



The Role of Lattice Displacement on Brillouin Polarization in a Magnetized n-InSb Crystal

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Abstract

For the study of parametric excitation, we have considered that origin of stimulated Brillouin scattering (SBS) of the Stoke's component of the scattered mode lies in the third order optical susceptibility. The optical polarization in SBS system arises due to nonlinear current density and acousto-optical strain of the medium. The lattice displacement vector or amplitude, induced polarization, acousto-optical polarization and threshold expressions are obtained. Using induced and acousto-optical polarization and coupled mode scheme, the effective Brillouin polarization, the electric amplitude of transmitted wave and efficiency of the Brillouin cell have also been calculated. The lattice displacement is slightly enhanced at wave number $k = 2.2 \times 10^7 \text{ m}^{-1}$. The required threshold electric field for the onset of SBS is also reduced significantly in the presence of magnetic field. The doping level also appreciably influences the effective Brillouin polarization. The study shows that a large effective polarization can be achieved at relatively low pump threshold in a magnetized doped semiconductor, which makes promising for the fabrication of nonlinear devices based on SBS.

Keywords: Nonlinear optics, Stimulated Brillouin scattering, parametric excitation, nonlinear interaction.

Introduction

Stimulated Brillouin Scattering (SBS) is a process in which an intense laser field couples to the Brillouin active normal modes and produces exponentially growing scattered fields. The process is limited by the usual attenuation mechanism of the normal modes. Once a limiting threshold power condition for the incident laser beam is exceeded, the exponential gain of the process leads to large conversion efficiency into the scattered fields.

With the advent of high power lasers and optoelectronics, it has become important to study not only the linear optical properties of a material but also its nonlinear optical properties. Parametric interactions have been studied extensively in solids¹ and several new fronts in the fields of research of nonlinear interaction of optical beams with matter such as harmonic generation, stimulated scattering processes, etc.^{2,4}. The origin of SBS lies in the effective third-order optical susceptibility of the medium arising from nonlinear induced current density and acoustic perturbations due to electrostriction. In SBS phenomenon, the signal wave may propagate in any direction but the maximum scattering occurs in the backward direction of a characteristic frequency downshift of the acoustic frequency⁵. The SBS is widely used in high power laser-system for improvement of amplifiers, pulse suppression, energy extraction and pulse compression etc.^{5,6}. Semiconductors are particularly the most promising materials showing large third-order nonlinear optical state and are used for fabrication of sophisticated optical devices⁷.

The SBS mechanisms have been employed in high power laser system for improvements in amplifiers, isolators, pre-pulse suppression, energy extraction and pulse compression, etc. Semiconductors are the most promising material showing third-order nonlinear optical state and are used for fabrication of sophisticated optoelectronic devices.

Neogi and Ghosh⁸ analytically investigated the possibility of optical phase conjugation via SBS in a transversely magnetized semiconductor crystal. The transmitted intensity and efficiency of the Brillouin cell have also been studied in an n-type highly doped centrosymmetric semiconductor crystal subjected to a transverse magnetic field⁹. Recently, Nimje et al.¹⁰ investigate the effects of material parameters on interaction length to occur OPC-SBS in semiconductors and found that when cyclotron frequency tuned with applied pump frequency interaction length is found to be 10^4 time smaller than in absence of magnetic field. In another study Nimje and Ghosh¹¹ analytically investigated the optical phase conjugation (OPC) reflectivity via stimulated Brillouin scattering (SBS) in acousto-optic diffusive semiconductor plasma crystal and 72% OPC reflectivity obtained when cyclotron frequency is in resonance with applied pump frequency.

The present paper deals with the study of SBS through third-order optical susceptibility, originating from finite induced current density or induced polarization and acousto-optical polarization, in a transversely magnetized n-type highly doped semiconductor. Electrostriction is the phenomenon of change in

optical dielectric constant of the medium induced by the strain whereas the acousto-optic phenomenon is the change in refractive index of the medium under the influence of an externally applied electric field. In the coupled mode scheme of plasmas, we have studied threshold condition for the onset of Brillouin instability, lattice displacement (u), acousto-optic polarization (P_{ao}), effective Brillouin polarization (P_{eff}) in highly doped semiconductor. The theory is based on parametric coupling of three finite time varying fields, which gives rise to an acousto-optic force that in turn drives an acoustic wave in the medium.

