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To write some parameters related to photon - a note

Rampada Misra
Ex-Reader in Physics, P.K.College, Contai, West Bengal, India
ramapadamisra@gmail.com

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Abstract

Photon is considered to have a rotation over and above its linear motion. The axis of rotation, the sense of rotation and the direction of propagation of photon may, assumed to, be arbitrarily chosen. The effect of these on fundamental matrix $(\bar{g}_{ij})$ where $(i, j = x, y, z, t)$, *the elements* $\bar{g}^{ij}$, *the curvature tensor* R_{ij}, *the scalar curvature tensor R and the energy-momentum tensor* T_{ij} *has been mentioned in this work. Here, these parameters have been obtained in a short cut way only by simple consideration of some relations and imposing certain conditions on them. The expressions obtained are compared with those calculated rigorously and found to be the same.*

Keywords: Photonic system, axis of rotation, sense of rotation, direction of propagation.

Introduction

Photons consist of a small mass concentrated in a ring which is rotating about an axis along which it has a linear velocity (c)[1-3]. Hence, over and above the linear velocity of the photon there is a rotation of it with angular velocity (ω, say) about the axis of rotation[4-7].

Now, the effect of change of i. axis of rotation (X, Y or Z- axis), ii. the sense of rotation (clockwise or anti-clockwise) and iii. the direction of propagation (positive or negative direction of the co-ordinate axis of propagation) could be studied as done in the previous paper[8]. It has been shown there that above three cases have effects on transformation of co-ordinates from photonic system to the frame of observer. But, there is no effect of case iii on $(\bar{g}_{ij}), \bar{g}^{ij}, R_{ij}, R$ and T_{ij}. Again, the effect of the sense of rotation on $(\bar{g}_{ij})$ would affect the metric for the solution of Einstein's equation for photonic system. But, it has no effect on $\bar{g}^{ij}, R_{ij}, R$ and T_{ij}. Thus, these parameters are dependent, only, on the change of axis of rotation of the photon which axis could be arbitrarily chosen. It is to note that these parameters are those of a photonic system as observed by an observer in a rest frame. These parameters have already been calculated rigorously and presented properly[8].

In the present dissertation trial would be made to obtain the expressions for these parameters with different axes of rotation in a short cut way with the help of some assumed expressions imposing certain boundary conditions. Mention must be made to the fact that one could verify the expressions shown here by rigorous calculations of the same.

To Find the Parameters

Following are the procedures used for finding $(\bar{g}_{ij}), \bar{g}^{ij}, R_{ij}, R$ and T_{ij} when X, Y or Z- axis is taken as the axis of rotation. To have the expressions of the above parameters let us assume a general expression

$$X = \frac{\omega^{\alpha}}{kc^{\alpha}}\left[\frac{\omega^2}{c^2}(\bar{x}^{\alpha} + \bar{y}^{2} + \bar{z}^{\alpha}) - c^{\alpha}\right] \qquad (1)$$

For different parameters some appropriate conditions are to be applied. We shall try to write the afore-said parameters one by one.

To write the expression for $(\bar{g}_{ij})$: In the above matrix $\bar{g}_{ii}$ where $(i = x, y, z)$, in general, would be unity. All other elements solely dependent on space co-ordinates would be zero. $\bar{g}_{it}$ may not be zero.

To write the entire matrix let $X = \bar{g}_{it} = \bar{g}_{ti}$. Then we shall put $kc^{\alpha} = 1$ and $\frac{\omega^2}{c^2} = 1$ in (1) so that we obtain

$$\bar{g}_{it} = \omega^{\alpha}(\bar{x}^{\alpha} + \bar{y}^{\alpha} + \bar{z}^{\alpha}) - c^{\alpha} \qquad (2)$$

It is to be noted that when $i = x, y, z$ then $\alpha = 1$ and $c = 0$ in (2) whereas for $i = t, \alpha = 2$ and $c \neq 0$.

