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A Markov chain analysis of rainfall distributions in three southwestern cities of Nigeria

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Abstract

In this study, the pattern of distribution of rainfall in Lagos, Ibadan and Abeokuta was investigated using a three state Markov chain. Monthly rainfall data from these three stations for 31yrs (1984-2014) were categorized into dry, wet and rainy month. The categorization was based on an average daily rainfall of dry day (≤2.5mm), wet day (2.51mm-5mm) and rainy day (≥ 5.0mm). Relevant Statistical tests were conducted to ascertain the validity of the Markov Chain used in this work. The results from this study corresponds with the establish fact that 7months of a calendar year will be rainy or wet and 5 months will be dry in the three geographical locations. It was also observed from the results of this work that the incidence of rainy month will approximately be 4 months in Lagos and Ibadan on the long run.

Keywords: Markov chain analysis, rainfall, distribution, Southwestern cities of Nigeria.

Introduction

Climate change has become one of the most important issues plaguing the earth in its entirety. Changes in rainfall pattern and other forms of precipitation constitute one of the most critical factors determining the overall impact of climate change. One of the indicators of climate change is increase or decrease in rainfall[1]. Rainfall is much more difficult to predict than temperature but scientist can still make some statements with confidence about the future pattern of rainfall distribution. A change in statistical distribution of rainfall patterns when that change last for an extended period of time has been used by scientist to explain climate change.

Climatic condition in Nigeria is usually tropical wet and dry[2]. The pattern of weather during a year is separated into wet/rainy season and dry season. In Southern Nigeria, the wet/rainy season is from April to October and the dry season is from November to March. In northern Nigeria, the wet/rainy season is generally between May and September and the dry season is from October to April. Lately, large and extended weather and climate extremes were recorded in different parts of the country, causing significant socio-economic impacts.

A research on climatic variability in Nigeria centered on the start, recede and extent of the rainy season was conducted; it was observed that the variation of date at which rainfall recedes over the whole country is spatially coherent where as spatial coherence in variation for the date at which rainfall start is restricted to southern Nigeria[3]. A study of rainfall pattern in 28 stations within Southern and Northern Nigeria between 1970 and 2002 showed that the amount of rainfall is generally decreasing in stations that are not within the southern coastal area whereas the stations in the southern coastal area are experiencing increasing rainfall[4]. Data from 9 stations in Niger/Delta region was used to investigate seasonality in rainfall distribution; the results from the study revealed that there is a significant seasonal pattern in the study of seasonality in rainfall distribution between February/March and November and a short dry season between December and January/February[5]. An investigation on the prospective problems of flooding in Lagos showed that the state capital (Ikeja) is susceptible to intense rainfall that could develop into flooding[6]. Climatic and statistical analyses were used to investigate two extreme events, flood and drought using data covering four decades[7]. The results showed variations in rainfall across the various climatic zones in the country[7]. In particular, flood and drought episodes were identified[7]. It was further noted that these variations in rainfall usually showed completely different patterns within the same climatic zones and that stations located relatively close to one another exhibited different patterns of rainfall[7]. A study of variation in rainfall data from 1971-2000 in 25 Nigeria meteorological stations was conducted in 2014[8]. The results from most station studied show proofs of substantial upsurge in rainfall anomaly[8]. A work on seasonality; long range dependency and time trend in the monthly rainfall data of 37 stations across Nigeria from 1981-2013 was done[9]. It was observed that the trend and persistence of long memory are fairly distributed across the country such that no particular zone can be identified with extreme or abnormal rainfall distribution[9].

This present study will enable practitioners and policy makers acquire additional information on rainfall distribution in Nigeria. This work describes and compare monthly pattern of rainfall distribution in three south west states of Nigeria namely, Lagos, Abeokuta and Ibadan. Markov chain was used to obtain probabilities of transition between three main weather conditions namely dry month, wet month and rainy month and to obtain the long run equilibrium probabilities for the three stations.

