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Statistical Material Balance Analysis of Water Drive Reservoirs

Adeloye Olalekan Michael[1*], Ejiofor Chinonso Declan[2] and Abu Robin Nyemenim[3]
Port Harcourt, Nigeria
adeloye.olalekan@yahoo.com

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Abstract

Statistical approach of advancing material balance analysis of undersaturated reservoir under water drive mechanism was investigated in this research study. Formulated model was solved by using rotational discrimination technique's algorithm arising from the weighted steepest descent model to pertub the objective function of the formulated model.The algorithm was applied to determine the stock tank oil initial in place, cumulative production from the reservoir and water influx into the reservoir. The regressed results from the algorithm history matched production history based on stock tank oil initial in place, cumulative production and water influx into the reservoir with a deviation of 0.1456 percent for the stock tank oil initial in place.

Keywords: Water drive reservoir, Model formulation, Rotational discrimination, Regression analysis.

Introduction

The material balance equation was presented in general form by Schilthuis in 1941. In the equation, the cumulative production of reservoir fluids is equated to the combination of factors such as fluid expansion, pore volume compaction, and water influx. Due to production of reservoir fluids, the initial pressure of the reservoir decreases and a differential pressure emanates from the aquifer surrounding into the reservoir[1]. According to the principle of fluid flow in porous media, the reaction of the aquifer leads to encroachment across the original hydrocarbon–water contact. In several cases, water encroachment occurs due to hydrodynamic conditions and recharge of the formation by surface waters at an outcrop[2]. In some cases, the significance of or variation between the aquifer pore volume and the reservoir pore volume itself is not large. Therefore, water expansion in the aquifer is neglected relative to the overall energy system, and the reservoir behaves volumetrically. As a result of this, the influence of water influx can be neglected. Besides, permeability of the aquifer may be adequately low such that the required pressure differential is very large before a reasonable quantity of water can encroach into the reservoir. As a result of this, the influences of water influx can be neglected too and the material balance is assumed to be a straight line equation[3].

Havlena and Odeh analysed material balance equation as a straight line equation that easily permits application of graphical technique. In particular, extrapolation of a straight line allows the prediction of future performance of the reservoir, while the line parameters are often related to in-place volumes or water influx performance. The Campbell Plot has shown to be an incredible diagnostic tool for differentiating between depletion-drive gas reservoirs and reservoir production under a water drive effect[1].

A major inherent assumption in the deduction of the classical oil material balance equation is that reservoir's solution gas remains as gas through the separator. This assumption is true with black oil, but the solution gas from the volatile oil condenses to liquid in the separator. Thus, the solution gas from a volatile oil exhibits retrograde behaviour similar to the behaviour of vapour condensing to liquid due to reduction in pressure. This is an anomalous behaviour in the sense that basic science teaches that fluid tends to ideal gas behaviour with decline in pressure. To properly perform material balance analysis on volatile oil below bubble-point, reservoir engineer performs a compositional material balance to account for production of retrograde condensate as well as oil from the reservoir[4].

When calculating water influx, the properties of aquifer such as thickness, permeability, compressibility, radius, porosity and angle of encroachment are difficult to deduce since wells are normally not drilled into the aquifer. When these properties cannot be accurately defined, oil initial in-place is hard to evaluate uniquely because the material balance of water drive reservoir is an inverse problem. The inverse nature of water drive reservoir problem stems from the nonlinear character of water drive material balance equation. To resolve this difficulty, Havlena and Odeh replaced the material balance equation by a linear function, with a slope equal to aquifer constant and an intercept equal to stock tank oil initial in place. This technique simplified mathematics, but failed to preserve the physical meaning of material balance parameters and its iteration procedure. Thus, this study helps to resolve or tackle these problems while ensuring reasonable and accurate predictions[5].

The efficiency of material balance analysis is dependent on the accuracy of data available and the extent to which the underlying assumptions are made[3]. The material balance equation is a zero-dimensional model. This implies that the material balance is a tank model and does not account for the geometry of the reservoir, drainage areas, the position and orientation of the wells. Also, the regional variations of the rock and fluid properties are not considered. It assumes that the PVT data applied in the expression are determined with the same gas-liberation process that is active in the reservoir. The material balance equation possesses high degree of inaccuracies of the measured reservoir pressure, and the model fails when decrease in reservoir pressure is not feasible, such as in pressure-maintenance operation[6].