Now, if i be the axis of rotation then the matrix will not contain $\bar{i}$ and $\bar{g}_{it}$ would be zero. Also, in $\bar{g}_{jt}(j = x, y, z; j \neq i)$ j would be absent. The non-zero $\bar{g}_{it}(= \bar{g}_{ti})$ elements appearing in the matrix prior to the other will have $\pm$ sign attached to it whereas the sign would be $\mp$ for the later one. The upper sign is for clockwise whereas the lower one is for anti-clockwise rotation of photon.

Again, rigorous calculations could show that $\bar{g}^{ii}, R_{ii}, R$ and T_{ii} will not be affected for assumed clockwise or anti-clockwise rotations of photon.

To write expression for $\bar{g}^{ii}$: Let X represent $\bar{g}^{ii}$. Then, $\frac{\omega^{\alpha}}{kc^{\alpha}}$ outside the bracket and c^{α} within the bracket are to be taken respectively as -1 and $+1$ whereas $\alpha = 2$. Thus, we obtain from (1)

$$\bar{g}^{ii} = -\frac{\omega^2(\bar{x}^2 + \bar{y}^2 + \bar{z}^2)}{c^2} + 1 \tag{3}$$

To find $\bar{g}^{xx}, \bar{g}^{yy}$ and $\bar{g}^{zz}$ we proceed as follows.

If i be the axis of rotation then $\bar{g}^{ii} = 1$ and there would be no $\bar{i}$ in other two expressions. Moreover, that co-ordinate in the expression would be zero which appears as superscript of $\bar{g}$. It is to be mentioned that for all the cases $\bar{g}^{tt} = -\frac{1}{c^2}$.

To write the expressions for R_{ii} and R: Let X in (1) represents the Riemann-Christofell tensor R_{ii} then we put $k = 1, \alpha = 2$ and $c^{\alpha} = 1$ in (1) so that

$$R_{ii} = \frac{\omega^2}{c^2}\left[\frac{\omega^2(\bar{x}^2 + \bar{y}^2 + \bar{z}^2)}{c^2} - 1\right] \tag{4}$$

when R_{tt} and R should be written respectively as

$$R_{tt} = -\frac{2\omega^6 \bar{x}^2 \bar{y}^2 \bar{z}^2}{c^4} + 2\omega^2 \tag{5}$$

and $$R = \frac{2\omega^4(\bar{x}^2 + \bar{y}^2 + \bar{z}^2)}{c^4} - 4\frac{\omega^2}{c^2} \tag{6}$$

Let us proceed as stated below.

For rotation about $i(= x, y, z)$ axis $R_{ii} = 0$ and $\bar{i}$ would disappear from all the expressions (4), (5) and (6). Moreover, only that co-ordinate will appear in the expression of other elements of R_{ii} which is present as suffix in it.

To write the expressions for T_{ii}: Let $T_{ii}(i = x, y, z)$ and T_{tt} represent the elements of energy-momentum tensor. Let $X = T_{ii}$ in (1) with $\alpha = 2$ and $c^{\alpha} = 1$ so that

$$T_{ii} = \frac{\omega^2}{kc^2}\left[\frac{\omega^2(\bar{x}^2 + \bar{y}^2 + \bar{z}^2)}{c^2} - 1\right] \tag{7}$$

Whereas: we write

$$T_{tt} = \frac{\omega^4}{kc^2}\left[\frac{4\omega^2\bar{x}^2\bar{y}^2\bar{z}^2}{c^2} + \frac{\omega^2(\bar{x}^4 + \bar{y}^4 + \bar{z}^4)}{c^2} - 3(\bar{x}^2 + \bar{y}^2 + \bar{z}^2)\right] \tag{8}$$

Where: k is the gravitational constant.

The elements of energy-momentum tensor for different axes of rotation could be obtained from (7) and (8) as follows.

If rotation be about i axis then $\bar{i}$ would be absent in all the expressions of T and we replace 1 by 2 in T_{ii}. In other elements the co-ordinate appearing as suffix in the expression would disappear.

Verification

Considerations are to be made for transformation of Cartesian co-ordinates from photonic system to that of an observer[10,11]. Now all the parameters mentioned earlier could be rigorously calculated for all cases and the expressions found out in previous section could be easily verified. For ease of comparison the calculated expressions for different axes of rotation are presented.

