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Aspect of Finite Element Analysis Methods for Prediction of Fatigue Crack Growth Rate

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Abstract

An attempt is made to predict the fatigue crack path using the finite element analysis to design a body against fatigue failure. Till now consideration is taken that the fatigue failures is always straight i.e. (Straight extending crack) but in practice the cracks fond in any body deviate or extend more in a zig zag manner due to mixed mode stress produced. The change in stress state mode caused by the deviation of a crack affect the succeeding crack path and its growth rate. Because of these reasons prediction of crack path and growth rate is more important for fatigue life evaluations. The use of the finite element method enabled the subsequent tracking of deflecting crack extension. In this paper we try to demonstrate the capability and its limitations, in predicting the crack propagation trajectory and the SIF values under linear elastic fracture analysis.

Keywords: Fatigue crack growth, finite element method, mixed mode, finite element method, crack path.

Introduction

Fatigue cracks are the major cause of fatal blow in any body. Failure analysis are mainly based on experimental analysis recently Finite element analysis has become a novel and supplementary approach to failure analysis. FEA can represent how fatal blows occurs in realistic conditions and with the development in fracture mechanics, the accuracy of FEM is highly appreciable. Fatigue crack initiation and propagation behaviors under mixed-mode conditions have to be studied for the fracture of materials subjected to loads in various directions. Fatigue crack growth simulation consists of two key parts: Numerical analysis and mesh generation. A crack in real structure is always under mixed-mode condition and grows into a complex geometry. This paper is an attempt to predict the fatigue crack path in combined mode I and mode II, using finite element method with the purpose of serving for an optimum design of structure against fatigue failures. The majority of the fatigue crack growth studies are usually performed under mode-I loading conditions. Although analytical and experimental studies have also been extensively conduced in mixed-mode loading. Most crack analyses up to now have treated the problem of a straight extending crack. But majority of cracks founds in real practice are known to be in mixed mode stress state, such as $K_I - K_{II}$ or K_I - K_{III}. This is due to Following facts: (i) other cracks and flaws present in the neighborhoods. (ii) The loading itself is a combined loading such as tension-shear, and (iii) the stress t a point of interest is in mixed-mode stress state due to structural configuration although loading itself is uni-axial. Therefore cracks in structure more or less deviate in zigzag manner. In a fracture Mechanics representation of a typical mode I case, the extending Fracture surfaces are idealized as flat planes. On a microscopic scale, however the fracture surface is not smooth, as asperities, which are related to micro structural details, are developed. In contrast to mode I crack growth, where there is little fracture surface interaction, mode II or mode III fracture surfaces interact in a complicated manner. Fracture surface interaction occurs when macroscopically rough Fracture surfaces are displaced relative to one another in shear. By adopting the finite element analysis, we can pursue successively each stage of deflecting crack extension. In the 1950s, many investigators mentioned how early in the fatigue life they could observe micro cracks. Since then it was clear that the fatigue life under cyclic loading consisted of two phases, the crack initiation life followed by a crack growth period until failure. This can be represented in a block diagram, see figure 1. Two different geometries were used on this finite element model in order, to analyze the reliability of this program on the crack propagation in linear and nonlinear elastic fracture mechanics. These geometries were namely; a rectangular plate with crack emanating from square-hole and Double Edge Notched Plate (DENT). Where, both geometries are in tensile loading and under mode I conditions.

Terminology Used: 2a= projected crack length, 2c= actual crack length, 2w= plate width, T= plate thickness, E= young's modulus, σ_0= stress amplitude, θ= direction of crack extension, υ = Poisson's ratio, K_I , K_{II} and K_{III} = stress intensity factor for Mode I, Mode II and Mode III respectively. X= symmetric axis of specimen in horizontal direction, Y= symmetric axis of specimen in vertical direction, x, y = crack tip coordinate parallel to X and Y.