This research study therefore resolves these difficulties by developing a statistical material balance simulator that would predict oil initial in place and reservoir aquifer properties under water drive. This is achieved by defining the properties of reservoir fluid using PVT models, evaluating volume of water influx into the reservoir from the aquifer using Van Everdingen and Hurst aquifer model and developing an inverse statistical method to estimate stock-tank initial oil in place and aquifer properties from material balance.

Materials and Methods

The technique of material balance analysis proposed by this research study follows the flowchart given below.

Model Formulation: The least-square minimization problem objective function $F(c)$ is expressed as

$$F(c) = r^T r \quad (1)$$

Where the vector $c = [c_1, c_2, c_3, \cdots, c_N]$.

The residue expresses the relative variation between the production history (p, N_p) and the corresponding material balance predicted cumulative production $\hat{N}_p(p, c)$.

$$r_i = \frac{w_i \left[\hat{N}_p(p,c)_i - N_p(p)_i\right]}{N_p(p)_i}, \quad i = 1, 2, 3, \cdots, M \quad (2)$$

The objective function $F(c)$ is also expressed as the summation of squares of the residues as:

$$F(c) = \sum_{i=1}^{M} r_i^2 \quad (3)$$

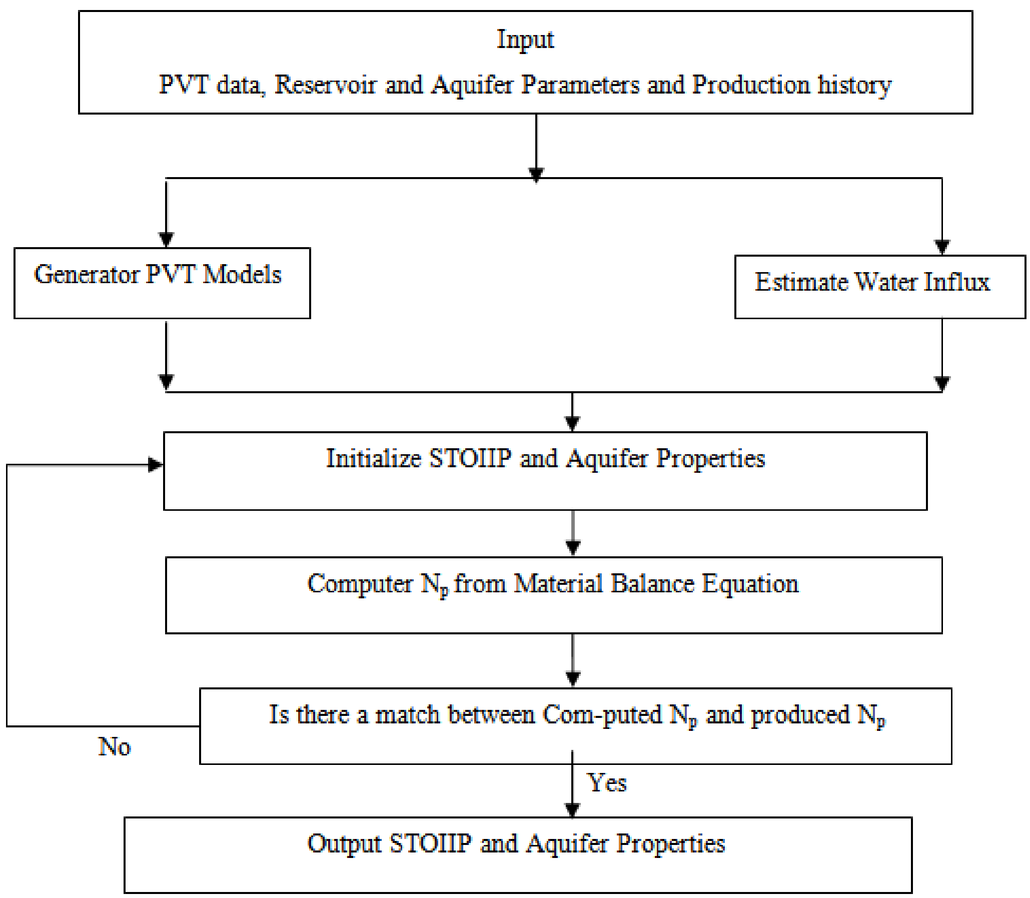

Figure-1
Material Balance Analysis flowchart

Usually the predicted cumulative production and the objective function are non-linear function of regression parameters, and as such an iterative process is required.