It is seen that the determinant of the fundamental matrix is $-c^2$ in all the cases.

Case I. When X axis is the axis of rotation.

$$(\overline{g}_{ij}) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & \pm\omega\overline{z} \\ 0 & 0 & 1 & \mp\omega\overline{y} \\ 0 & \pm\omega\overline{z} & \mp\omega\overline{y} & \omega^2(\overline{y}^2+\overline{z}^2)-c^2 \end{pmatrix}$$

$$\overline{g}^{xx} = 1, \overline{g}^{yy} = -\frac{\omega^2 z^2}{c^2}+1, \overline{g}^{zz} = -\frac{\omega^2 y^2}{c^2}+1, \overline{g}^{tt} = -\frac{1}{c^2}$$

$$R_{xx} = 0, R_{yy} = \frac{\omega^2}{c^2}(\frac{\omega^2\overline{y}^2}{c^2}-1), R_{zz} = \frac{\omega^2}{c^2}(\frac{\omega^2\overline{z}^2}{c^2}-1)$$

$$R_{tt} = -\frac{2\omega^6\overline{y}^2\overline{z}^2}{c^4}+2\omega^2, R = \frac{2\omega^4(\overline{y}^2+\overline{z}^2)}{c^4}-4\frac{\omega^2}{c^2}$$

$$T_{xx} = \frac{\omega^2}{kc^2}[\frac{\omega^2(\overline{y}^2+\overline{z}^2)}{c^2}-2], T_{yy} = \frac{\omega^2}{kc^2}[\frac{\omega^2\overline{z}^2}{c^2}-1],$$

$$T_{zz} = \frac{\omega^2}{kc^2}[\frac{\omega^2\overline{y}^2}{c^2}-1],$$

$$T_{tt} = \frac{\omega^4}{kc^2}[\frac{4\omega^2\overline{y}^2\overline{z}^2}{c^2}+\frac{\omega^2(\overline{y}^4+\overline{z}^4)}{c^2}-3(\overline{y}^2+\overline{z}^2)]$$

Case II. When Y axis is the axis of rotation.

$$(\overline{g}_{ij}) = \begin{pmatrix} 1 & 0 & 0 & \pm\omega\overline{z} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \mp\omega\overline{x} \\ \pm\omega\overline{z} & 0 & \mp\omega\overline{x} & \omega^2(\overline{x}^2+\overline{z}^2)-c^2 \end{pmatrix}$$

$$\overline{g}^{xx} = -\frac{\omega^2\overline{z}^2}{c^2}+1, \overline{g}^{yy} = 1, \overline{g}^{zz} = -\frac{\omega^2\overline{x}^2}{c^2}+1, \overline{g}^{tt} = -\frac{1}{c^2}$$

$$R_{xx} = \frac{\omega^2}{c^2}(\frac{\omega^2\overline{x}^2}{c^2}-1), R_{yy} = 0, R_{zz} = \frac{\omega^2}{c^2}(\frac{\omega^2\overline{z}^2}{c^2}-1)$$

$$R_{tt} = -\frac{2\omega^6\overline{x}^2\overline{z}^2}{c^4}+2\omega^2, R = \frac{2\omega^4(\overline{x}^2+\overline{z}^2)}{c^4}-4\frac{\omega^2}{c^2}$$

$$T_{xx} = \frac{\omega^2}{kc^2}[\frac{\omega^2\overline{z}^2}{c^2}-1], T_{yy} = \frac{\omega^2}{kc^2}[\frac{\omega^2(\overline{x}^2+\overline{z}^2)}{c^2}-2],$$

$$T_{zz} = \frac{\omega^2}{kc^2}[\frac{\omega^2\overline{x}^2}{c^2}-1],$$

$$T_{tt} = \frac{\omega^4}{kc^2}[\frac{4\omega^2\overline{x}^2\overline{z}^2}{c^2}+\frac{\omega^2(\overline{x}^4+\overline{z}^4)}{c^2}-3(\overline{x}^2+\overline{z}^2)]$$