Methodology

The data used for this research was obtained from the Nigerian Meteorological Agency, Lagos and it consists of the monthly rainfall data of three rainfall stations in the south western part of Nigeria (Ikeja, Abeokuta and Ibadan). These stations are located in the three most industrialized states within the south west geo-political zones of Nigeria. Abeokuta is the capital of Ogun State, geographically located between latitude 7.15N and longitude 3.36E with a population of about one million people and a land mass of 879 square kilometer. Ibadan is the capital of Oyo State, geographically located between latitiude 7.38N and longitude 3.95E with a population of over three million people and a land mass of about 3080square kilometer. It is the country largest city by geographical area. Ikeja is the capital of Lagos State, geographically located between latitiude 6.60N and longitude 3.35E. The mean annual rainfall in Lagos Oyo and Ogun States are 1603mm, 1253mm and 1156mm respectively.

Markov Chain is a stochastic model in which the state space and index are discrete in nature[10]. It is a discrete–time process where the future pattern of a process given the past and the current only depends on the current but does not depend on the past[10]. In other words, the probability of transitioning to any specific state is dependent solely on the present state and not time passed. Another significant characteristic of a Markov chain is that the transition probabilities remain unchanged regardless of time or space[10]. Some researchers have used either the two-state Markov Chain or the three-state Markov Chain in the study of pattern of rainfall distribution.

In this present work, a three-state Markov chain was used to describe the monthly pattern of rainfall distribution in these three stations. The three states considered were dry (d), wet (w) and rainy(r). The conditions of occurrence of rainfall for the three states are stated in the Table-1.

Table-1: Conditions of Rainfall Occurrence for the three states based on the number of days per month.

	28 day	29 days	30 days	31 days
Dry Month	≤70mm	≤72.5mm	< 75mm	<77.5mm
Wet Month	70.1mm -140mm	72.6mm -145mm	75.1mm -150mm	77.6mm -155mm
Rainy	≥140mm	≥ 145mm	≥ 150mm	≥ 155mm

The conditions of occurrence defined above were based on the classification of dry day (≤2.5mm), wet day (2.51mm-5mm) and rainy day (≥ 5.0mm)[11].

Validity of the Markov Chain Model: To investigate the validity of the Markov chain approach used in this work, the data collected will be subjected to appropriate statistical test. The basic assumption of using the Markov chain that occurrence of rainfall in successive months is related will be investigated using the chi-square test of independence and the WS test of Wang and Maritz[12]. The null and the alternative hypothesis are stated i. H_0: Rainfall incidence on consecutive months is independent, ii. H_1: Rainfall incidence on consecutive months is related.

Chi-Square Test of Independence: The test statistic for chi-square is

$$\chi^2_{calculated} = \sum\left(\frac{(O_i - E_i)^2}{E_i}\right) \quad (1)$$

O_i (i=1….n) are the observed frequencies, E_i (i=1….n) are the expected frequencies.

The null hypothesis is rejected if $\chi^2_{calculated} \geq \chi^2_{[(r-1)(c-1),\alpha]}$,

For a three state Markov chain, r=c=3.

WS test: The test statistic for the WS test of Wang and Maritz[12] is

$$WS = \frac{S_a + S_b - 1}{\sqrt{V(S_a + S_b - 1)}} \to N(0,1) \quad (2)$$

Where: $S_a = f_{dd} + f_{ww} + f_{rr}$

$$S_b = f_{rd}f_{dr} + f_{wr}f_{rw} + f_{dw}f_{wd} - f_{dd}f_{ww} - f_{dd}f_{rr} - f_{ww}f_{rr}$$

Where: $f_{dd} = f(d/d)$: Probability of a dry month given the previous month was dry, $f_{ww} = f(w/w)$: Probability of a wet month given the previous month was wet, $f_{rr} = f(r/r)$: Probability of a rainy month given the previous month was rainy: etc.