At iteration *k*, the base point location in the parameter space is defined by c^k. The iteration step is repeated until it converges, that is, $\left[F\left(c^{k+1}\right)-F\left(c^k\right)\right]/F\left(c^k\right)<\varepsilon$, where $\varepsilon=10^{-6}$.
The procedure is also terminated for other reasons, for instance, when the elements of $\Delta c^k = c^{k+1}-c^k$ are very small.
If it is assumed that $F\left(c^k\right)$ is continuous, and that its first and second derivatives exist, a minimum of $F\left(c^k\right)$ can only be found at a stationary point.

$$\frac{\partial F\left(c^k\right)}{\partial c_j}=0 \qquad j=1,\,2,\,\cdots,\,N \tag{4}$$

$F\left(c^k\right)$ is expressed to Taylor series around the *k*th iteration c^k and by ignoring third and higher order terms.

$$F\left(c^k+\Delta c^k\right)=F\left(c^k\right)+\Delta c^k\cdot\nabla F\left(c^k\right)+0.5\left(\Delta c^k\right)^2\cdot\nabla^2F\left(c^k\right) \tag{5}$$

The first and second derivatives of *F* are described as follows:

$$\nabla F\left(c^k\right)=2J\left(c^k\right)^T r\left(c^k\right) \tag{6}$$

$$\nabla^2 F\left(c^k\right)=2\left[J\left(c^k\right)^T J\left(c^k\right)+Q\left(c^k\right)\right] \tag{7}$$

Where the elements of the Jacobian matrix are

$$J\left(c^k\right)_{ij}=\frac{\partial\, r\left(c^k\right)_i}{\partial\, c_j} \tag{8}$$

Matrix Q is defined from

$$Q\left(c^k\right)=\sum_{i=1}^{M} r_i\left(c^k\right)\cdot\nabla^2 r_i\left(c^k\right) \tag{9}$$

The elements of ***Q*** are difficult to obtain, and since these are assumed to be small, ***Q*** is neglected.

Gauss-Newton Model: The Gauss-Newton model always assumes ***Q*** to be neglected. The least-squares problem is then analysed by the expression

$$\left(J^T J\right)\Delta c^k = J^T r \tag{10}$$

Since the Hessian matrix is approximated by first derivatives of the residuals and simplifies to $H=J^T J$, the solution to Equation 10 is expressed as:

$$\Delta c^k=-H^{-1}J^T r \tag{11}$$

The important factor for a minimum to exist is that *H* is positive definite, that is, all eigenvalues of *H* are positive. When some or all eigenvalues of *H* are less than zero, the parameter space is a saddle point or a maximum. When Hessian matrix is singular or nearly so, its inverse does not exist, and Equation 11 has no solution. Base on this, the Hessian matrix may have one or more eigenvalues equal to zero. Even when the eigenvalues tends to zero, the Hessian matrix approaches singularity. This leads to round-off errors in the inversion of Hessian matrix, and results in unrealistically large values for Δc^k.

The algorithm called the QR decomposition with Gram-Schmidt orthogonalization procedure yields a numerically stable solution to the linear Equation-11.

Steepest Descent Model: A steepest descent model with solution vector given by

$$\Delta c^k=-J^T r \tag{12}$$

Always guarantees the property of the truncation convergence, that is, reduction of the summation of squares of the residuals. But the convergence of this model may not be as fast as the Gauss-Newton model when the parameters are near to their maximal values. The Gauss-Newton model tends not to converge when the solution at the current iteration point is far from the solution and the objective function is large. In this situation, the optimal steepest descent model is deduced.

Rotational Discrimination: The technique of rotational discrimination arises from the weighted steepest descent model and was proposed by Law and Fariss[7]. Transformation of Δc^k into a new coordinate system may be achieved by transforming the Hessian matrix into a diagonal form such that

$$S^T H\, S = D \tag{13}$$

$D=\left[d_{11},\cdots d_{ii},\cdots d_{NN}\right]$.

The matrix ***S*** contains column vector V_i, $i=1,\,2,\,\cdots,\,N$ which are orthogonal to the remaining vectors V_j, $i\neq j$.

Since $S^T S = I$
Equation 11 becomes

$$S^T H\, S\cdot S^T\Delta c^k=-S^T\left(J^T r\right) \tag{14}$$

When: S^T has been pre-multiplied on both sides.

