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Determination of the optimal tilt angle for a tilted solar panel in Kathmandu, Nepal using isotropic model

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Abstract

Different types of geographical and climatological parameters are necessary to evaluate the performance of a solar panel. A solar device will perform efficiently if it receives maximum solar radiation throughout the day. To achieve this, the angle of tilt of the panel should be optimized periodically. This project aims to calculate the optimal angle oftilt for any solar panel located in Kathmandu (27.7^{o}N latitude) for every month using Liu and Jordan's Isotropic Model. The monthly and the annual optimum tilt angles are calculated for a south facing tilted surface which are almost equivalent to the latitude of the site. Further, the optimum tilt angle decreases from a peak value of 28^o in January to a minimum of 20^o in July and again rises to a maximum of 28^o in December. It is found that the tilt angle should be optimized annually or monthly in order to boost the effectiveness of the sun oriented collector as opposed to setting it on the level ground which is the usual practice. However, there will be an annual loss of 0.0714% in irradiance if the optimization process is conducted annually instead of monthly.

Keywords: Parameters, solar radiation, optimum tilt angle, Isotropic Model, latitude, efficiency, irradiance.

Introduction

Solar radiation is an electromagnetic wave whose wavelength ranges from about 0.25 to 4.5 µm. It incorporates the near ultraviolet (UV), visible light and near Infra – red radiation. Its perpetual energy serves to be a benediction to humankind because of the short – lived nature of the fossil fuel[1]. For more than a century, scientists have been studying the contrasts in solar energy[2]. Solar irradiance data is required to adjudge the efficiency of a solar collector having different orientation and tilt[3]. A limitation is posed on the advancement of solar energy applications in those locations where the necessary solar energy measuring devices are in under supply. It proves to be extremely valuable to gather information regarding solar radiation for those sites where no sources suffice to gauge any meteorological information, but only the collection of geographical parameters are possible. Solar radiation models offer aid in evaluating the solar energy using various input parameters[4]. For estimating the global solar radiation for a location, various empirical models have been established which make use of the parameters like: sunshine hours, declination angle and latitude, daylight span, relative humidity, most extreme temperature, altitude and latitude[5-7].

A Photovoltaic (PV) cell is utilized to convert the solar energy incident on the surface of the earth to electric energy while it is converted into thermal energy via solar collector. The intensity of the beam falling on the solar panel depends upon the day of the year, latitude, azimuthal angle, slope or angle of tilt and the angle of incidence. This intensity can be measured by using devices called Pyranometers and Pyrheliometers which are placed alongside the solar device. For locations in the Northern hemisphere, the solar device should be oriented with its face towards the equator (or due south).

In case of developing countries like Nepal, the dependence on energy sources should be more inclined towards solar energy rather than the short – lived fossil fuel. The latitude of Nepal is 28.40^{o}N, which is quite suitable to gather sufficient solar energy all over the country as the sunlight duration, in an average, is300 days per year. According to Pant and Poudyal, the average Global Solar Radiation in Nepal ranges from 3.6 – 6.2 $KWhr.m^{-2}.day^{-1}$, the national average sunshine hour is 6.8/day and the national average solar irradiance is 4.7 $KWhr.m^{-2}.day^{-1}$[8].

A solar device will have maximum efficiency if it is facing towards the sun throughout the day. The tilt of a surface (β) plays a key role to influence the sunlight based radiation received by it[9]. So, in order to enhance the power output of the solar devices, we require a sun oriented following framework, which pursues the sun powered directions to upgrade the falling radiation on that surface.

However, because of their sophisticated mechanical parts and greater installation costs, such tracking devices don't prove to be cost – effective in context of Nepal. Alternatively, one can also incline the devices at an optimum angle on an everyday, month to month or yearly premise.

