov-Smirnov to judge the applicability of the distributions for modeling recorded Annual 1-Day Maximum Rainfall (ADMR) data.

Bigerelle and et al[19], for example, use generalized lambda distribution and Bootstrap analysis to the prediction of fatigue lifetime and confidence intervals. In this research, the lambda distributions associated with the Bootstrap technique were first employed to model the Paris coefficients PDF and turned out to be able to estimate accurately the experimental values. Then, lambda distributions were used to model the PDF lifetime of a basic structure under fatigue loading.

The characterizing of generalized lambda distribution has been studied by Karvanen, and Nuutinen[20]. It has been introduced as a reversed Probability Cumulative Distribution Function Q as below:

$$x = Q(y, \lambda_1, \lambda_2, \lambda_3, \lambda_4) = \lambda_1 + \frac{y^{\lambda_3} - (1-y)^{\lambda_4}}{\lambda_2} \tag{3}$$

where y implies the relative Density Probability in point x and it is obvious that its extent would fall between zero and one. λ_1 and λ_2 are the co efficiencies related to the measurement and place, respectively and λ_3, λ_4 referred to the prominence and suspense of the distribution[21].

The generalized lambda distribution is a distribution that can organize, simulate and estimate all distributions through changing the parameters. It is flexible enough to exactly simulate and accordingly the quality control operation is carefully done. This task is done via accessing 100 data.

Fitting a probability distribution to data is an important task in any statistical data analysis. The data to be modeled may consist of observed events, such as quality characteristic of components. When fitting data, one typically first selects a general class, or family, of distributions and then finds values for the distributional parameters that best match the observed data.

By changing the values of λ, the GLD can cover any distribution in distinct forms and match them. The details of the particulars of this distribution, the applications and the way of computing the parameters have been precisely explained in[21].

Eventually, Tarsitano[22] raised the number of the parameters of this distribution up to 5 and studied the characteristics. This is done to increase the capability and exactness to fit the panel data distribution. The distribution of a new five parameters is an accumulated opposite as below:

$$X(y, \lambda) = \lambda_1 + \lambda_2 y^{\lambda_3} - \lambda_4 (1-y)^{\lambda_5}, 0 \le y \le 1 \tag{4}$$

In this distribution, λ_1 is place indicator, λ_2, λ_3 are measuring indicators and λ_4 and λ_5 related to the distribution feature. The two measuring indicators display different weights for the distribution extent and provide this place for a new distribution to be well- adjusted with the data without symmetric extents are well – matched.

The PDF of this distribution is:

$$f(y) = \frac{dx}{dy} = \lambda_2 \lambda_4 y^{\lambda_4 - 1} + \lambda_3 \lambda_5 (1-y)^{\lambda_5 - 1} \tag{5}$$

The m[th] moment of this distribution can be computed by using the relations (4) and (5) and also the total computations of the moments are as follow:

$$\begin{aligned} M_m &= E(X^m) = \int_{-\infty}^{+\infty} X^m f(x)dx \\ &= \int_0^1 (\lambda_1 + \lambda_2 y^{\lambda_3} - \lambda_4 (1-y)^{\lambda_5})^m dy \end{aligned} \tag{6}$$

Estimation the GLD parameters: Several methods for estimating the parameters of the GLD, such as: Percentile Matching (PM), the moment matching (MM), Probability-Weighted Moment (PWM), Minimum Cramér-Von Mises (MCM), Maximum Likelihood (ML), Pseudo Least Squares (PLS), Downhill simplex method, and starship methods, have been reported in the literature[22, 23]. Fournier and et al[7], for example, developed a new method for estimating the parameters of a GLD based on the minimization of the Kolmogorov–Smirnov distance in a two-dimension space. In this article, we will briefly review the moment matching method.

The moment-matching method, described in this paper, was proposed in Ramberg and Schmeiser[24]. The method can be described in a straightforward manner as follows: given the GLD distribution with quartile function Q(u), find parameters λ_1, λ_2, λ_3, λ_4 and λ_5 so that the mean μ and variance σ^2 of the GLD match the corresponding mean μ^*, and variance $(\sigma^*)^2$ of the sample (i.e., the first five moments of the theoretical GLD match those of the data). More formally, if such a method denotes the probability density function of the random variable X with distribution 4, we compute the parameters λ such that satisfying equations 6. Finally, after determining the 5 parameters of distribution, we can show that the mean and variance of GLD can be calculated as bellow:

$$\mu = E(X) = \int_{-\infty}^{+\infty} xf(x)dx$$
$$= \int_0^1 (\lambda_1 + \lambda_2 y^{\lambda_3} - \lambda_4 (1-y)^{\lambda_5})dy \quad (7)$$
$$= \lambda_1 + \frac{\lambda_2}{(\lambda_4+1)} - \frac{\lambda_3}{(\lambda_5+1)}$$

$$\sigma^2 \equiv E(X^2) - [E(X)]^2$$
$$= \int_{-\infty}^{+\infty} x^2 f(x)dx - \left[\int_{-\infty}^{+\infty} xf(x)dx\right]^2 \quad (8)$$
$$= \int_0^1 (\lambda_1 + \lambda_2 y^{\lambda_3} - \lambda_4 (1-y)^{\lambda_5})^2 dy - \left(\lambda_1 + \frac{\lambda_2}{(\lambda_4+1)} - \frac{\lambda_3}{(\lambda_5+1)}\right)^2$$

By using this method, after we have collected data from production line, and then calculated the mean, variance, and standard deviation of the underlying distribution of the qualitative specification, we can determine the reliability of product.

Research Method

First, the statistical model of computing reliability is developed; afterwards, the frequency table of data of product life is prepared using random numbers. Next, the generalized lambda distribution is plotted on random data through Excel and MATLAB software and the reliability is calculated.

The research was done in the following steps: i. Data collection: Since it is costly and time-consuming to collect data concerning the real life of products, the random numbers were utilized. To this end, 100 sets of data among 5 and 7 were created, using the [RAND ()*2+5] formula in Excel environment. It was also assumed that the generated data is the lifetime of products. That is, all 100 products have worked until they broke down. For example, the lifetime of a product is a number as 5.594, that is, this product has worked 5.594 years and then it broke down. ii. Frequency table: The frequency table of data was prepared in the form of 10 class intervals. To better estimate the parameters, the frequency table has to have at least 7 class intervals. iii. Computation of experience moments: Experience moments of data for the first and fifth times were estimated by the following formula.

$$M_m = \sum_{i=1}^{k} C^m f_i \quad (9)$$

Where: M_m= M^{th} moment, C= Midpoint or mean of the Class Interval, and f= relative frequency of the Class Interval.

Estimating theoretical moments: In GLD, theoretical moments are estimated using equation 6.

Forming equation system and determining distribution parameters: A five- variable non-linear equations system can be derived from calculating steps 3 and 4. MATLAB and Excel software programs help us to estimate the unknown parameters.

Goodness of fitness test: Solving non-linear equations usually yields more than one series of answers, so it is quite necessary to run goodness of fitness test to attain the acceptable answer. To this end, Chi-square statistics was deployed:

$$\chi_0^2 = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i} \quad (10)$$

H_0: GLD with the obtained parameters fits the data. H_1: GLD with the obtained parameters does not fit the data.

H_0 is rejected if $\chi_0^2 > \chi_{\alpha,k-i-1}^2$ where α is the level of significance of the test, K, the number of class intervals and I, the number of distribution parameters.

Computing reliability: Since the probability density function of the Lambda distribution is not available, the reliability function cannot be calculated; therefore, the possible solution to this problem is that computations can be done by using the original definition of reliability.

As we showed in equation 1, and by definition, reliability is the probability that a product works at least for a certain time designated by the designer. Thus, the table of the reliability of the product can be calculated according to the definition.

Data Gathering and Results

To collect the data, random generation method by Excel was deployed, and 100 random numbers among 5 and 7 were created by. In the next step, the frequency table of generated random numbers in the form of 10 class intervals was supplied, in which the interval between each class was considered as 0.2 (table 2).

Table-2
The frequency table of random numbers

Class Interval	Range	Midpoint (C)	Frequency	Relative frequency
1	5-5.2	5.1	11	0.11
2	5.2-5.4	5.3	10	0.1
3	5.4-5.6	5.5	13	0.13
4	5.6-5.8	5.7	8	0.08
5	5.8-6	5.9	9	0.09
6	6-6.2	6.1	10	0.1
7	6.2-6.4	6.3	12	0.12
8	6.4-6.6	6.5	13	0.13
9	6.6-6.8	6.7	8	0.08
10	6.8-7	6.9	6	0.06
		Total	100	1

Calculating the empirical moments: Then, the empirical moments of the data are estimated. The results are in table 3.