The variance of

$$V(S_a + S_b - 1) = 2(\phi_1\phi_2\phi_3)\left[\frac{1}{n_{d.}n_{w.}} + \frac{1}{n_{w.}n_{r.}} + \frac{1}{n_{r.}n_{d.}}\right] \quad (3)$$

Where: n_d: Total number of dry month, n_w: Total number of wet month, n_r: Total number of rainy month

Since ϕ_1, ϕ_2, ϕ_3 are the stationary probabilities that are calculated as follows:

$$\phi_1 = [(1+p)+(1+s)\,p/q]^{-1}$$
$$\phi_2 = (r + ps/q)\phi_1$$
$$\phi_3 = [p/q]\phi_1$$
$$p = \left[f_{dr} + \frac{f_{wr}(1-f_{dd})}{f_{wd}}\right]\left(\frac{1}{1-f_{rr}}\right),$$
$$q = 1 + \left[\frac{f_{wr}f_{rd}}{f_{wd}(1-f_{rr})}\right],\ r = \frac{f_{dw}}{1-f_{ww}},\quad s = \frac{f_{rw}}{1-f_{ww}}$$

The null hypothesis is rejected if $\left|\hat{WS}\right| \geq Z_{(1-\alpha/2)}$,

The Long Run (Equilibrium) probabilities: The long run equilibrium probabilities of the dry, wet and rainy months can be determined by the product of the matrix

$$\begin{bmatrix}\pi_1\\ \pi_2\\ \pi_3\end{bmatrix} = [x_1 \quad x_2 \quad x_3]\begin{pmatrix} f_{dd} & f_{dw} & f_{dr}\\ f_{wd} & f_{ww} & f_{wr}\\ f_{rd} & f_{rw} & f_{rr}\end{pmatrix} \tag{4}$$

Subject to $\pi_1 + \pi_2 + \pi_3 = 1$

The Expectation of different length of Seasons: The expectation of lengths of different seasons is defined in this section.

Dry Period (DP): Dry period of lengths 'D' is an order of successive dry months in between wet or rainy months. The probability of the dry period of lengths 'D' is

$$P(DP = D) = (f_{dd})^{d-1}(1-f_{dd})$$

The expectation of length for DP is

$$E(DP) = 1/(1-f_{dd})$$

Wet Period (WP): A wet period of lengths 'W' is an order of successive wet months in between rainy or dry months. The probability of the wet period of lengths 'W' is

$$P(WP = W) = (f_{ww})^{w-1}(1-f_{ww})$$

The expectation of length for WP is

$$E(WP) = 1/(1-f_{ww})$$

Rainy Period (RP): A rainy period of lengths 'R' is an order of successive rainy months in between dry or wet months. The probability of the rainy period of lengths 'R' is

$$P(RP = R) = (f_{rr})^{r-1}(1-f_{rr})$$

The expectation of length for RP is

$$E(RP) = 1/(1-f_{rr})$$

Results and discussion

The analysis of the data on rainfall from the three southwestern stations namely Lagos, Ibadan and Abeokuta using the methodologies described in the last section are presented.

Results on the Validity of the Markov Chain Model: The contingency tables of observed number of months of being in a particular current atmospheric condition immediately after leaving a particular previous atmospheric condition are presented on Tables-2, 3 and 4 also presented are the chi-square value and the probability value for the chi-square test of independence.

Table-2: Contingency table of Observed Frequency (Lagos).

		Present			Aggregate
		Dry Month	Wet Month	Rainy Month	
Preceding	Dry Month	85	32	28	145
	Wet Month	32	24	43	99
	Rainy Month	28	43	56	127

$\chi^2_{lag} = 42.55$, P-value=0.000.

Table-3: Contingency table of Observed Frequency (Ibadan).

		Present			Aggregate
		Dry Month	Wet Month	Rainy Month	
Preceding	Dry Month	99	35	15	149
	Wet Month	17	26	48	91
	Rainy Month	33	30	68	131

$\chi^2_{ibd} = 87.19$, P-value=0.000

Table-4: Contingency table of Observed Frequency (Abeokuta).

		Present			Aggregate
		Dry Month	Wet Month	Rainy Month	
Preceding	Dry Month	117	28	26	171
	Wet Month	34	34	29	97
	Rainy Month	20	35	48	103

$\chi^2_{abk} = 72.28$, P-value=0.000.

A further test to show the validity of the Markov model is the WS statistic. The estimated value of WS statistic is presented on Table-5.

Table-5: Estimate value of WS Test Statistic.