According to a study by Danny and Tony in Hong Kong, 2004, the optimal tilt angle was found to be 20^o due South[10]. Doing so, the solar panel would yield, in a year, solar energy over 1598 KWh/m^2. The outcomes hold up that when a solar collector was tilted in such a way that its tilt angle was made nearly equal to the latitude of the site, it could acquire solar energy to its fullest. Islam, Alam and Sharker found that for a 20.83^o tilted surface at Dhaka Station (23.81^oN), the annual average gain of the solar radiation was 5.977 $KWhr.m^{-2}.day^{-1}$ [11]. Ertekin, Evrendilek and Kulcu reported that the optimal tilt angle assumes greater numbers during autumn and winter and lower values in the summer[12]. When a study was done on a solar panel tilted at three different angles for six distinct azimuth angles, Gopinathan revealed that he obtained different optimum tilts for three aforementioned seasons as well[13].

In order to determine the solar irradiance - the power per unit area received by the tilted solar panel, various solar angles serve as useful parameters.

Declination Angle (δ): The solar declination angle at any time of the year is the angle made by the lines of a solar beam with the plane of the equator. The declination angle has a range of $-23.45^0<\delta<+23.45^0$ during its yearly cycle. The positive values indicate the entrance of the solar rays into the equator through the Northern hemisphere. The maximum value of δ corresponds to June 21st-22nd and the minimum value corresponds to December 20th-21st. It assumes zero value on March 22nd and September 22nd.

The approximation of the declination angle given by Cooper is as follows[14]:

$$\delta = 23.45 * \sin\left(\frac{360}{365} * (284 + n)\right) \quad (1)$$

Where: 'n' is the day number.

The values of 'n' were chosen very specifically as suggested by Klein so that the extra – terrestrial radiation will be equal to the monthly mean values[15].

Latitude (φ): It is defined as the angle made at the earth's center, between the line drawn from the point of observation and the equatorial plane. Its value ranges from $-90^\circ \le \varphi \le 90^\circ$. For the regions lying in the Northern hemisphere, it will have positive values and for those located in the Southern hemisphere, it will assume negative values.

Hour Angle (ω): The hour angle at any point is the angle made by the plane of a great circle containing the point of observation and through the geographical North and South poles with the plane of another great circle through these geographical poles and also through an external celestial body, which in this case, is the sun. The hour angle changes by 360/24 = 15 degrees every hour, given that it takes 24 hours for the earth to rotate about its geographical axes. Two equations are used to calculate the specific hour angles when various solar angles are known.

Sunset Hour Angle:

$$\omega_s = \cos^{-1}(-\tan\phi\tan\delta) \quad (2)$$

Sunset Hour angle for the mean day on a tilted surface:

$$\omega_m = \min\{\cos^{-1}(-\tan\phi\tan\delta), \cos^{-1}(-\tan(\phi-\beta)\tan\delta)\} \quad (3)$$

Table-1: Specific days of the months and their day numbers.

Months	Corresponding Day of the year (*n*)
January	17 (17th day)
February	47(16th day)
March	75(16th day)
April	105(15th day)
May	135(15th day)
June	162(11th day)
July	198(17th day)
August	228(16th day)
September	258(15th day)
October	288(15th day)
November	318(14th day)
December	344(10th day)

Tilt Angle (β): It is simply the angle made by the plane of the solar device with the horizontal ground. The tilt for a horizontally inclined surface is 0^o. Moreover, the panel receives maximum radiation for a particular value of tilt angle known as optimal tilt angle (β_o).

Determination of monthly averaged - daily solar irradiance on a tilted surface: The method for estimating the monthly averaged- daily irradiance in isotropic conditions on a south – facing panel was developed by Liu and Jordan and was described in a review paper by Klien[16].

The Monthly average daily solar radiation on a tilted surface depends upon the beam, diffuse and reflected components and is given by the following relation.

$$G_T = G_b + G_d + G_r \quad (4)$$

Where: G_T is the monthly averaged - daily Radiation on a tilted collector. G_b is the monthly averaged - daily Beam solar radiation on a tilted collector. G_d is the monthly averaged - daily Diffused solar radiation on a tilted collector. G_r is the monthly averaged - daily Reflected solar radiation on a tilted collector.