Equation 14 in closed pattern

$$DY=P \tag{15}$$

Where: $Y=S^T\Delta c^k$ (16)

And $P=-S^T\left(J^T r\right)$ (17)

The solution to Equation 15 is simply

$$y_i=-1/d_{ii}\cdot p_i \tag{18}$$

and y_i may be changed back into a Δc^k vector as

$$\Delta c^k = SY \tag{19}$$

The algorithm for implementing the rotational discrimination model is outline thus: At iteration *k*, a base point c^k is defined. Also, an optimum allowable distance factor is set, example, $y_h = 0.2$, Calculate $F\left(c^k\right)$ and the forward differences, and set up the Jacobian and Hessian matrix, Determine matrix *D* and *S* so that d_{ii} are given in descending order, Calculate y_i, $i = 1, \cdots, N$ from Equation 3.18 until one of the following occurs: $i = N$, b $d_{ii} < 0$, c. $|y_i| > y_h$.

If step 4 is terminated at $i = j$ for reason b, set $y_i = 0$, $i = 1, \cdots, N$, and continue from step 7. If step 4 is terminated for reason c at $i = M$ then

$$y_M = -y_h \operatorname{Sin}(p_M) \quad (20)$$

p_M is defined by Equation 17

Parameters are determined as a result of the weighted steepest descent technique

$$y_i = y_M s_i p_i / p_M \quad (21)$$

Scale factor is defined as $s_i = A^r$ and

$$r = 1 - \exp\left(\frac{-0.5}{\ln A} \cdot \ln \frac{d_{MM}}{d_{ii}}\right) \quad (22)$$

If $|y_i| > y_h$ when calculated from Equation 21, it should be truncated so that $|y_i| = y_h$

Conversion of *Y* into the original x-coordinate system with equation 19 and update the parameters of regression

$$c^{k+1} = c^k + \Delta c^k \quad (23)$$

Scaling: In statistical material balance analysis, some of the points of the solution vectors are too large such as oil initial-in place), while others are too small such as compressibilities. To this extent, some members of the Hessian matrix are too large while others are too small. This may indeed mislead the direction and step range of the solution. Thus, an effective regression algorithm requires that the regression paramters are properly scaled. Commonly, a regression parameter can be scalled with the equation below

$$s_i = \frac{c_{i\,\max} - c_i}{c_{i\,\max} - c_{i\,\min}} \quad (24)$$

Numerical Differentiation: The dependence of the derivatives of the objective function on the parameters of regression are normally determined by numerical differentiation, such as the central differences. This is due to its complexity or almost impossibility in determining exact analytical derivatives. Selecting a proper perturbation size may be troublesome, and can influence the solutions. If the deviation is too small, round-off errors will overshadow the influence of the perturbation, thereby forcing the algorithm to abandon the iteration premature. However, if the deviation is too large, the Hessian matrix at that point will be incorrect. Agarwal et al. proposed that a deviation of one percent in the parameters of regression is adequate to compute derivatives by numerical differentiation. In this research, 0.1 percent of regression parameters were used[8].

Error Analysis: Specific problems arise in the application of material balance technique due to the inherent correlation between rock and fluid properties. Since most fluids expand as pressure is lowered, there is need to compensate for pressure dependent model errors by combining the effect in the fluid expansion terms.

Although statistical tests can sometimes show a modeling inadequacy, there is no alternative to engineering judgment, and only by fully understanding the physical properties and geological setting of the reservoir, can such modeling errors normally be identified[8].

Once the optimal curve-fit parameters are determined, parameter statistics are determined for the converged result using the weight values. The weight value equals the mean square measurement of error.

$$w_i = \sqrt{\frac{\left(N_p - \hat{N}_p\right)^T \left(N_p - \hat{N}_p\right)}{m - n + 1}} \quad (25)$$

The covariance matrix is then computed from

$$COV = \left[J^T w J\right]^{-1} \quad (26)$$

and the asymptotic standard parameter error is given as

$$\sigma_p = \sqrt{\operatorname{diag}\left[J^T w J\right]^{-1}} \quad (27)$$

The asymptotic standard parameter error is a degree of how unexplained variability in the data leads to variability in the parameters, and is essentially an error measure for the parameters. The standard error of the fit is shown below

$$\sigma_{N_p} = \sqrt{\operatorname{diag}\left[J^T w J\right]^{-1} J^T} \quad (28)$$