The important property attached to solid state plasma is that it retains its own physical properties and yet attains the plasma state of free carriers present in the medium. The elastic property associated with the lattice determines characteristics of the propagating waves. For the present analytical study, we have considered n-InSb crystal owing to high mobility carriers. Moreover, its single-valley energy band gap matches well with wavelength of 10.6 μm from CO₂ laser.

Theoretical Formulation: We have considered well-known hydrodynamic model of a homogeneous one-component (electron) plasma irradiated with spatially uniform ($|\mathbf{k}_0| \rightarrow 0$) pump field under thermal equilibrium. In order to make an analytical study of the effective polarization of centrosymmetric crystal, we derive expression for total current density \mathbf{J} for the resonant Stokes' component induced due to nonlinear interaction of the waves. The total current density \mathbf{J} and acousto-optic strain in the medium produce effective polarization. The theoretical analysis has been divided into the following subsections.

Total induced current density: In order to calculate total current density \mathbf{J} , we consider pump electric field $\mathbf{E}_0 \exp(k_0x - \omega_0t)$ parallel to acoustic wave vector \mathbf{k}_a (along x-axis) in an n-InSb semiconductor subjected to dc magnetic field along z-axis. For analytical simplicity, we neglect effects of pseudo-potential by assuming the sample to be centro-symmetric. For the present study, we employ following set equations;

$$\frac{\partial^2 u}{\partial t^2} - \frac{c}{\rho} \frac{\partial^2 u}{\partial x^2} + 2\Gamma_a \frac{\partial u}{\partial t} = \frac{\varepsilon}{2\rho} (\eta^2 - 1) \frac{\partial}{\partial x} (\mathbf{E}_{eff} \cdot \mathbf{E}_1^*), \quad (1)$$

$$\frac{\partial \mathbf{v}_0}{\partial t} + \mathbf{v}_0 \cdot \frac{\partial \mathbf{v}_1}{\partial x} = \frac{e}{m} \mathbf{E}_{eff}, \quad (2)$$

$$\frac{\partial \mathbf{v}_1}{\partial t} + \nu \mathbf{v}_1 + \mathbf{v}_0 \cdot \frac{\partial \mathbf{v}_1}{\partial x} = \frac{e}{m} [\mathbf{E}_1 + \mathbf{v}_1 \times \mathbf{B}_0], \quad (3)$$

$$\frac{\partial n_1}{\partial t} + \nu_0 \frac{\partial n_1}{\partial x} + n_1 \frac{\partial \nu_0}{\partial x} + n_e \frac{\partial \nu_1}{\partial x} = 0, \quad (4)$$

$$\mathbf{P}_{ao} = -\varepsilon_0 (\eta^2 - 1) \frac{\partial u}{\partial x} \mathbf{E}_{eff}, \quad (5)$$

$$\frac{\partial \mathbf{E}_1}{\partial x} = \frac{n_1 e}{\varepsilon} + \frac{(\eta^2 - 1)}{\varepsilon_1} \mathbf{E}_{eff} \cdot \nabla^2 \mathbf{u}^*, \quad (6)$$

where $\mathbf{E}_{eff} = \mathbf{E}_0 + \mathbf{v}_0 \times \mathbf{B}_0$.

Equation (1) represents lattice vibration in the medium. ρ , c and Γ_a are the material density, elastic constant and phenomenological damping parameters of the medium. All the equations and notations are explained elsewhere⁹.

The acousto-optic force gives rise to a carrier density perturbation within the medium where Brillouin scattering process is active. In a doped semiconductor, this density perturbation can be obtained by using the standard approach¹². Differentiating equation (4) with respect to time and using Equations (2) and (6), one gets on simplification

$$\frac{\partial^2 n_1}{\partial t^2} + \nu \frac{\partial n_1}{\partial t} + \omega_p^2 n_1 - \frac{n_0 e k^2}{m \varepsilon_1} (\eta^2 - 1) \mathbf{E}_{eff} \cdot \mathbf{u}^* = -\bar{E} \frac{\partial n_1^*}{\partial x}, \quad (7)$$

where $\omega_p^2 = \frac{\nu^2 \omega_p^2}{\nu^2 + \omega_c^2}$ and $\bar{E} = \left(\frac{e E_0}{m} + \omega_c \nu_{0y} \right) = \frac{e}{m} (E_{eff})_x$ in

which $\omega_c = (e/m) B_0$ the cyclotron frequency and $\omega_p = \sqrt{n_0 e^2 / m \varepsilon}$ the electron plasma frequency. The perturbed

electron density concentration n_1 will have two components, which can be distinguished as fast (n_{1f}) and slow (n_{1s}) such that $n_1 = n_{1f} + n_{1s}$. The slow component is associated with the low frequency acoustic wave (ω_a); while the fast one oscillates at high frequency electromagnetic wave ($\omega_0 + \omega_a$).