For obtaining the optimal tilt angle, equation 4 is maximized with respect to β.

$$\left(\frac{d(G_T)}{d\beta}\right)_{\beta=\beta_o} = 0 \quad (5)$$

The beam radiation on the tilted surface (G_b) in equation 4 was given by Liu and Jordan as:

$$G_b = (G - G_D)R_b \quad (6)$$

Where: G is the monthly averaged - daily Global solar radiation on a horizontally leveled collector. G_D is the monthly averaged - daily Diffused solar radiation on a horizontally levelled collector. R_b is the monthly averaged geometric factor or the ratio of the monthly averaged - daily beam radiation on the tilted collector to that on the horizontally levelled collector.

Mathematically,

$$R_b = \frac{\cos(\phi-\beta)\cos\delta\cos\omega_m + (\pi/180)\omega_m \sin(\phi-\beta)\sin\delta}{\cos\phi\cos\delta\cos\omega_s + (\pi/180)\omega_s \sin\phi\sin\delta} \quad (7)$$

The diffused irradiance on the tilted surface is

$$G_d = G_D R_d = G_D \frac{(1+\cos\beta)}{2} \quad (8)$$

Where: R_d is called diffusion factor and is obtained as the ratio of diffuse solar irradiance on a surface tilted at some angle 'β' to that on a horizontal surface.

Moreover, the irradiance for the radiation that has been reflected and which falls on the collector surface is

$$G_r = GR_r \quad (9)$$

And, $$R_r = \rho_g \frac{(1-\cos\beta)}{2} \quad (10)$$

Where: R_r is the Reflectance factor. ρ_g is the ground reflectance; generalized to be 0.2 for tropical locations by Muneer and Saluja[17].

Finally, equation 4 becomes

$$G_T = (G - G_D)R_b + \left(\frac{1+\cos\beta}{2}\right)G_D + \rho_g\left(\frac{1-\cos\beta}{2}\right)G \quad (11)$$

Methodology

The data used in this research is extracted from the NASA website. The 22 year monthly and yearly averages (July 1983 – June 2005) for Global Solar Radiation and Diffused Solar Radiation have been provided by NASA Surface Meteorology and Solar Energy (SSE) for public use. The data for these parameters are extracted for 27°N latitude and 85°E longitude – the geographical co – ordinates of Kathmandu.

After the solar declination angle (δ) and the specific hour angles (ω_s and ω_m) are calculated, the irradiance falling on a tilted collector is determined as a function of tilt angle (β) using equation 11. The values of β is discretized from 0^o (horizontal surface) to 90^0 (collector surface being perpendicular to the horizontal) and the G_T values are calculated for each month. Then, using the β vs. G_T graph, the optimal tilt angle (β_o) is determined for each month.

After calculating twelve values of the optimal tilt angle, the G_T values at β_o are compared with the G_T values at $\beta = 0^o$ i.e. for a horizontal surface and the corresponding gains are calculated using the differences in the G_T values. Furthermore, the arithmetic mean of the twelve values of β_o is calculated. The G_T values for this averaged β_o are determined for each month and are again compared with the G_T values at $\beta = 0^o$.Finally, the G_T values for the averaged β_o for each month are compared with the G_T values at β_o. Here, the losses (not the gains) are calculated for each month.

Results and discussion

G_T is calculated as a function of β for each month and the optimal tilt angles are determined as shown in Figure-1.

Variation of the optimal tilt angle (β_o): The value of β_o is in decreasing trend from January to July. From July, β_o increases to December. Therefore, the optimal tilt angle is larger during winter and smaller during summer. Moreover, the yearly averaged optimum tilt is 25.33°. This value nearly equals the latitude of Kathmandu (27.7°N).

Analysis of the results: On comparing the irradiance values on a collector tilted at monthly optimum tilt angles ($\beta = \beta_0$) with a horizontally aligned collector surface ($\beta = 0^o$), we get an annual average gain of 8.169% (Table-2); the gain being maximum in November with the value 10.932% and minimum in July with the value 4.322%. The range of the gain is (10.932 – 4.322) % = 6.610%. Hence, the efficiency of the solar collector will increase when its tilt angle is optimized monthly instead of placing it horizontally.