Related Work: Finite element analysis (FEA) has become commonplace in recent years, and is now the basis of a multibillion dollar per year industry. Numerical solutions to even very complicated stress problems can now be obtained routinely using FEA, and the method is so important that even introductory treatments of Mechanics of Materials. In spite of the great power of FEA, the disadvantages of computer solutions must be kept in mind when using this and similar methods. There is a lot of software available for Finite Element Analysis, such as Algor (FEMPRO), ANSYS, ABAQUS, and MSC Patran. All this software widely used nowadays, in order to analyze the part. In solving partial differential equations, the primary challenge is to create an equation that approximates the equation to be studied, but is numerically stable, meaning that errors in the input and intermediate calculations do not accumulate and cause there sulting output to be meaningless. There are many ways of doing this, all with advantages and disadvantages. The Finite Element Method is a good choice for solving partial differential equations over complicated domains. Finite element method; FEM is a numerical technique for finding approximate solutions of partial differential equation as well as of integral equations. The solution approach is based either on eliminating the differential equation completely (steady state problem) or rendering the PDE into an approximating system of ordinary differential equations, which are then numerically integrated using standard techniques such as Euler's method and Runge-Kutta. Skorupa and Skorupa were studied crack growth predictions for structural steel using constraint factors. At positive stress ratios, structural steel shows significant crack growth retardation under variable amplitude. It also studied load interaction effects in crack growth are negligible. The crack growth response of structural steel in some of the experiments is very different from Al-alloys used in the aircraft industry were studied two different geometries were used on this finite element model in order, to analyze the reliability of this program on the crack propagation in linear and nonlinear elastic fracture mechanics. These geometries were namely; a rectangular plate with crack emanating from square-hole and double edge notched plate (DENT). Where, both geometries are in tensile loading and under mode I conditions. Predict the crack propagations directions and calculate the stress intensity factors and the results are the application of the source code program of 2-D finite element model showed a significant result on linear elastic fracture mechanics detail level.

Fatigue Life Curve: The fatigue behaviour of a material is graphically illustrated with the aid of a stress-life (*S-N*) curve as shown by figure 2. The curve is obtained by testing a number of specimens at various stress levels under sinusoidal loading conditions and is composed of an initiation and a propagation component. The fatigue tests are conducted on smooth un-notched specimens that make it practically impossible to distinguish between the crack initiation and propagation phase. Thus, the *S-N* curve usually represents only the total fatigue life of a specimen as shown by the solid line of figure 2