Taking into account the Stokes component of the scattered electromagnetic wave, ($\omega_1 = \omega_0 - \omega_a$) and $k = k_0 - k_a$, we obtain the following coupled equation under rotating wave approximation

$$\frac{\partial^2 n_{1f}}{\partial t^2} + \nu \frac{\partial n_{1f}}{\partial t} + \omega_p^2 n_{1f} - \frac{n_0 e k^2}{m \varepsilon_1} (\eta^2 - 1) \mathbf{E}_{eff} \cdot \mathbf{u}^* = -\bar{E} \frac{\partial n_{1s}^*}{\partial x}, \quad (8a)$$

and

$$\frac{\partial^2 n_{1s}}{\partial t^2} + \nu \frac{\partial n_{1s}}{\partial t} + \omega_p^2 n_{1s} = -\bar{E} \frac{\partial n_{1s}^*}{\partial x}. \quad (8b)$$

Equations (8a) and (8b) indicate that fast and slow components are coupled to each other via the pump electric field, which is the fundamental necessity of SBS to occur. Using equations (1) and (8), we obtain,

$$n_{1s}^* = \frac{n_0 \varepsilon_0 k k_a (\eta^2 - 1)^2 \mathbf{E}_0 \cdot \mathbf{E}_1}{2\rho (\omega_a^2 - k_a^2 \nu_a^2 - 2i\Gamma_a \omega_a)} \left[1 - \frac{(\delta_1^2 - i\nu\omega_1)(\delta_2^2 + i\nu\omega_a)}{k_1^2 |\bar{E}|^2} \right]^{-1} \quad (9)$$

Where $\delta_1^2 = \omega_p^2 - \omega_1^2$ and $\delta_2^2 = \omega_p^2 - \omega_a^2$.

From above expression, it is clear that n_{1s} depends on the pump intensity $I_{in} = \left[0.5 \eta \varepsilon_0 c_0 |E_{eff}|^2 \right]$, c_0 being the speed of light.

The resonant Stokes component of acousto-optic current density due to finite nonlinear induced polarization can be given by

$$\mathbf{J}(\omega_1) = n_e e v_{1x} + n_{1s}^* e v_{0x} \quad (10)$$

Using equations (4) and (9), Equation (10) becomes

$$\mathbf{J}(\omega_1) = \frac{i \varepsilon \omega_p^2 \omega_1 \mathbf{E}_1}{(\omega_1^2 - \omega_c^2)} + \frac{i \omega_p^2 \varepsilon \varepsilon_0 k k_a (\eta^2 - 1) \omega_0 |\mathbf{E}_0|^2 \mathbf{E}_1}{2 \rho (\omega_a^2 - k_a^2 v_a^2 - 2 i \Gamma_a \omega_a) (\omega_0^2 - \omega_c^2)} \times \left[1 - \frac{(\delta_1^2 - i v \omega_1)(\delta_2^2 + i v \omega_a)}{k_1^2 |\bar{E}|^2} \right]^{-1} \quad (11)$$

The first term in above expression represents the linear component induced current density and the other term represents the nonlinear current density due to three-wave interactions.

Lattice displacement vector : The lattice displacement motion is represented by equation (1), which leads to lattice displacement amplitude $|\mathbf{u}|$ as

$$|\mathbf{u}| = \frac{\varepsilon k \omega_0^2 (\eta^2 - 1) |\mathbf{E}_0| |\mathbf{E}_1^*|}{2 \rho (\omega_0^2 - \omega_c^2) [(\omega_a^2 - k_a^2 v_a^2)^2 + 4 \Gamma_a^2 \omega_a^2]^{0.5}}, \quad (12)$$

in which $k_a = [k_0^2 + k^2 - 2 k_0 k \cos \phi]^{0.5}$ with ϕ being the scattering angle between \mathbf{k}_a and \mathbf{k} .

