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Short Review Paper

Vacuum in relativistic theory

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Abstract

Characteristics of vacuum (and or vacuum energy) have been discussed qualitatively in the first half of this work. Mathematical considerations of the above have, also, been made with reference to different mathematical relations in their appropriate form applicable to vacuum.

Keywords: Cosmological constant, Dark energy, Dirac Sea, Horizon of the World, Latent Background Energy, Massless Virtual Particle, Vacuum energy.

Introduction

Vacuum plays an important role in the study of science especially the relativistic physics and the phenomenon of wave propagation. It is interesting to see that different characteristics have been adhered to vacuum in different times by different scientists. But, the most common idea is that vacuum is a space which contains no material and through which light can travel with maximum possible velocity.

In this dissertation, both qualitative and mathematical discussions of the characteristics of vacuum have been included.

Qualitative Discussion

It is well known that, New Cartesian Physics believes the physical vacuum as condition of the physical space that differs from the movable geometric space[1]. Also, quantum field theory described vacuum as being filled with “massless virtual particles”. In this concept the vacuum ground state is a seething mass of these virtual particles and fields of probabilities. Again, according to quantum theory vacuum contains some energy called Vacuum Energy which, of course, is difficult to describe. Yet, let us describe the nature of vacuum energy. It has been assumed that[2] the vacuum energy is: i. constant in space and time, and ii. positive as indicated by present observations.

The first law of thermodynamics, then, implies a pressure $p_v = -\rho_v$ which is constant in space and time but is negative[3,4]. It is well known that a negative pressure resembles a tension.

Again, the vacuum energy density could, often, be written as $\rho_v = \frac{C^4}{8\pi G}\Lambda$ where, Λ is a constant with the dimensions of an inverse squared length called the Cosmological Constant.

It is seen from the above that if there is a non-zero vacuum energy, it remains constant while the energy densities are considered in matter and radiation decay away as the Universe expands. The long-term future of a Universe that expands indefinitely is dominated by vacuum energy.

It may be mentioned here that the vacuum energy is related to the Concept of Dirac Sea. The Dirac Sea has been assumed to consist of mass-less virtual particles. But this simple idea of a sea, full of particles which are not quite real, is not sufficient to describe the vacuum and these virtual particles are only used when it is mathematically convenient to do so.

Again, it has been claimed that in case of pair production half of the energy of electron or positron comes from the vacuum energy and the other half from the waves of the source field.

Another way to model the vacuum energy that is responsible for the mass of the electron is to treat it as an ideal gas. Density Functional Theory (DFT) takes this approach and has its origin in the Thomas Fermi Model and the two Hohenberg-Kohn (H-K) theorems[5].

Thus, vacuum could be modelled, as an ideal gas or liquid depending on density, without discrete and real particles having mass. The density of vacuum must be considered, as well, in free space in addition to its contribution to mass in particles.

It is interesting to mention that vacuum energy is not potential energy, as it has neither lower state to fall to, nor kinetic energy of motion like a wave. But it is a form of energy. So, this can best be described as Latent Background Energy.

Mention must, also, be made to the fact that waves of the source field are quite simpler to explain compared to the vacuum energy.

Mathematical Considerations

Let us consider the nature of vacuum mathematically. Here, some mathematical formulae related to vacuum (hence to vacuum energy) would be discussed. It is to be mentioned here that to have an idea of vacuum we shall mention some equations in presence of matter which would be converted to the case of vacuum.

Einstein's Field Equations in presence of matter: We have, Einstein's equations in general relativity are given by[2]

$$G^{\mu}_{\nu} = -\frac{8\pi G}{C^4} T_{\mu\nu} \quad (1)$$

In relativistic units, this yields

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -8\pi T_{\mu\nu} \quad (2)$$

Where: $R_{\mu\nu}$ = curvature tensor, $g_{\mu\nu}$ = fundamental tensor, R = the contracted form of curvature tensor $R_{\rho\mu\nu\sigma}$, $T_{\mu\nu} =$ the components of material-energy tensor, $G^{\mu}_{\nu} =$ the mixed Einstein tensor.