The standard error of the fit indicates how parameter variability affects the curve-fit variability. The asymptotic standard prediction error reflects the standard error of the fit as well as the mean square measurement error.

$$\sigma_{\hat{N}_p} = \sqrt{\sigma_{N_p}^2 + \operatorname{diag}\left[J^T w J\right]^{-1} J^T} \quad (29)$$

Results and Discussion

The procedure developed by this research study is applied to analyze an undersaturated reservoir under water drive mechanism. The PVT report of the reservoir is shown below

Table-1
Fluid PVT Properties for Field Data

Pressure (Psia)	**Solution GOR (SCF/STB)**	**Oil Formation Volume Factor (RB/STB)**	**Gas Formation Volume Factor (RB/SCF)**
4000	500	1.29111	0.000824
3885.64	500	1.29213	0.000841
3836.75	500	1.29259	0.000849
3762.57	500	1.29331	0.000862
3705.21	500	1.29389	0.000872
3655.34	500	1.2944	0.000881
3707.56	500	1.29386	0.000872
3647.76	500	1.29448	0.000883
3602.54	500	1.29496	0.000892
3565.38	500	1.29537	0.000899
3531.51	500	1.29575	0.000906
3502.16	500	1.29608	0.000912
3471.13	500	1.29644	0.000914
3442.89	500	1.29677	0.000925

Deterministic approach of reserve estimation was applied in deducing the initial oil in-place as 206MMSTB from representative parameters of the reservoir. The STOIIP and the reservoir properties are shown below in Table-2.

For $r_D = 4$ and based on reservoir pressure history (Table 4) in combination with the reservoir and aquifer properties, the initial estimate of water influx into the reservoir using the unsteady state theory of Van Everdingen and Hurst model was calculated by the studies software as shown in Table-5.

Table-2
Reservoir Input for Field Data

Parameters	Value
STOIIP	206 MMSTB
Rock compressibility	3.3 E-06 /Psi
Porosity	0.23 Fraction
Gas cap size	0
Reservoir radius	2500 Feet
Connate water saturation	0.15 Fraction
Water formation volume factor	1.045 RB/STB
Water viscosity	0.2344 CP
Water compressibility factor	3.44E-06/Psi

Based on seismic and geological evidence, the aquifer properties are shown in Table 3, and this aquifer properties are also inferred from the seismic and geological evidence derived from the reservoir.

Table-3
Aquifer input for field data

Parameters	Value
Aquifer-Reservoir ratio	4
Aquifer thickness	250 Feet
Aquifer permeability	4 milli Darcy
Encroachment angle	245.497 Degree

Once the initialization is completed, the material balance function is regressed to match production history with the following output as given in Table-6.

The nature of the resulting match is highlighted in Table 7 and shown in Figure 2.

Table-4
Production History for Field Data

Time (Days)	Reservoir pressure (Psia)	Cum. oil production (MMSTB)	Cum. gas oil ratio (SCF/STB)	Cum. Water production (MMSTB)	Cum. water injection (MMSTB)	Cum. gas injection (MMSCF)
0	4000	0	0	0	0	0
31	3885.64	0.356222	500.000	0	0	0
59	3836.75	0.586151	501.150	0	0	0
90	3762.57	0.927019	499.990	0	0	0
120	3705.21	1.249420	499.990	0	0	0
151	3655.34	1.576490	500.000	0	0	0
181	3707.56	1.665470	500.000	0	0	0
212	3647.76	1.894020	500.002	0	0	0
243	3602.54	2.205270	499.990	0	0	0
273	3565.38	2.501730	500.002	0	0	0
304	3531.51	2.803950	500.002	0	0	0
334	3502.16	3.092820	500.000	0	0	0
365	3471.13	3.398310	500.001	0	0	0

Table-5
Initial Estimate of Water Influx at $r_D = 4$

Time (Days)	Pressure (Psia)	Delta P	Dimensionless Time	Dimensionless Water Influx	Water Influx (MMSTB)
0	4000	57.180	0	0	0
31	3885.64	81.625	0.3461669374	0.8227413208	0.0869538858
59	3836.75	61.535	0.6588338486	1.2110399743	0.2521199026
90	3762.57	65.770	1.0005000786	1.5732113786	0.4425564942
120	3705.21	53.615	1.3400010480	1.8874748970	0.6745922703
151	3655.34	-1.175	1.6861679854	2.1881345993	0.9237101959
181	3707.56	-6.210	2.0211682474	2.4623422919	1.1145154359
212	3667.76	52.510	2.3673351848	2.7131653441	1.2803979417
243	3602.54	51.190	2.7135021222	2.9629379480	1.5181269782
273	3565.38	35.515	3.0485023842	3.1919355195	1.7823618101