Induced polarization: To begin with, let us treat the induced polarization $\mathbf{P}_1(\omega_1)$ as time integral of the current density $\mathbf{J}_1(\omega_1)$, then one gets

$$\mathbf{P}_{cd}(\omega_1) = \int \mathbf{J}_1(\omega_1) dt = -\frac{\mathbf{J}_1(\omega_1)}{i \omega_1} \quad (13)$$

Using equations (11) and (13), one gets

$$|\mathbf{P}_{cd}(\omega_1)| = \frac{\varepsilon_0 k_a A \omega_p^2}{\omega_0 \omega_1} (\eta^2 - 1) |\mathbf{E}_0| |\mathbf{u}_0|, \quad (14)$$

$$\text{where } A = \left[1 - \frac{(\delta_1^2 - i v \omega_1)(\delta_2^2 + i v \omega_a)}{k_1^2 |\bar{E}|^2} \right]^{-1}.$$

The above equation, on setting $\mathbf{P}_{cd}(\omega_1) = 0$, enables us to evaluate threshold for the onset of SBS as

$$E_{0th} = \frac{e}{mk} \left(\frac{\omega_0^2 - \omega_c^2}{\omega_0^2} \right) [(\delta_1^4 + v^2 \omega_1^2)(\delta_2^4 + v^2 \omega_a^2)]^{0.5} \quad (15)$$

Acousto-optical polarization: Besides the induced polarization $\mathbf{P}_{cd}(\omega_1)$, the crystal possesses acousto-optic polarization \mathbf{P}_{ao} arising due to interaction between the pump and the generated acoustic wave. The scattering of electromagnetic wave from the acoustic mode affords a convenient way of controlling frequency, intensity and direction of the pump. This type of control makes possible

large number of applications involving transmission display and processing of information¹³.

The acousto-optic polarization is obtained from equations (1) and (5) as

$$|\mathbf{P}_{ao}| = \varepsilon_0 k_a (\eta^2 - 1) |\mathbf{E}_0| |\mathbf{u}| \quad (16)$$

Effective Brillouin polarization: The effective Brillouin polarization at the Stoke's shifted frequency in Brillouin active medium is given by

$$\mathbf{P}_{eff}(\omega_1) = \mathbf{P}_{cd}(\omega_1) + \mathbf{P}_{ao}(\omega_1). \quad (17)$$

From Equations (13) and (15), one gets

$$|\mathbf{P}_{eff}(\omega_1)| = \varepsilon_0 k_a (\eta^2 - 1) \left(1 - \frac{\omega_p^2}{\omega_1 \omega_0} \right) |\mathbf{E}_0| |\mathbf{u}|. \quad (18)$$

Now this relation may be used to computing the effective Brillouin polarization of the medium.

Results and Discussion

For numerical appreciation of the above analysis is applied to a specific case of centrosymmetric crystal irradiated by a 10.6 μm CO₂ laser. The set of parameters has been used in the analysis is taken from ref.¹⁴ and are as follows:

$$m = 0.015 \times 9.1 \times 10^{-31} \text{ kg}, \quad \eta = 3.9, \quad \rho = 5.8 \times 10^3 \text{ kg m}^{-3}, \\ \Gamma_a = 2 \times 10^{10} \text{ s}^{-1}, \quad \nu = 3 \times 10^{11} \text{ s}^{-1}, \quad v_a = 4.8 \times 10^2 \text{ ms}^{-1}, \\ \omega_0 = 1.78 \times 10^{14} \text{ s}^{-1}, \quad \omega_a = 10^{12} \text{ s}^{-1}, \quad k_0 = 5.92 \times 10^5 \text{ m}^{-1}, \\ k_a = 2.08 \times 10^8 \text{ m}^{-1}, \quad n_e = 2.44 \times 10^{24} \text{ m}^{-3} \text{ and } \varepsilon_1 = 15.8.$$

It's well known that the important properties attached to the solid state plasma is that it retains its own physical properties and yet attains the plasma state due to the presence of free carriers. The elastic properties which are related to the material density of the medium determine the propagation properties. The first experimental observation of propagation of sound wave and other properties in In Sb are made by Libhaber and Veilex¹⁵. For being a single-valley semiconductor and having a large density of large density of mobile particles compared to other semiconductors, n-InSb has been extensively investigated. The band gap of InSb also matches well with the wavelength 10.6 μm of pulsed CO₂ laser.