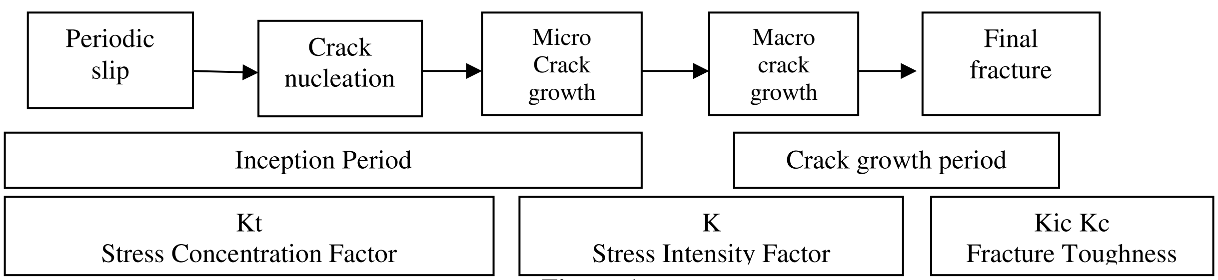


Figure-1
Different phase of the fatigue life and relevant factors

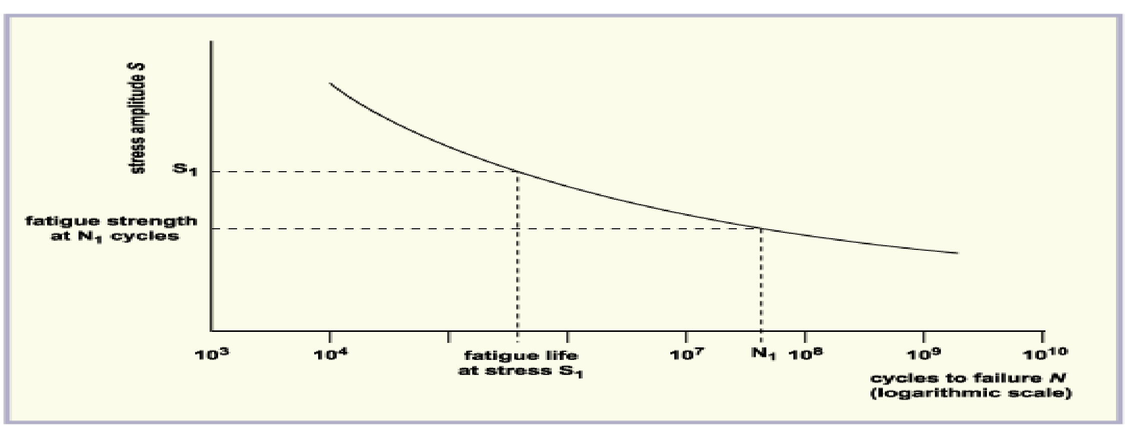


Figure-2
S-N curves

A typical *S-N* curve for some materials appears to become flat for large number of cycles representing a distinct stress level below which fatigue failure will not occur. This limit is called the fatigue limit or the endurance limit. A given *S-N* curve is valid for the specific conditions under which it was tested. One may extrapolate the results from an *S-N* curve to include the influence of other factors provided that sufficient knowledge of how they affect the *S-N* curve is known. These factors include member geometry, chemical environment, cyclic frequency, temperature, residual stress and mean stress.

Fatigue Rate Curve: A typical fatigue rate curve, commonly referred to as a da/dN versus Δ*K* curve, is illustrated by figure 3 The curve is defined by regions A, B and C which are commonly referred to as region I, II and III respectively

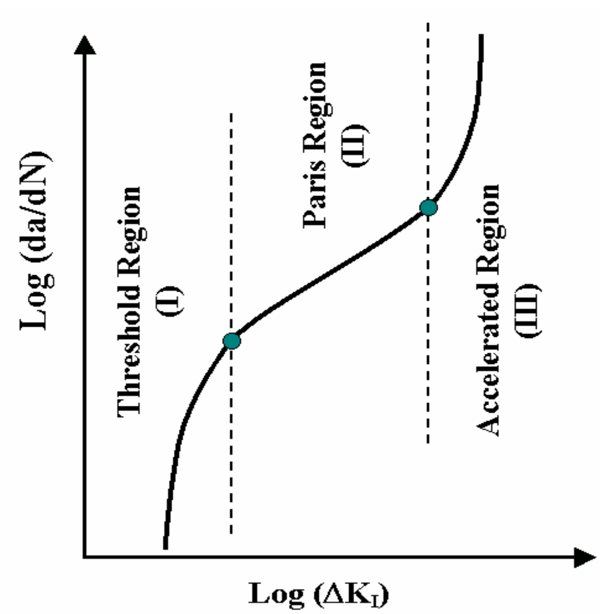


Figure-3
A typical fatigue crack growth rate Curve (FCG curve)

Region I represents the early development of a fatigue crack and the crack growth rate; da/dN is typically of the order 10-6 mm/cycle or smaller of the test data result from ASTM E647. This region is extremely sensitive and is largely influenced by the microstructure features of the material such as grain size, the mean stress of the applied load, the operating temperature and the environment present.

The most important feature of this region is the existence of a stress intensity factor range below which fatigue cracks should not propagate. This value is defined as the fatigue crack growth threshold and is represented by the symbol ΔKth. Its value is experimentally determined by using the decreasing K test as described in the ASTM E647 documents.

Region II represents the intermediate crack propagation zone where the length of the plastic zone ahead of the crack tip is long compared with the mean grain size, but much smaller than the crack length. The use of linear elastic fracture mechanics (LEFM) concepts is acceptable and the data follows a linear relationship between log da/dN and log Δ K. The crack growth rate is typically on the order of 10-6 to 10-3 mm/cycle, which corresponds to the majority of the test data results from ASTM E647.

Region III represents the fatigue crack growth at very high rates, da/dN > 103 mm/cycle due to rapid and unstable crack growth just prior to final failure. The da/dN versus ΔK curve becomes steep and asymptotically approaches the fracture toughness Kc for the material. The corresponding stress level is very high and causes a large plastic zone near the crack tip as compared with the specimen geometry.

Paris Mode1: A simple and well known method for predicting fatigue crack propagation is a power law described by Paris and Erdogan, and it is also known as the Paris Law. The equation represents the first application of fracture mechanics to fatigue and is given by the following relationship $da/dN = Cp\,(\Delta K)^{m}{}_{p}$

Where Cp is the intercept and mp is the slope on the log-log plot of da/dN versus Δ K. Equation represents a straight line on the log-log plot of da/dN versus ΔK and thus describes region II of the fatigue rate curve. The Paris law is simple to use and requires the determination of two curve fitting parameters which are easily obtained. As long as the data follows a straight line relationship, the Paris law is commonly used. The limitation of the Paris law is that it is only capable of describing data in region II (figure 3). If the data exhibits a threshold (region I) or an accelerated growth (region III) Paris law cannot adequately describe these regions. Depending upon the analysis being undertaken, this approximation may not be adequate. Finally, the Paris law does not consider the effect of stress ratio and it depends upon the material used. For steels tested at various stress ratios, a family of straight lines parallel to each other is produced. This means that the value of mp is the same for all stress ratios but the value of Cp is specific for a particular stress ratio. Therefore one must ensure that the fatigue rate data used is for the stress ratio of interest.