Table-3
The empirical moments of the data

Midpoint or Mean (C)	Relative Frequency f	C*f	C2*f	C3*f	C4*f	C5*f
5.1	0.11	0.561	2.8611	14.59161	74.417211	379.5277761
5.3	0.1	0.53	2.809	14.8877	78.90481	418.195493
5.5	0.13	0.715	3.9325	21.62875	118.958125	654.2696875
5.7	0.08	0.456	2.5992	14.81544	84.448008	481.3536456
5.9	0.09	0.531	3.1329	18.48411	109.056249	643.4318691
6.1	0.1	0.61	3.721	22.6981	138.45841	844.596301
6.3	0.12	0.756	4.7628	30.00564	189.035532	1190.923852
6.5	0.13	0.845	5.4925	35.70125	232.058125	1508.377813
6.7	0.08	0.536	3.5912	24.06104	161.208968	1080.100086
6.9	0.06	0.414	2.8566	19.71054	136.002726	938.4188094
Total	1	5.9540	35.7588	216.5842	1,322.5482	8,139.1953

The system of equations: After obtaining the moments of first to fifth order, now it is turn to construct the five equations and five variables of system in which with the help of the theoretical moment of GLD and the obtained empirical moments, the equations system are derived:

$$\int_0^1 (\lambda_1 + \lambda_2 y^{\lambda_3} - \lambda_4 (1-y)^{\lambda_5}) dy = 5.9540$$

$$\int_0^1 (\lambda_1 + \lambda_2 y^{\lambda_3} - \lambda_4 (1-y)^{\lambda_5})^2 dy = 35.7588$$

$$\int_0^1 (\lambda_1 + \lambda_2 y^{\lambda_3} - \lambda_4 (1-y)^{\lambda_5})^3 dy = 216.5842$$

$$\int_0^1 (\lambda_1 + \lambda_2 y^{\lambda_3} - \lambda_4 (1-y)^{\lambda_5})^4 dy = 1322.5482$$

$$\int_0^1 (\lambda_1 + \lambda_2 y^{\lambda_3} - \lambda_4 (1-y)^{\lambda_5})^5 dy = 8,139.1953$$

Figure-2
The equations system of moments

It is feasible to solve the nonlinear equation above by Newton's method. Of course, due to the complexity it is necessary to employ advanced software programs. In the present research, two software programs, Excel and MATLAB were used. With regard to the fact that the solution is of practical importance, the steps done by the software are presented in detail.

For deriving the variables or unknown parameters by Newton Method, the first and foremost step is to obtain the primary answer that fulfills a crucial function in the final answer. To obtain the primary answer, the first equation can serve as the basis and its resolving leads to the primary answer.

$$\int_0^1 (\lambda_1 + \lambda_2 y^{\lambda_3} - \lambda_4 (1-y)^{\lambda_5}) dy = 5.9540$$

The equation above after being expanded:
$\lambda_1+\lambda_2/(\lambda_3+1)-\lambda_4/(\lambda_5+1)=5.9540$

There will be one primary answer for the equation as: λ_1= 5.9540, λ_2=1, λ_3=1, λ_4=1 and λ_5=1. Given the complexity of the equation, manual computation is not possible, so it is essential to utilizing a robust software program for mathematical computations. This research uses MATLAB, software program. As a result, the final answer to the parameters of GLD is presented in table 4.

Table-4
The final answer to the unknowns of the equations system of moments

λ_1	λ_2	λ_3	λ_4	λ_5
5.9947	0.9367	1.0218	1.0272	1.0191

Consequently, the quartile cumulative distribution function of the lifetime of products in the study equals:

$$X(\lambda, y) = 5.9947 + 0.9367 y^{1.0218} - 1.0272(1-y)^{1.0191} \quad (11)$$

Analyzing the parameters: Due to the nonlinearity of the equations, there will be more than one series of parameters, so the goodness of fit test has to be done on the derived parameters to ensure that the obtained distribution coincides with the real data. To do so, first the expected values (E_i) are obtained. For each class interval i^{th}, the cumulative amount of relative frequency is positioned in the distribution relationship; hence, the value of the mean of the class interval in question is calculated, which is the very E_i. Similarly, the observed value is the mean of the data class intervals. Chi-square test statistic is with K-6 degrees of freedom, where K is the number of class intervals. Table 5 is the hypothesis testing of the research data.

With respect to the relationship $\chi_0^2(0.0141) < \chi_{0.05,4}^2(9.488)$, it can be claimed that the derived Generalized Lambda Distribution fits the data.

