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Short Communication

L.R.S. Bianchi type II Stiff Fluid Cosmological model with Decaying Λ in General Relativity

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Abstract

Einstein's field equations with variable cosmological constants are considered in the presence of stiff fluid for LRS Bianchi type-II universe. To get the deterministic model of the universe, we have assumed a condition A=B. It is shown that the vacuum energy density Λ is positive and proportional to $1/t^2$. The model represent accelerating, shearing and non-rotating universe. The physical and geometrical behavior of these models are also discussed.

Keywords: LRS bianchi type II models, cosmological constant, stiff fluid.

Introduction

Cosmological (or vacuum energy) constant[1-3] is one of the most theoretical candidate for dark energy. Unfortunately there is a huge difference of order 10^{120} between observational ($\Lambda \sim 10^{-55}$ cm^{-2}) and the particle physics prediction value for Λ. This discrepancy is known as cosmological constant problem. There have been several ans¨atz suggested in which the ⊄ term decays with time[4-9]. Several authors have argued in favor of the dependence $\Lambda \alpha \frac{1}{t^2}$ in different context[6-8]. The relation $\Lambda \alpha \frac{1}{t^2}$ seems to play a major role in cosmology[10]. The cosmological consequences of this decay law are very attractive. This law provides reasonable solutions to the cosmological puzzles presently known. One of the motivations for introducing Λ term is to reconcile the age parameter and the density parameter of the universe with recent observational data.

Stiff fluid cosmological models create more interest in the study because for these models, the speed of light is equal to speed of sound and its governing equations have the same characteristics as those of gravitational field[11]. Barrow[12] has discussed the relevance of stiff equation of state ρ = p to the matter content of the universe in the early state of evolution of universe. Wesson[13] has investigated an exact solution of Einsteins field equation with stiff equation of state. G¨otz[14] obtained a plane symmetric solution of Einsteins field equation for stiff perfect fluid distribution and Λ.

In this paper, a new anisotropic L.R.S. (locally rotationally symmetric) Bianchi type II stiff fluid cosmological model with variable Λ has been investigated by assuming a supplementary condition A=B where A and B are metric potentials. The out line of the paper is as follows: Basic equations of the model are given in Sec. 2 and their solution in Sec. 3. We discuss the model and conclude our results in Sec.4.

Metric and Filed Equations

The metric for LRS Bianchi type II in an orthogonal frame is given by

$$ds^2 = g_{ij}\theta^i\theta^j \ , \ g_{ij} = \text{diag}(-1, 1, 1, 1) \tag{1}$$

where the Cartan bases θ^i are given by

$$\theta^0 = dt,\ \theta^1 = B\,\omega^1,\ \theta^i = A\,\omega^2,\ \theta^i = A\,\omega^3 \tag{2}$$

Here, A and B are the time-dependent metric functions. Assuming (x, y, z) as local coordinates, the differential one forms ω^i, are given by

$$\omega^1 = dy + xdz,\ \omega^2 = dz,\ \omega^3 = dx \tag{3}$$

We consider the energy-momentum tensor in the form

$$T_{ij} = (\rho + p)\, v_i v_j + p g_{ij} \tag{4}$$

where ρ is the energy density of the cosmic matter and p is its pressure, v_i is the four velocity vector such that $v_i v^i = 1$.

The Einstein's cosmological field equations are given by (with 8πG = 1 and c = 1)

$$R_{ij} - ½ R g_{ij} = - T_{ij} - \Lambda (t) g_{ij} \tag{5}$$

For the metric (1) and energy - momentum tensor (2) in co-moving system of co-ordinates, the field equation (3) yields.

$$2\frac{\ddot{A}}{A} + \frac{\dot{A}^2}{A^2} - \frac{3}{4}\frac{B^2}{A^4} = -p + \Lambda \tag{6}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} + \frac{1}{4}\frac{B^2}{A^4} = -p + \Lambda \tag{7}$$

$$2\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{A}^2}{A^2}-\frac{1}{4}\frac{B^2}{A^4}=\rho+\Lambda \tag{8}$$

where an overdot stands for the first and double overdot for second derivative with respect to t. The usual energy conservation equation $T^{j}_{i;j}=0,$ yields.

$$\left[\dot{\rho}+(\rho+p)\left(2\frac{\dot{A}}{A}+\frac{\dot{B}}{B}\right)\right]=-\dot{\Lambda}G \tag{9}$$

Let R be the average scale factor of Bianchi type -I universe i.e.

$$R^3 \quad = A^2B \tag{10}$$

The Hubble parameter H, volume expansion θ, sheer σ and deceleration parameter q are given by

$$H=\frac{\theta}{3}=\frac{\dot{R}}{R},\quad q=-1-\frac{\dot{H}}{H^2}=-\frac{-R\ddot{R}}{\dot{R}^2} \tag{11}$$

$$\theta = 2\frac{\dot{A}}{A}+\frac{\dot{B}}{B} \tag{12}$$

we consider a perfect-gas equation of state

$$p=\omega\rho, 0\le\omega\le 1 \tag{13}$$

$$\sigma^2=\frac{1}{2}\left(\sum H_i{}^2-\frac{1}{3}\theta^2\right)$$

Solution of the Field Equations

The system of equations (6)-(8) supply only three equations in five unknowns (A, B, ρ, p and Λ). Two extra equation is needed to solve the system completely.Therefore we propose a phenomenological decay law for Λ of the form[15,16].

$$\Lambda=\beta\frac{\ddot{R}}{R} \tag{14}$$

where β is constant.

As the second condition, we assume that
A=B (15)

In case of stiff fluid, equation (7) and (8), with the use of (14) and (15), reduces to

$$\frac{\ddot{A}}{A}+2\frac{\dot{A}^2}{A^2}=-\Lambda \tag{16}$$

Since A=R the above equ. reduces to

$$\frac{\ddot{R}}{R}+2\frac{\dot{R}^2}{R^2}=-\Lambda=-\beta\frac{\ddot{R}}{R} \tag{17}$$

Therefore $\frac{\ddot{R}}{R}=-\frac{2}{1+\beta}\frac{\dot{R}^2}{R^2}=K\frac{\dot{R}^2}{R^2}$ (18)

Integrate it,we get

$$R(t)=\left(\frac{at}{c}\right)^c \quad \text{where } c=\frac{1}{1-K} \tag{19}$$

By using (19) in (14) we obtain

$$\Lambda(t)=\frac{K_0}{t^2} \qquad \text{where } K_0=2\beta\frac{(1+\beta)}{(3+\beta)}$$

(20)

$$q=-\frac{R\ddot{R}}{\dot{R}^2}=-K \tag{21}$$

Since $\theta=3\frac{\dot{A}}{A}=3\frac{\dot{R}}{R}=3a\left(\frac{at}{c}\right)^{c(K-1)}$ (22)

Conclusion

In this paper we have analyzed the Bianchi type II stiff fluid cosmological models with varying Λ term of the form $\Lambda=\beta\frac{\ddot{R}}{R}$. It is observed that the Λ vanish at $t\to\infty$ and infinite at $t\to 0$. we observe that the cosmological term Λ is a decreasing function of time and it approaches a small positive value at late time. A positive cosmological constant or equivalently the negative deceleration parameter is required to solve the age parameter and density parameter. The model starts with a big bang at t = 0. The scale factors also vanish at t = 0 and hence the model has a point-type singularity at initial epoch.In this model we have q = -K this indicates that expansion rate of our model is constant .We also observe that $\frac{\sigma}{\theta}=0$ therefore model approach isotropy .

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