



Short Communication

Deviation from Tri-bi-maximal mixings with Charged Lepton correction in case of Inverted and Normal Hierarchical Neutrino Mass Models

Das Mrinal Kumar and Sarma Jayanta Kumar

Department of Physics, Tezpur University, Napaam, Tezpur-784028, Assam, INDIA

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Abstract

Deviation from tri-bi-maximal (TBM) mixing is an important topic in the recent neutrino physics. In this paper we try to get non-zero U_{e3} without disturbing the atmospheric and solar mixing angles. Present analysis is in case of inverted and normal hierarchical mass models using the charge lepton correction in the PMNS (Pontecorvo-Maki-Nakagawa-Sakata) matrix without CP violating phase. Here we have shown that in this case one can perturb the TBM condition in order to generate a seizable value of U_{e3} in the range of 0.6 to 0.8, while at the same time keeping solar neutrino mixing near its measured value, which is close to $\sin^2 \theta_{12} = 1/3$ and the atmosphere neutrino mixing close to $\sin^2 \theta_{23} = 1/2$.

Keywords: Tri-bi-maximal mixing, Neutrino masses, charged lepton correction.

Introduction

The latest experimental data of the solar mixing angle points towards the specific form of lepton mixing matrix. This pattern is known as tri-bi-maximal mixing (TBM) or Harrison-Perkins-Scott (HPS)¹ pattern, proposes flavour mixing matrix U_{PMNS} as:

$$U_{PMNS} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \end{pmatrix}. \quad (1)$$

Predictions of equation (1) viz. $\sin^2 \theta_{23} = 1/2$, $\sin^2 \theta_{12} = 1/3$, are consistent with atmospheric neutrino oscillation data. It also leads to $\sin \theta_{13} = 0$. Recent values of $\sin^2 \theta_{12}$ and $\sin^2 \theta_{23}$ obtained from the global fits of neutrino oscillation are very close to those predicted by TBM are²⁻⁷:

$$\sin^2 \theta_{23} = 0.466_{-0.058}^{+0.073} \quad (2)$$

$$\text{and } \sin^2 \theta_{12} = 0.312_{-0.018}^{+0.019} \quad (3)$$

Even if nature has chosen TBM as lepton mixing scheme, one can expect deviation from it on very general grounds specially on vanishing θ_{13} , which is against the recent experimental data. To achieve experimental data deviation from TBM is required

special on θ_{13} either by charged lepton corrections, renormalization effects.

If we give interest on the third mixing then we can see many author have reached none vanishing θ_{13} in model independent way in the 3σ range:

$$\sin^2 \theta_{13} = 0.016 \pm 0.010 \quad (4)$$

Similar values and ranges have been found in other independent analysis⁶⁻⁹. In the present paper, we try to investigate the possibility of breaking TBM with seizable value of U_{e3} as a result of deviation from this mixing in normal and inverted hierarchical models. Same type of work had been reported⁵ in model independent way. In this paper, our aim is to implement the idea in different neutrino mass models which will be useful for any neutrino mass model using SO (10) GUT with seesaw mechanism. In our calculation, we play around the value suggested by equation (4) where we have not considered the CP phases. Main idea in the calculation is that, we try to keep the values of mixing fixed. For our analysis we have used the inverted and normal hierarchal mass models from ref¹⁰⁻¹⁴. These mass matrices are given by,

$$m_{LL} = \begin{pmatrix} 1+2\delta_1 & \delta_1 & \delta_1 \\ \delta_1 & \frac{1}{2} & \frac{1}{2} + \delta_2 \\ \delta_1 & \frac{1}{2} + \delta_2 & \frac{1}{2} \end{pmatrix} m_o, \quad (5)$$

Similarly, normal hierarchical mass matrix with TBM has the following form,

$$m_{LL} = \begin{pmatrix} -\delta_2 & -\delta_1 & -\delta_1 \\ -\delta_1 & 1-\delta_1 & -1+\delta_2 \\ -\delta_1 & -1+\delta_2 & 1-\delta_1 \end{pmatrix} m_o \quad (6)$$

Methodology

The PMNS matrix is, in general, product of two unitary matrices $U = U_l^+ U_\nu$ (7)

where U_ν diagonalizes the neutrino mass matrix and U_l is associated with the diagonalization of the charged lepton mass matrix. Analysis on TBM has been carried out considering diagonal charged lepton mass¹⁵. Also, analysis on charged lepton correction unlike the present work have been carried out in different aspects¹⁶⁻²². For the present analysis neutrino mixing matrix U_ν without CP violation phase can be written as:

$$U_\nu = \begin{pmatrix} c_{13}^v c_{12}^v & c_{13}^v s_{12}^v & s_{13}^v \\ -c_{23}^v s_{12}^v - c_{12}^v s_{13}^v s_{23}^v & c_{12}^v c_{23}^v - s_{12}^v s_{13}^v s_{23}^v & c_{13}^v s_{23}^v \\ s_{12}^v s_{23}^v - c_{12}^v s_{13}^v c_{23}^v & -c_{12}^v s_{23}^v - c_{23}^v s_{13}^v s_{12}^v & c_{13}^v c_{23}^v \end{pmatrix} \quad (8)$$