We now focus our attention on the dependence of threshold field, lattice displacement and the effective Brillouin polarization on wave number, magnetic field, carrier density.

Figure 1 exhibits the variation of threshold electric field E_{0th} with the applied dc magnetic field in terms of cyclotron frequency ω_c at fixed wave number and carrier density.

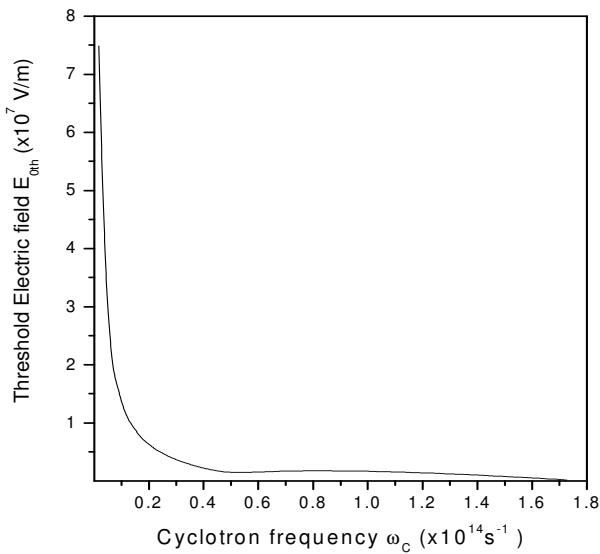


Figure-1

Dependence of threshold pump electric field (E_{0th}) on cyclotron frequency (ω_c) for $k = 10^7 m^{-1}$ and $n_e = 5 \times 10^{22} m^{-3}$

It can be seen from the figure that E_{0th} decreases sharply with increase in ω_c and becomes nearly constant at $\omega_c \geq 5 \times 10^{13} s^{-1}$. Thus, the threshold pump field necessary for the onset of Brillouin instabilities can be reduced significantly by immersing the crystal in transverse magnetic field.

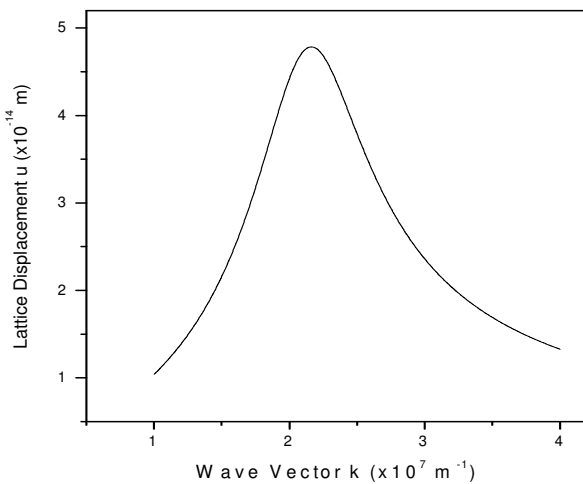


Figure-2

Dependence of lattice displacement (u) with wave number (k) for $E_0 = 10^7 V/m$, $\omega_c = 0.9\omega_0$ and $n_e = 5 \times 10^{22} m^{-3}$

The dependence of lattice displacement (u) with wave number (k) is shown in figure 2. The magnitude of lattice displacement increases with increasing the value of wave number k and

attained its maximum value at $k = 2.2 \times 10^7 m^{-1}$. Further, if one can increase the value of k in the range $k \geq 2.2 \times 10^7 m^{-1}$ the magnitude of lattice displacement decreases sharply with k . This can be used to achieve large amounts of Brillouin polarization, transmission intensity of Brillouin mode and efficiency of Brillouin cell. Thus, at appropriate value of k , the lattice displacement maximizes, which in turn, produces large amplification of the scattered wave.