Furthermore, when the irradiance values on a collector tilted at yearly optimum tilt angle (β=25.33°) is compared with a horizontally aligned collector surface (β=0°), we get an annual average gain of 8.093% (Table-3). The gain becomes maximum in November with the value 10.826% and minimum in July with the value 4.077%. The range of the gain is (10.826 – 4.077) % = 6.749%. Therefore, the efficiency of the solar collector will also increase when its tilt angle is optimized yearly instead of placing it horizontally.

When the irradiance values at the yearly optimum tilt (β = 25.33°) is compared with that of $\beta = \beta_0$, there is a loss in efficiency which averages out to be 0.071%. This loss is greatest in July (0.234%) and least in April (0.000%).

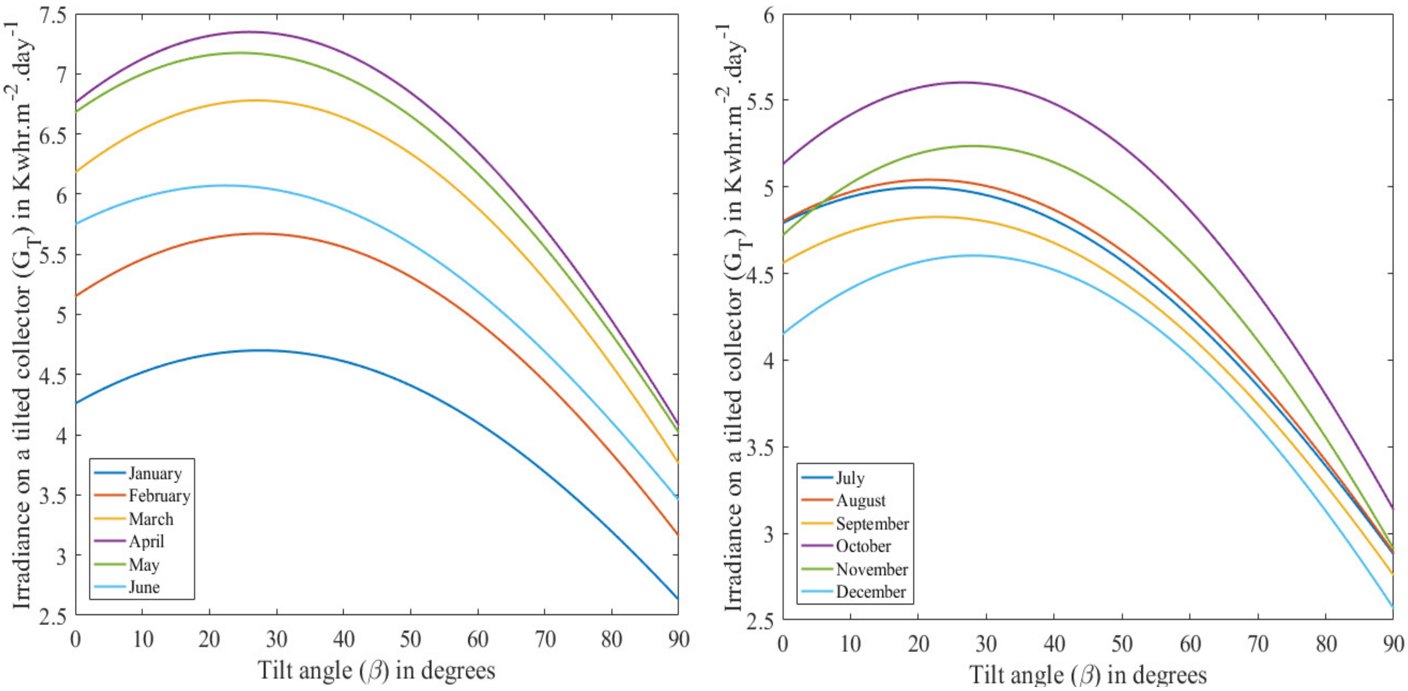

Figure-1: Graph showing the variation of G_T with β throughout the year. The first graph contains β *vs.* G_T plot for the first six months and the second shows the same plot for the last six months.