	Lagos	Ibadan	Abeokuta
WS Statistic Value	9.62	-9.14	-12.91

The Probability Matrices: The three estimated transition probability matrices for the three rainfall stations of Lagos, Ibadan and Abeokuta are shown below.

$$\underset{\text{Lagos}}{\begin{pmatrix} 0.586 & 0.221 & 0.193 \\ 0.323 & 0.242 & 0.435 \\ 0.220 & 0.339 & 0.441 \end{pmatrix}} \underset{\text{Ibadan}}{\begin{pmatrix} 0.664 & 0.235 & 0.101 \\ 0.187 & 0.286 & 0.527 \\ 0.252 & 0.229 & 0.519 \end{pmatrix}} \quad \underset{\text{Abeokuta}}{\begin{pmatrix} 0.684 & 0.164 & 0.152 \\ 0.351 & 0.351 & 0.298 \\ 0.194 & 0.340 & 0.466 \end{pmatrix}} \tag{5}$$

The Long Run (Equilibrium) Probabilities and the Expected Length of Seasons: The Estimated Long term Equilibrium State Probabilities of the distribution pattern of monthly rainfall in the three stations are presented on Table-6. The expectation of length for different periods and the weather cycle for the three stations are presented on Table-7.

Table-6: The long run equilibrium Probabilities for Lagos, Ibadan and Abeokuta.

	Current Month		
	Dry	Wet	Rainy
Lagos	0.390	0.267	0.343
Ibadan	0.402	0.245	0.353
Abeokuta	0.478	0.316	0.206

Table-7: The expectation of length for different Periods for Lagos, Ibadan and Abeokuta.

	Expected Length			
	Dry Period	Wet Period	Rainy Period	Weather Cycle
Lagos	2	1	2	5
Ibadan	3	1	2	6
Abeokuta	3	2	1	6

Discussion: The pattern of distribution of rainfall in three stations in southwest Nigeria namely Lagos, Ibadan and Abeokuta were studied using the Markov chain model. Specifically a Markov model having three states was used to conduct the investigation using monthly rainfall data from 1984 to 2014. The long term equilibrium probabilities for each of the three states of dry month, wet month and rainy month were determined for the three stations. Also obtained where the expectation of length for different periods and the weather cycle.

Initially, two statistical tests (the chi-square test of independence, WS test) were used to validate the appropriateness of the Markov chain model. The results from these tests as presented on Table-1-4 indicate that the rainfall distribution pattern of the current month is dependent on the rainfall distribution pattern of the preceding month, thus validating the assumption of dependency.

The transition probability matrices of rainfall occurrence in Lagos Ibadan and Abeokuta were used to calculate the long run (equilibrium) state probabilities of a month being dry, wet or rainy. The equilibrium state probabilities for the three stations as presented on Table-6 shows that the long run (equilibrium) state probabilities of a month being dry in Lagos, Ibadan and Abeokuta respectively were 0.39, 0.402 and 0.478 respectively. The equilibrium state probabilities of a month being wet in Lagos Ibadan and Abeokuta were 0.267, 0.245 and 0.316 while the probabilities for a month being rainy are 0.343, 0.353 and 0.206 respectively.

These results suggest that in Lagos and Ibadan approximately about 5 months in a calendar year will be dry, 3 months will be wet and 4 months will be rainy. In Abeokuta approximately about 6 months will be dry, about 4 months will be wet and about 2 months will be rainy.

The results on expected length of different seasons presented on Table-7 indicate that the expected length of dry, wet and rainy month (rounded up to nearest integer) are 2 months, 1 month and 2 months respectively with a weather cycle of 5months for Lagos; 3 months, 1 month and 2 months respectively with a weather cycle of 6 months for Ibadan; 3 months, 2 months and 1 month respectively with a weather cycle of 6months for Abeokuta.

Conclusion

The findings from this work on the monthly rainfall pattern of Lagos, Ibadan and Abeokuta did not show a major shift from the well establish belief that the wet or rainy season is from April to October (7months) and the dry season is from November to March (5months) in Southern Nigeria. It was observed from the results of this work that the incidence of rainy month i.e. precipitation $\geq$155mm for a 31 day calendar month will approximately be 4 months in Lagos and Ibadan on the long run.

This suggests that Ibadan and Lagos could experience heavy downpour for more than one-third of a calendar year.

The appropriate use of the results from this work would help governments and citizens of Lagos, Ibadan and Abeokuta make adequate future plan on matters that require useful knowledge on pattern of rainfall distribution in their immediate environment.

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