Taking into account the cosmological constant Λ, one could write Einstein's modified field equations as

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = -8\pi T_{\mu\nu} \quad (3)$$

Since, Λ is negligibly small, so, (3) reduces to (2) when Λ is taken to be zero. These are the required field equations in the General Theory of Relativity in presence of matter and represent Einstein's law of gravitation for naturally curved material world.

Equation of continuity: Since, vacuum may be thought of as an ideal gas then $\frac{\partial T_{\mu\nu}}{\partial x^{\nu}} = 0$ will lead to the fact that $T_{\mu\nu}$ is either a constant or zero. This means that the medium is either isotropic and homogeneous throughout or it is absent (i.e. vacuum).

According to this conjecture the equation of continuity becomes

$$\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} + \frac{\partial(\rho)}{\partial t} = 0 \quad (4)$$

which leads to the idea of vacuum.

Einstein's Field Equations in Vacuum: Multiplying (3) by $g^{\mu\nu}$ we obtain after simplification

$$R - 4\Lambda = 8\pi T \quad (5)$$

In absence of matter, $T_{\mu\nu} = 0$, so that, T = 0. Hence, R = 4Λ. Thus, the field equations describe the empty space, i.e. vacuum, when T = 0. Here the cosmological constant Λ plays the same role as the intrinsic energy density of the vacuum.

Using the above conditions, (3) gives {by putting R = 4Λ in (3)}

$$R_{\mu\nu} = \Lambda g_{\mu\nu} \quad (6)$$

which represents Einstein's field equations in General Theory of Relativity in the absence of matter or Einstein's Law of Gravitation in empty space[6].

In general relativity, the cosmological constant was taken to be proportional to the energy density of empty space. Several workers set the vacuum energy equal to the cosmological constant Λ and obtained satisfactory results.

As with Newtonian case the field equations for gravity in vacuum may lead to describe geodesic deviation. The Ricii curvature can be expressed directly in terms of the Christoffel symbol by

$$R_{\mu\nu} = \frac{d\Gamma^{\gamma}_{\mu\nu}}{dx^{\gamma}} - \frac{d\Gamma^{\gamma}_{\alpha\gamma}}{dx^{\nu}} + \Gamma^{\gamma}_{\mu\nu}\Gamma^{\delta}_{\gamma\delta} - \Gamma^{\gamma}_{\mu\delta}\Gamma^{\delta}_{\nu\gamma} \quad (7)$$

The Einstein equation in vacuum, for relativistic generalization of $\nabla^2\varphi = 0$, will give $R_{\mu\nu} = 0$ where, $R_{\mu\nu}$ is symmetric in μ and ν.

Again, we know that in presence of matter the elements like $R_{\mu\mu}$ of the tensor $R_{\mu\nu}$ are non-zero, but all the off-diagonal elements are zero. Hence, Einstein's field equations for empty space would be $R_{\mu\nu} = 0$ i.e. all the elements of $R_{\mu\nu}$ are zero. Unlike those of Newtonian gravity or electromagnetism the field equations of general relativity are non-linear. Flat space-time is one solution of the vacuum Einstein's equation ($R_{\mu\nu} = 0$) here all the Γ's vanish in usual rectangular co-ordinates; therefore, so does $R_{\mu\nu}$.

It is noteworthy that the energy density of empty space could be measured by the curvature of space.

Line element: Schwarzchild's exterior solution yields the line element due to static and spherically symmetric distribution of perfect fluid outside the sphere in empty space to be[7]

$$ds^2 = -\left(1 - \frac{2m}{r} - \frac{1}{3}\Lambda r^2\right)^{-1} dr^2 - r^2 d\theta^2 - r^2 \mathrm{Sin}^2\theta \, d\varphi^2 + \left(1 - \frac{2m}{r} - \frac{1}{3}\Lambda r^2\right) dt^2 \quad (8)$$

Again, the Schwarzchild line element due to a static, isolated gravitating mass point (m) is

$$ds^2 = -\left(1 - \frac{2m}{r}\right)^{-1} dr^2 - r^2 d\theta^2 - r^2 \mathrm{Sin}^2\theta \, d\varphi^2 + \left(1 - \frac{2m}{r}\right) dt^2 \quad (9)$$

It is obvious that in the limit $r \to \infty$ the Schwarzchild line element reduces to the line element of flat space-time of special relativity which is that of vacuum. Also, at infinite distance from a mass point the gravitational effect becomes negligibly small. So, we may consider the space at a distance from the mass point to be vacuum.