Walker Mode1: The major limitation of the Paris law is its inability to account for the stress ratio. This drawback notified Walker to improve the Paris model by including the effect of stress ratio. Walker proposed a parameter ΔK, which is an equivalent zero to maximum (R = 0) stress intensity factor that causes the same growth rate as the actual Kmax, and R combination. It is expressed by the following relationship:

$\Delta K = K_{max}(1-R)^{y}{}_{w}$

Where Kmax = Δ K/(1-R),and equation reduces to

$da/dN = Cw\,(\Delta K)^{m}{}_{w}$

Which is equivalent to the Paris law with *Cp* = *Cw* and *mp* = *mw*.

Which is equivalent to the Paris law with Cp = Cw and mp = mw. The significance of this equation is that a log-log plot of da/dN versus ΔK should result in a single straight line regardless of the stress ratio for which the data was obtained.

The ability to account for this results in the introduction of a third curve fitting parameter γω. γω is determined by trial and error and its value is the one that best consolidates the data along a single straight line on the log-log plot of da/dN versus ΔK . It is possible that no value of γω can be found, and in this situation the Walker equation cannot be used. If the value of γω, is qua1 to one. ΔK equals ΔK which indicates that the stress ratio has no effect on the data. In summary, the Walker law is a modification of the Paris law that accounts for the stress ratio effect at the expense of introducing a third curve fitting parameter.

Forman Mode1: Walker improved the Paris model by taking account of the stress ratio, neither model could account for the instability of the crack growth when the stress intensity factor approaches its critical value. Forman improved the Walker model by suggesting a new model, which is capable of describing region III of the fatigue rate curve and includes the stress ratio effect. The Forman law is given by this mathematical relationship:

$$da/dN = {}^{C}{}_{F}\,(\Delta K)^{m}{}_{y}/(1-R)Kc-\Delta K = {}^{C}{}_{F}\,(\Delta K)^{m}{}_{y}/(1-R)(Kc-K_{max})$$

Where, Kc is the fracture toughness for the material and thickness of interest. Equation indicates that as Kmax. Approaches Kc and then da/dN tends to infinity. Therefore, the Forman equation is capable of representing stable intermediate growth (region II) and the accelerated growth rates (region III).

Stress intensity factor and crack propagation: The Stress Intensity Factor (SIF) is one the most important parameters in fracture mechanics analysis. It would, define sufficiently the stress field close to the crack tip and provide fundamental information of how the crack is going to propagate. In this study, the displacement extrapolation method was employed to, predict the crack propagation trajectory and to, calculate the stress intensity factors. In addition, for further details about this method[1].

Mesh generation and adaptive refinement: In this study, the unstructured triangular mesh is automatically generated, by employing the advancing front method[2]. The latest review of this method can be found in the previous research[3]. In order to represent the field singularity correctly at the crack tip, the singular elements have to be constructed. In our implementation, these special elements as shown in figure 1 are generated separately from the conventional ones. The singular elements have to be constructed, in order to get a proper field of singularity around the crack tip. An impressive discussion on the adaptive mesh generators was stated, by a group of Research[4]. The number of elements depends on the distributed nodes around the crack tip[2-5], which can beset by the user as shown in figure 4. Here the natural triangular quarter point elements are used instead of the collapse quadrilateral element.