Computing reliability: Since the cumulative distribution function of the lambda distribution is not available, to calculate the reliability, a table is deployed in such a way that the probability that a product works more than a specified time is estimated. By definition, reliability (equation 1), and similarly, the reverse cumulative distribution function of the data (equation 11), and with regard to the fact that the lifetime of a product does not lead us to its reliability; thus, we have to use the assumed reliability to calculate the lifetime of a given product. The research associated with the reliability of the product life of 0.1 to 0.99 is calculated. In this research, the corresponding outcome with reliability interval of 0.1—0.99 has been computed (table 6). Which we have: R(t)=1-Y, and t=X(R(t)). In this table, if, for example, it is assumed that the reliability of a product is 0.62, solving the reverse cumulative equation function yields the lifetime of the product as 5.71211 years.

Table-5
The goodness of fit test

Class Interval	Observation value O_i	Cumulative Relative Frequency Y_i	Expected Observation Value E_i	$(O_i-E_i)^2$	$\frac{(O_i-E_i)^2}{E_i}$
1	5.1	0.11	5.18	0.0064	0.0012
2	5.3	0.21	5.37	0.0049	0.0009
3	5.5	0.34	5.63	0.0169	0.0030
4	5.7	0.42	5.79	0.0081	0.0013
5	5.9	0.51	5.96*	0.0036	0.0006
6	6.1	0.61	6.16	0.0036	0.0005
7	6.3	0.73	6.40	0.0100	0.0015
8	6.5	0.86	6.65	0.0225	0.0033
9	6.7	0.94	6.81	0.0121	0.0017
10	6.9	1.00	6.93	0.0009	0.0001
Total					0.0141

*$5.96=5.9947+0.9367*0.51^{1.0218}-1.0272*0.49^{1.0191}$

Table-6
Reliability of the product

	0	1	2	3	4	5	6	7	8	9
0	-	6.9124	6.8932	6.8739	6.8545	6.8351	6.8156	6.7961	6.7766	6.7571
10	6.7375	6.7179	6.6983	6.6787	6.6591	6.6395	6.6198	6.6002	6.5805	6.5609
20	6.5412	6.5215	6.5018	6.4821	6.4624	6.4427	6.4230	6.4033	6.3836	6.3639
30	6.3442	6.3244	6.3047	6.2850	6.2652	6.2455	6.2257	6.2060	6.1862	6.1665
40	6.1467	6.1270	6.1072	6.0875	6.0677	6.0480	6.0282	6.0085	5.9887	5.9689
50	5.9492	5.9294	5.9097	5.8899	5.8702	5.8504	5.8306	5.8109	5.7911	5.7714
60	5.7516	5.7319	5.7121	5.6924	5.6727	5.6529	5.6332	5.6134	5.5937	5.5740
70	5.5543	5.5346	5.5148	5.4951	5.4754	5.4557	5.4360	5.4163	5.3967	5.3770
80	5.3573	5.3377	5.3180	5.2984	5.2787	5.2591	5.2395	5.2199	5.2003	5.1807
90	5.1612	5.1416	5.1221	5.1026	5.0831	5.0637	5.0443	5.0249	5.0056	4.9864

Conclusion

Reliability is one of the critical parameters in designing production systems and also of crucial importance for the customer to find out. Therefore, its determination is of special significance. For those products whose lifetime is in accordance with the known life distributions, it is essential to figuring out the relevant distribution with spending lots of money and then the concerned reliability is calculated. Instead, the Generalized Lambda Distribution can be deployed, which applies to every known or unknown distribution. After estimating the parameters of the distribution, reliability can be calculated in the next step. As a result, for every product, a reliability table can be devised. To do so, first, the frequency table of the lifetime of the product in question is contrived at least in 7 class intervals. Then, the moments in order of 1 to 5 are computed and derived. By means of the function of Generalized Lambda Distribution moments and the calculated empirical moments, 5 nonlinear equations are yielded. From the first equation (the first moment), one primary answer is derived for the 5 parameters of the distribution. Afterwards, MATLAB is deployed to solve the equations and determine the parameters with Newton method. The obtained answers are examined for their goodness of fitness. To compute and estimate the parameters, there are various methods that are as: Percentile Matching (PM), the moment matching (MM), Probability-Weighted Moment (PWM), Minimum Cramér-Von Mises (MCM), Maximum Likelihood (ML), Pseudo Least Squares (PLS), Downhill simplex method, and starship methods, In this study, due to the limited access to an appropriate software program, Moment Matching (MM) Estimates were employed without considering the target function. But for the future research, Maximum likelihood can be utilized and Simplex method can be used for solving equations and estimating parameters. Since the Generalized Lambda Distribution is still an innovative approach, it is necessary to investigate different aspects of this distribution. For example, we can seek to find out whether probability density function can be computed. If it can be done, to estimate the reliably, we can make use of the definition of reliability rather than do varied computations.

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