Case III. When Z axis is the axis of rotation.

$$(\overline{g}_{ij}) = \begin{pmatrix} 1 & 0 & 0 & \pm\omega\overline{y} \\ 0 & 1 & 0 & \mp\omega\overline{x} \\ 0 & 0 & 1 & 0 \\ \pm\omega\overline{y} & \mp\omega\overline{x} & 0 & \omega^2(\overline{x}^2+\overline{y}^2)-c^2 \end{pmatrix}$$

$$\overline{g}^{xx} = -\frac{\omega^2\overline{y}^2}{c^2}+1, \overline{g}^{yy} = -\frac{\omega^2\overline{x}^2}{c^2}+1, \overline{g}^{zz} = 1, \overline{g}^{tt} = -\frac{1}{c^2}$$

$$R_{xx} = \frac{\omega^2}{c^2}(\frac{\omega^2\overline{x}^2}{c^2}-1), R_{yy} = \frac{\omega^2}{c^2}(\frac{\omega^2\overline{y}^2}{c^2}-1), R_{zz} = 0$$

$$R_{tt} = -\frac{2\omega^6\overline{x}^2\overline{y}^2}{c^4}+2\omega^2, R = \frac{2\omega^4(\overline{x}^2+\overline{y}^2)}{c^4}-4\frac{\omega^2}{c^2}$$

$$T_{xx} = \frac{\omega^2}{kc^2}[\frac{\omega^2\overline{y}^2}{c^2}-1], T_{yy} = \frac{\omega^2}{kc^2}[\frac{\omega^2\overline{x}^2}{c^2}-1],$$

$$T_{zz} = \frac{\omega^2}{kc^2}[\frac{\omega^2(\overline{x}^2+\overline{y}^2)}{c^2}-2],$$

$$T_{tt} = \frac{\omega^4}{kc^2}[\frac{4\omega^2\overline{x}^2\overline{y}^2}{c^2}+\frac{\omega^2(\overline{x}^4+\overline{y}^4)}{c^2}-3(\overline{x}^2+\overline{y}^2)]$$

Conclusion

It is seen that the values of the parameters which would be obtained from section 2 are same as those given in section 3 obtained from direct calculations.

Hence, it could be concluded that the considerations made in section 2 are correct.

References

1. Rocard Y. (1957). Thermodynamique. Masson & Cie, Paris, 250.
2. Raman C.V. and Bhagavantam S. (1931). Experimental Proof of the Spin of the Photon. *Indian Journal of Physics*, 6, 353-366.
3. Das M.C. and Misra R. (2012). Fundamental Way of Charge Formation and Relation between Electromagnetic Field and Gravitational Field. *International Journal of Astronomy and Astrophysics*, 2(2), 97-100.
4. Das M.C. and Misra R. (2014). Lorentz Transformation in Super System and in Super System of Photon. *International Letters of Chemistry, Physics and Astronomy,* 19(1), 8-14.
5. He L., Li H. and Li M. (2016). Optomechanical measurement of photon spin angular momentum and optical torque in integrated photonic devices. *Science advances*, 2(9), e1600485.
6. Van Enk S.J. and Nienhuis G. (1994). Spin and orbital angular momentum of photons. *EPL (Europhysics Letters)*, 25(7), 497.
7. Hawking S.W. and Ellis G.F.R. (1973). The Large Scale Structure of Space-Time. Cambridge University Press, 117.
8. Misra R. (2018). Some Studies with Photonic System. Paper Communicated to *Journal of Science, Islamic Republic of Iran.*
9. Hartle J.B. (2009). Gravity : An Introduction to Einstein's General Relativity. *Pearson Education*, 63.
10. Iyer C. and Prabhu G.M. (2007). Comparison of Two Lorentz Boosts through Spatial and Space-time Rotations. *Journal of Physical and Natural Sciences*, 1(2), 2.
11. Iyer C. and Prabhu G.M. (2007). Lorentz Transformations with Arbitrary Line of Motion. *European Journal of Physics,* 28, 183-190.