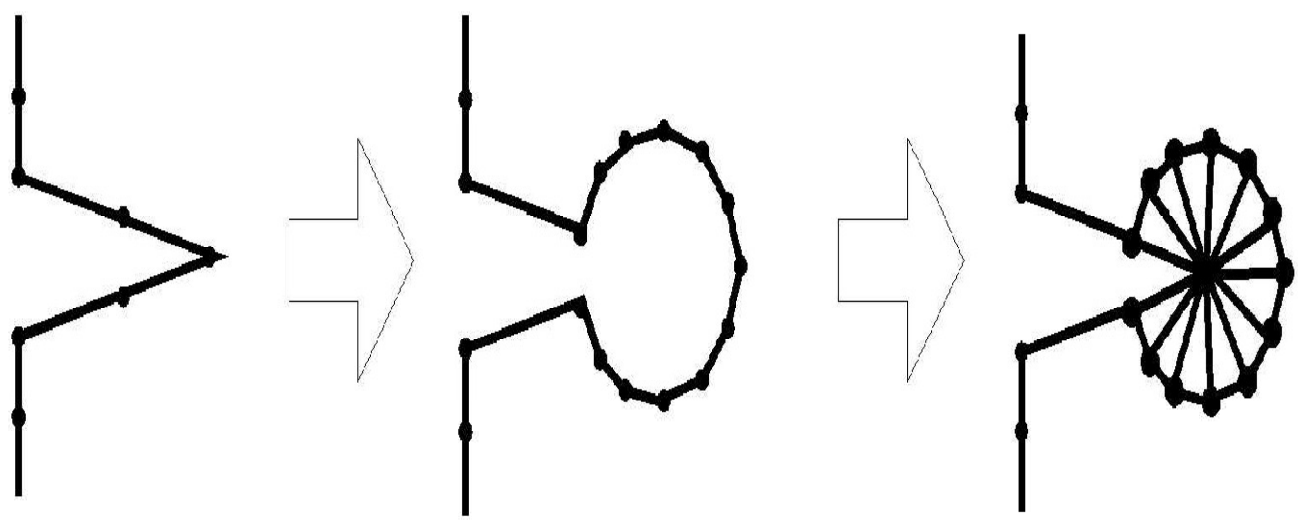


Figure-4
The cut and patch procedure of generating singular elements around a crack tip

Methodology

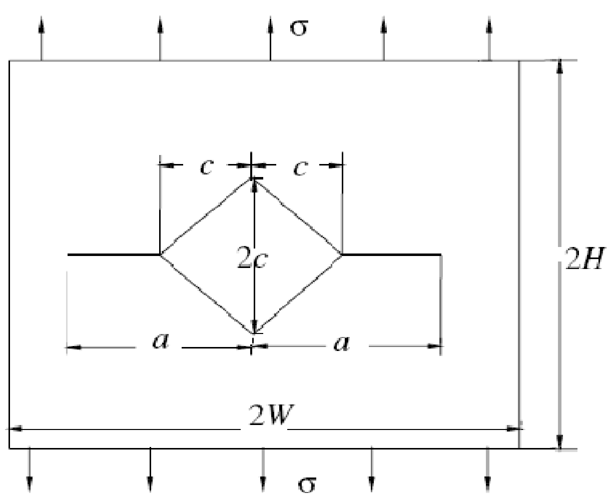
A numerical analysis of cracks emanating from a circular whole and square hole in rectangular plate intension was performed by means of the displacement discontinuity method with crack-tip elements[6]. The body force method to calculate their stress intensity factors, for a crack originating from a corner of a square hole in an infinite plate intension[4]. A mixed mode stress intensity factors of the bend specimen are computed by Finite Element Method (FEM) to determine the effects of different crack location and loading distances from the middle of the specimen[7]. Employed a statistical analysis based on weakest link theory, to describe the brittle fracture induced at singularities in ceramic materials[8]. Developed two-dimensional finite element Program, to determine SIF by using the displacement extrapolation method[9]. A new method based on the FE approach was developed for coupling the FEM and the BEM. This approach has been assessed by the evaluation of Stress Intensity Factors (SIF) using two examples of fracture mechanics, i.e., centre-cracked plate, cracks emanating from a circular hole[10]. The prediction of fatigue crack path was applied on tensile specimen with holes. Using the values of KI and KII as well as the stress were calculated, for the obtained curvilinear and reference crack path trajectories[11]. A finite element stress analysis program has been used to find values of the stress intensity factor, KI, for radial cracks at the boundary of a hole in a finite plate under tension[2]. Finite element static stress intensity factor calculations for an annular crack around a spherical inclusion (void) are presented and compared with those from approximate analytical methods[12]. Analysis of cracks emanating from a circular hole both in infinite Plate subjected to internal pressure and in rectangular plate in tension[3]. One gives solutions of a circular hole with a single edge crack and a pair of symmetrical edge cracks in a plate under tension[13].