The unitary matrix U_l which can be written as:

$$U_l = \begin{pmatrix} c_{13}^l c_{12}^l & c_{13}^l s_{12}^l & s_{13}^l \\ -c_{23}^l s_{12}^l - c_{12}^l s_{13}^l s_{23}^l & c_{12}^l c_{23}^l - s_{12}^l s_{13}^l s_{23}^l & c_{13}^l s_{23}^l \\ s_{12}^l s_{23}^l - c_{12}^l s_{13}^l c_{23}^l & -c_{12}^l s_{23}^l - c_{23}^l s_{13}^l s_{12}^l & c_{13}^l c_{23}^l \end{pmatrix} \quad (9)$$

where we have used $c_{ij} = \cos \theta_{ij}$ and $s_{ij} = \sin \theta_{ij}$. With this correction the small neutrino mass can be express considering non-diagonal charged lepton mass in seesaw mechanism. To incorporate this correction, first we calculate the elements of the U_ν from the relations (without CP phases)²³⁻²⁵.

$$\tan 2\theta_{23}^v = \frac{2 |m_{23}^v|}{|m_{33}^v| - |m_{22}^v|} \quad (10)$$

$$\text{and } \theta_{13}^v = \frac{\tilde{m}_{13}^v}{\tilde{m}_3^v} \quad (11)$$

where

$$\text{and } \tan 2\theta_{12}^v = \frac{2 |\tilde{m}_{12}^v|}{|\tilde{m}_{22}^v| - |\tilde{m}_{11}^v|} \quad (12)$$

where

$$\begin{pmatrix} \tilde{m}_{12}^v \\ \tilde{m}_{13}^v \end{pmatrix} = \left(R_{23}^v \right)^T \begin{pmatrix} |m_{12}^v| \\ |m_{13}^v| \end{pmatrix}. \quad (13)$$

$$\text{Here } R_{23}^v = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23}^v & s_{23}^v \\ 0 & -s_{23}^v & c_{23}^v \end{pmatrix}, \quad \tilde{m}_{11}^v = m_{11}^v - \frac{(\tilde{m}_{13}^v)^T}{\tilde{m}_3^v}$$

$$\text{and } \tilde{m}_{22}^v = (c_{23}^v)^2 |m_{22}^v| + 2s_{23}^v c_{23}^v |m_{23}^v| + (s_{23}^v)^2 |m_{33}^v|.$$

Different values of m_{ij} are taken from the m_{LL} given by equations (5) and (6). Elements of U_L have been calculated from the relations given by⁹
 $\sin \theta_{13}^l = \lambda^3, \sin \theta_{23}^l = \lambda^2 A$
 and $\sin \theta_{12}^l = \lambda^3 B$. (14)

Result and Discussion

Using equation (10-13) elements of U_ν are calculated for both the cases normal as well as inverted. Then by using the values of A, B and λ elements of U_L are calculated⁹. After getting all the elements one can get the PMNS matrix with θ_{13} nonzero as shown in table-1, which is deviated from TBM condition as given by the equation (1).

Table – 1
Values of A, B and λ

Items	Inverted hierarchical	Normal hierarchical
Range of A and B	0.2-5	0.2-5
Range of λ	0.104-0.247	0.104-0.247
λ value used	0.2	0.18
A and B value used	0.3	0.3
$ U_{e3} ^2$	0.064	0.049

The values of δ_1, δ_2 and m_o is taken from ref. as given in the Table-2 below¹⁴:

Table – 2
Values of δ_1, δ_2 and m_o

Type	δ_1	δ_2	m_o
Inverted hierarchical	0.0055	0.0165	0.05
Normal hierarchical	0.176	--	0.03

The values of θ_{13} are calculated for both the cases viz. inverted hierarchical and normal hierarchical cases by taking consideration that the values of θ_{12} and θ_{23} still satisfy the conditions given in equations (2) and (3).

Conclusion

Tri-bi-maximal mixing provides a very close description of neutrino mixing angles. However the present hint of nonzero

θ_{13} coming from analysis of global neutrino oscillation data may indicate that it is broken. Here, in this work, we try to use the charge lepton correction in PMNS matrix to get the deviation from TBM, which will be useful for model building ideas. There is a good scope for extension of this work with CP violating phases and also one can get the seizable values with running of renormalization group equation within this framework.

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