Table-6
Regressed STOIIP and Aquifer Properties for Field Data

Parameters	Initial Value	Regressed Value
STOIIP	206	209.054379997974
Aquifer-Reservoir Ratio	4	3.99980016531246
Rock Compressibility	3.3E-06	3.31448961305514
Aquifer Thickness	250	252.136472371549
Aquifer Permeability	4	4.01209269203304
Porosity	0.23	0.22930478256111
Encroachment Angle	245.497	247.095331362132

Table-7
Regressed Output for Field Data

Time (Days)	Reservoir Pressure (Psia)	Cumulative Oil Production (MMSTB)	Regressed Oil Production (MMSTB)	Estimated Water Influx (MMSTB)	Regressed Water Influx (MMSTB)
0	4000	0	0	0	0
31	3885.64	0.356222	0.3413807861	0.0869538858	0.0886614266
59	3836.75	0.586151	0.5920507594	0.2521199026	0.2570819627
90	3762.57	0.927019	0.9282360790	0.4425564942	0.4512877397
120	3705.21	1.249420	1.2585581815	0.6745922703	0.6879253521
151	3655.34	1.576490	1.5838077634	0.9237101996	0.9419970534
181	3707.56	1.665470	1.5992803512	1.1145154359	1.1366252572
212	3667.76	1.894020	1.8327600249	1.2803979417	1.3058266038
243	3602.54	2.205270	2.1915838768	1.5181269782	1.5482627670
273	3565.38	2.501730	2.5001979609	1.7823618101	1.8177162939
304	3531.51	2.803950	2.8021086887	2.0483340424	2.0889251713

Conclusion

A statistical material balance technique was presented in this research study to give significant insight into reservoir performance and evaluation when under water drive. It showed that the application of a more extended material balance model allowed the evaluation of more complex reservoirs. It is worthy to note that when more complexities are involved, the common graphical techniques employed in the past cannot be relied upon. In such situation, the application of standard statistical methods can allow the implementation of material balance technique in cases where graphical methods are impractical.

The research study used the technique of rotational discrimination arises from the weighted steepest descent model to pertub the objective function in model formulation to determine the stochastic gradient. The damping factor was observed to most influence the convergence of the algorithm, especially during early iterations of history matching. The influences that control the direction and step range of the statistical algorithm were controlled using special considerations such as scaling and numerical differentiation.

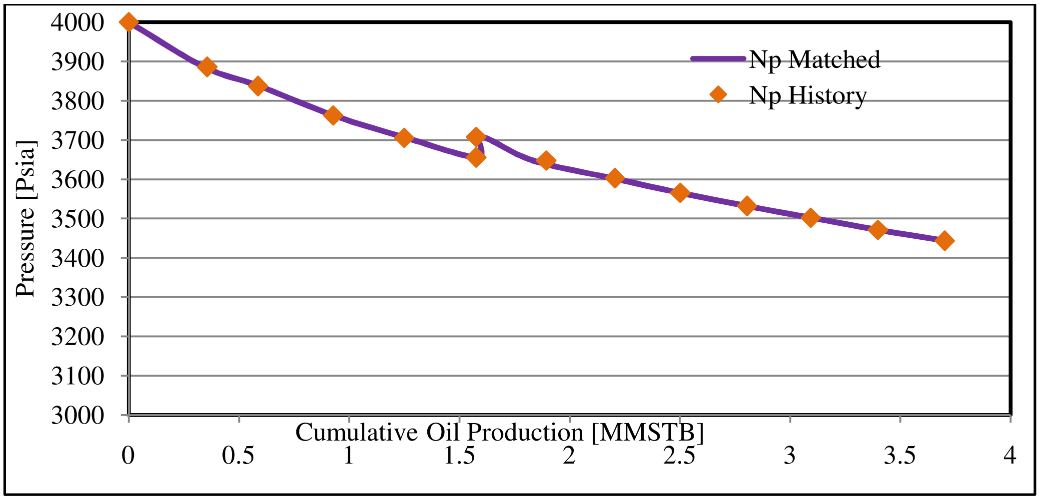

Figure-2
Material Balance Matching for Field Data

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