A numerical study has been considered[11], in order to model the singularity near the crack-tip. The aim was to estimate numerically the values of SIFs using different techniques (classical finite element, enriched finite element and deformed finite element) for different specimens Central Notched (CN) and Single Edge Notched (SEN). Various numerical methods have been used to derive SIF such as Finite Difference Method (FDM), Finite Element Method (FEM) and Boundary Element Method (BEM). Among them, FEM has been widely employed for the solution of both fracture problems linear elastic and elasto-plastic. A typical and practical point matching technique, called Displacement Extrapolation Method (DEM) is chosen for the numerical analysis method. To determine the SIFs of the two well-known geometries, i.e. a rectangular plate with crack emanating from circular hole and Double Edge Notched Plate (DENT).

Results and Discussion

Rectangular plate with crack emanating from square-hole in tensile loading: The geometry was imposed by plane strain condition and edge load (σ) applied under mode I loading condition. The square hole in rectangular plate together with the final adaptive mesh is shown in figure 5. For this problem, the present study of crack emanating from a square hole in rectangular plate in tension is compared and validated, with numerical solutions of[6]. Taking in consideration the specimen parameters respectively, the ratio of (a/W) with various values of the initial crack length (a/c) together with range values of the height per width (H/W). The following cases are:

a/c = 1.02, 1.04, 1.06, 1.08, 1.1, 1.15, 1.2, 1.5, 2.0
H/W = 0.5, 0.6, 0.7, 0.8, 0.9, 1.0, 1.5, 2.0
a/W = 0.2, 0.4, 0.7



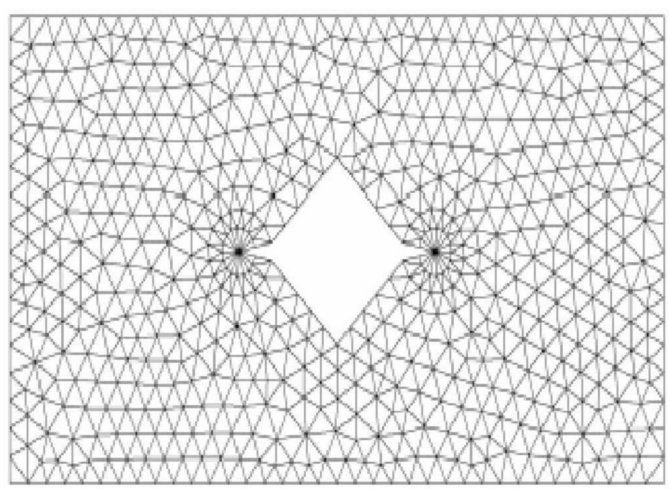
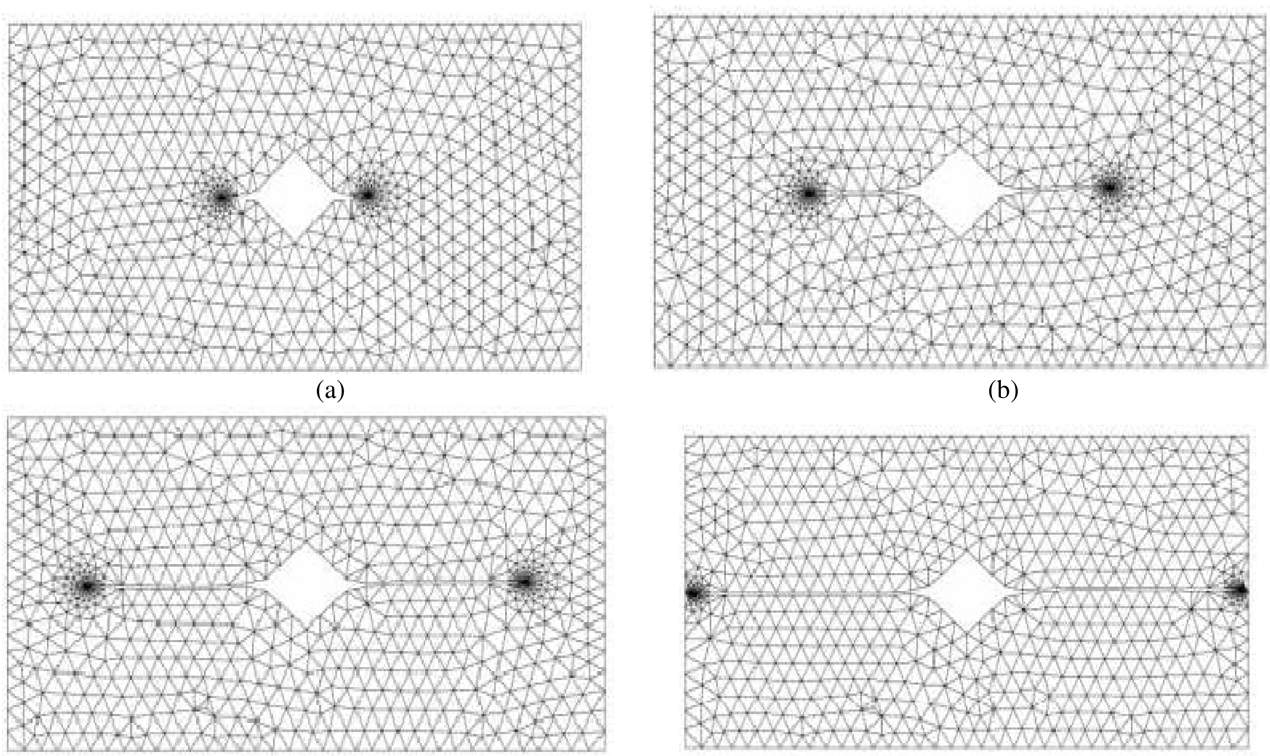