Now, we have Schwarzchild radius to be r (= 2 m).This represents the boundary of the isolated particle and the solution holds in empty space outside the spherical distribution of matter (or isolated particle) whose radius must be greater than 2m. For entirely empty world (vacuum), we have m = 0 . So, the line element or metric becomes

$$ds^2 = -(1 - \frac{1}{3}\Lambda r^2)^{-1}dr^2 - r^2 d\theta^2 - r^2 \mathrm{Sin}^2\theta \, d\varphi^2 + \left(1 - \frac{1}{3}\Lambda r^2\right) dt^2 \qquad (10)$$

It is to be noted that this solution of Schwarzchild has a singularity at $r = (\frac{3}{\Lambda})^{\frac{1}{2}}$. Since, Λ is very small hence; this value of r is very large. The space outside this boundary at r is called horizon of the world. The space outside the horizon may be considered as vacuum. Again, in absence of any mass point the space-time would be flat so that the line element in spherical polar co-ordinates would be

$$ds^2 = -dr^2 - r^2 d\theta^2 - r^2 \mathrm{Sin}^2\theta \, d\varphi^2 + dt^2 \qquad (11)$$

Poisson's Equation: We know that the components of energy momentum tensor are

$$T^{\mu\nu} = \rho \frac{dx^\mu}{ds} \frac{dx^\nu}{ds} \qquad (12)$$

which, in the limit of Newtonian approximation ($v \ll c$), yields

$$T^{44} = T = \rho \qquad (13)$$

For vacuum T = 0 = ρ. Hence, Poisson's equation $\nabla^2\varphi = 4\pi\rho$, in vacuum, yields

$$\nabla^2\varphi = 0 \qquad (14)$$

Cosmological Model: The three crucial tests of the general theory of relativity indicate that it has furnished an acceptable solution of the problem of the field of a star in the empty space surrounding it at least to the distances of the order of dimensions of the solar system. It is known that Λ has negligible effect for phenomena in solar system even in our galaxy, but it is important when the universe as a whole is considered.

Quantum Mechanical World View: Quarks and gluons are thought to be the building blocks of protons and neutrons like all the hadrons. These hadrons contribute more than 99% mass to the ordinary matter through their quasi-stable equilibrium states.

Similarly, the mass of electron is due to the excitation of an electron field of an infinite ocean of zero point energy of vacuum. Perhaps, the huge dark energy which provides repulsive gravitation and has some underlying relationship with that of vacuum energy of space[8] is responsible for the present acceleration in the expanding universe.

Empty Universe: Let the universe be entirely empty (i.e. it is vacuum). Then, $p_0 = 0 = \rho_0$ so that, the equations[7] $\Lambda = 4\pi(3p_0 + \rho_0)$ and $\frac{1}{R_0^2} = 4\pi(p_0 + \rho_0)$ respectively yield $\Lambda = 0$ and $\frac{1}{R_0^2} = 0$. Hence, $e^{-\lambda} = 1 - \frac{r^2}{R_0^2} = 1$. Thus, in this case, Einstein's line element for flat space-time i.e. Einstein's Universe degenerates to the flat space-time of special relativity.

Conclusion

From the above discussions it is revealed that although vacuum does not contain any matter yet it has some energy which is the vacuum energy. This energy is of special character. Again, vacuum in general theory of relativity satisfies some mathematical relations duly modified to their appropriate forms.

Hence, vacuum, in theory of relativity, may be considered as a special state of matter in its general form.

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