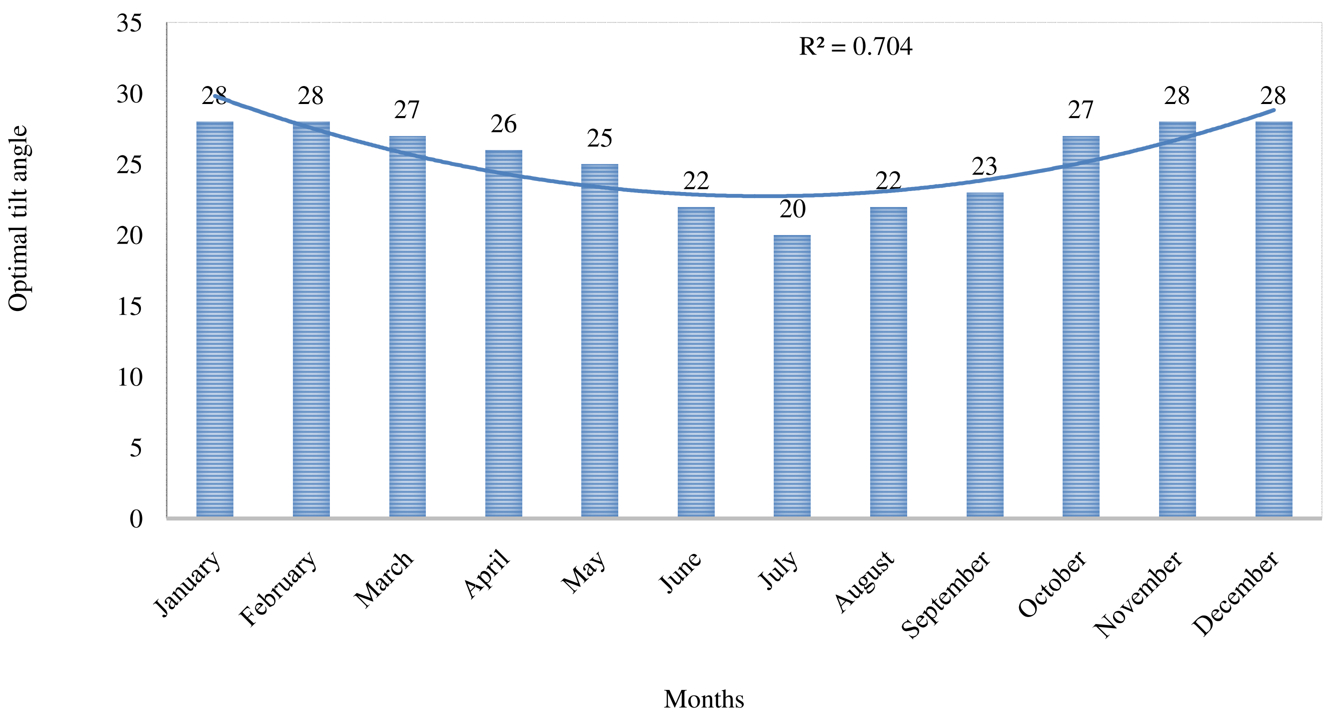

Figure-2: Variation of the optimal tilt angle (β_o) throughout the year with a polynomial fit whose R^2 = 0.704.

Table-2: Table showing the G_T values for $\beta = \beta_o$ and $\beta = 0^o$ for each month.

Months	Optimal Tilt angle (β_o)	G_T ($\beta = \beta_o$) (KWhr.m^{-2}.day^{-1})	G_T ($\beta = 0^0$) (KWhr.m^{-2}.day^{-1})	Difference	Gain (%)
January	28	4.701	4.260	0.441	10.352
February	28	5.671	5.150	0.521	10.117
March	27	6.778	6.180	0.598	9.676
April	26	7.347	6.760	0.587	8.683
May	25	7.173	6.680	0.493	7.380
June	22	6.072	5.750	0.322	5.600
July	20	4.997	4.790	0.207	4.322
August	22	5.041	4.800	0.241	5.021
September	23	4.826	4.560	0.266	5.833
October	27	5.602	5.130	0.472	9.201
November	28	5.236	4.720	0.516	10.932
December	28	4.603	4.150	0.453	10.916
Average	25.33	5.671	5.244	0.426	8.169

Table-3: Table showing the G_T values for $\beta = 25.33^o$ and $\beta = 0^o$ for each month.