Figure-5
The square-hole in rectangular plate (a): Specimen dimension; (b): Final adaptive mesh

The SIFs (denoted by F) were normalized by s pa. Therefore, the comparison results were illustrated in Table 1, of the current study with numerical solution of[6], for the ratio values of initial crack length per width (a/W = 0.2, 0.4 and 0.7) respectively. It's found that the results, that obtained by FEM are in very good agreement with those results of a numerical solution (BEM). The predictions of the crack propagation trajectories are considered on crack emanating from circular hole of rectangular plate. F, the stresses distribution is symmetric and the higher stress is concentrated at the crack tip because of the load effect. Therefore, four steps of the crack trajectories are shown in figure 6, which propagated as it was predicted straight forward from the both crack tips. Clearly, the results show that, the crack propagated gradually to the expected path under mode I loading condition. Furthermore, in this current study, we have come out with more results than the previous relevant studies in terms of, the crack propagation trajectory and the maximum principal stress.

Table-1
Shows the comparison of the normalized SIFs of the current study with those obtained by[6] (a/W = 0.2)

Data from[6]				Present work			
H/W				H/W			
a/c	0.5	0.8	1	a/c	0.5	0.8	1
1.02	1.2576	1.1422	1.1054	1.02	1.2332	1.1226	1.1001
1.04	1.2882	1.1698	1.1324	1.04	1.271	1.1301	1.1122
1.06	1.302	1.1823	1.1448	1.06	1.2894	1.1552	1.1252
1.08	1.3086	1.1883	1.1505	1.08	1.2998	1.1561	1.1385
1.1	1.3124	1.1917	1.1541	1.1	1.3097	1.1634	1.1396
1.15	1.314	1.1931	1.1558	1.15	1.305	1.1689	1.1477
1.2	1.3108	1.1902	1.1533	1.2	1.3869	1.1878	1.1422
1.5	1.2779	1.1626	1.1276	1.5	1.2589	1.1536	1.1188
2	1.2361	1.1287	1.0963	2	1.2199	1.1189	1.0779



(a) (b) (c) (d)

Figure-6
Four steps of crack propagation trajectories for a crack emanating from square hole of rectangular plate

Conclusion

The prediction of fatigue crack path in the combined mode by the finite element method was attempted, with the purpose of serving for an optimum design of structures against fatigue failures and the fatigue crack path In the crack path prediction in combined mode, the reduction of computational efforts by the introduction of higher shape function is not so great as in other cases. A developed source code program of comprehensive finite element model has been performed in this research work with an advancing front method for crack propagation analysis. The prediction of SIF for a crack emanating from circular hole in rectangular plate and double edge notched plate were considered under mode I loading, using an adaptive mesh finite element strategy. Based, on the obtained by current studies, it's seen that are very close with other compared results. In addition, this developed source program shows that is capable of demonstrating the SIF evaluation and the crack path direction satisfactorily. Finally, the numerical finite element analysis with displacement extrapolation method, have been successfully employed for linear-elastic fracture Mechanics problems.

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