Months	G_T ($\beta = 25.33^o$) (KWhr.m^{-2}.day^{-1})	G_T ($\beta = 0^0$) (KWhr.m^{-2}.day^{-1})	Difference	Gain (%)
January	4.698	4.260	0.438	10.277
February	5.669	5.150	0.519	10.068
March	6.776	6.180	0.596	9.649
April	7.347	6.760	0.587	8.683
May	7.173	6.680	0.493	7.379
June	6.066	5.750	0.316	5.501
July	4.985	4.790	0.195	4.077
August	5.034	4.800	0.234	4.879
September	4.823	4.560	0.263	5.774
October	5.601	5.130	0.471	9.185
November	5.231	4.720	0.511	10.826
December	4.599	4.150	0.449	10.817
Average	5.667	5.244	0.423	8.093

Table-4: Table showing the G_T values for β = 25.33°and $\beta = \beta_o$ for each month.

Months	Optimal Tilt angle (β_o)	G_T ($\beta = \beta_o$) (KWhr.m^{-2}.day^{-1})	G_T (β = 25.33°) (KWhr.m^{-2}.day^{-1})	Difference	Loss (%)
January	28	4.701	4.698	0.003	0.068
February	28	5.671	5.669	0.003	0.044
March	27	6.778	6.776	0.002	0.025
April	26	7.347	7.347	0.000	0.000
May	25	7.173	7.173	0.000	0.001
June	22	6.072	6.066	0.006	0.094
July	20	4.997	4.985	0.012	0.234
August	22	5.041	5.034	0.007	0.135
September	23	4.826	4.823	0.003	0.056
October	27	5.602	5.601	0.001	0.014
November	28	5.236	5.231	0.005	0.095
December	28	4.603	4.599	0.004	0.089
Average	25.33	5.671	5.667	0.004	0.071

Conclusion

Nepal is a developing landlocked country located between China and India. It receives sunlight in almost all seasons. Nepal is heavily dependent on the short – lived fossil fuel for its daily energy use. Thus, it is necessary to make a sustainable utilization and optimization of solar energy as an alternative.

Due to the sophisticated mechanical parts and high cost of installation and maintenance of the advanced solar tracking devices, it is more economical and practical to optimize the tilt angle of the installed solar collector. For this, the Liu and Jordan Isotropic Model is used, where, the irradiance on a tilted solar collector (G_T) is calculated as a function of Global Solar Radiation, Diffuse Solar Radiation, Solar Declination angle (δ), Hour Angles (ω_m and ω_s), Latitude of the site of interest (φ) and the tilt of the collector (β). Using this model, it is evident that the optimal tilt angle attains values which are very close to the latitude of Kathmandu (27.7^0). It acquires maximum value of 28° in January and decreases to a minimum of 20° July and again rises to 28° in December. Thus, β_o attains greater values in winter and lower values in summer. The yearly average value of the monthly optimum tilt angles is 25.33°. Further, when the tilt angle is optimized monthly, a yearly gain of 8.169% is achieved as compared to the horizontally aligned panel. Also, a yearly gain of 8.093% is achieved as compared to the horizontally aligned panel when the tilt angle is optimized annually. So, optimization of tilt angle increases the efficiency considerably and gives better output than a usually placed horizontal collector. Moreover, when the tilt angle is optimized yearly instead of monthly, yearly average loss of 0.0714% occurs in the output. The yearly optimization is feasible only for those locations where the human reach is less frequent, like in the rural areas. Otherwise, monthly optimization is recommended for the urban areas of Kathmandu.

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