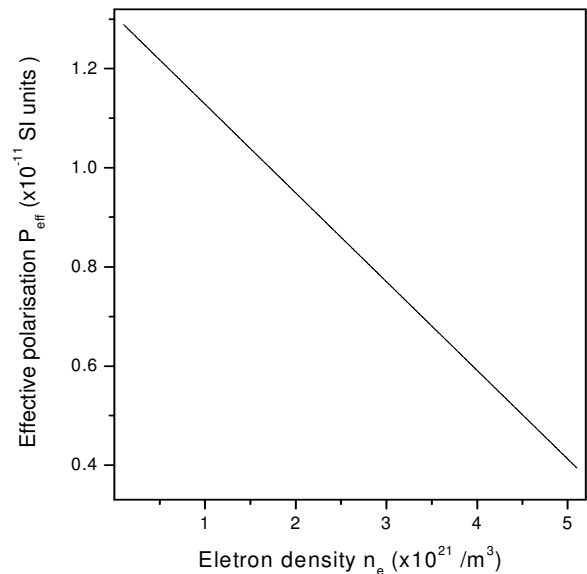


Figure-3

Variation of Effective polarization (P_{eff}) with electron density (n_e) for $E_0 = 10^7 V/m$, $\omega_c = 0.9\omega_0$ and $k_0 = 10^7 m^{-1}$

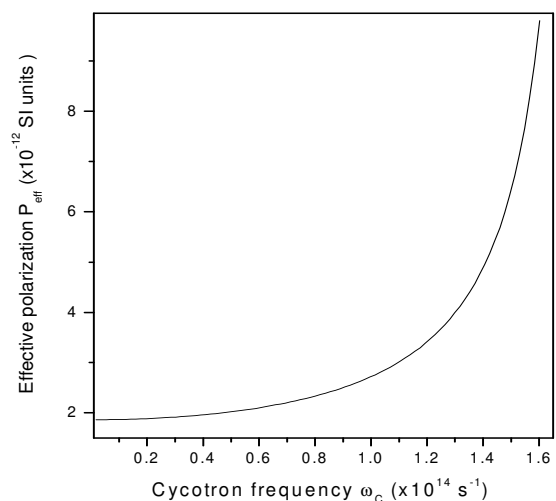


Figure-4

Variation of Effective polarization (P_{eff}) with Cyclotron frequency (ω_c) for $E_0 = 10^7 V/m$, $k_0 = 10^7 m^{-1}$ and $\omega_p = 0.5\omega_0$.

Figure 3 shows the effective Brillouin polarization \mathbf{P}_{eff} of the medium as a function of carrier concentration n_e in the presence of strong magnetic field. As can be seen that \mathbf{P}_{eff} decreases linearly with increase in carrier concentration. It is therefore inferred that larger amplification of the scattered waves can be obtained in a lightly doped semiconductor.

Figure 4 depicts the dependence of the effective Brillouin polarization \mathbf{P}_{eff} on cyclotron frequency ω_c . It is clear from this figure that as we increase the value of cyclotron frequency the effective polarization of the InSb crystal increases parabolically.

Conclusion

Thus, from the above analytical study we conclude that a large Brillouin polarization may be obtained when a low threshold pump field propagates without dispersion in a lightly doped III-V semiconductor immersed in strong transverse magnetic field, which makes promising for the fabrication of nonlinear devices based on SBS.

References

1. Flytzanis C.H., In: Rabin H. and Tang C.L. editors. Quantum electronics, **1**, 9, New York: Academic (1975)
2. Franken P.A., Hill A.E., Peters C.W. and Weinrich G., Generation of Optical Harmonics, *Phys. Rev. Lett.*, **7**, 118-119 (1961)
3. Lin C., Stolen R.H. and Cohen L.G., A tunable 1.1- μm fiber Raman oscillator, *Appl. Phys Lett.*, **31**, 97-98 (1977)
4. Yariv A., Quantum electronics, 49, New York: Wiley (1975)
5. Sharma G. and Ghosh S., Stimulated Brillouin scattering in a magnetoactive III-V semiconductor: effects of carrier heating, *Physica B.*, **322**, 42-50 (2002)
6. Carr I.D. and Hanna D.C., Performance of Nd:YAG oscillator/amplifier with phase conjugation via stimulated Brillouin scattering, *Appl. Phys B*, **36**, 83-92 (1985)
7. Jain R.K., Degenerate four wave mixing in semiconductor-application to phase conjugation and to picoseconds resolved study of transient carrier dynamics, *Opt. Engg.*, **21**, 199-2018 (1982)
8. Neogi A. and Ghosh S., Stimulated Brillouin Scattering in a Transversely Magnetised Doped Centrosymmetric Semiconductor, *Phys. Stat. Sol. (b)*, **156**, 705-715, (1989)
9. Jat K.L., Mukherjee S., Ghosh S. Effects of material parameter on interaction length to occur optical phase conjugation via stimulated Brillouin scattering in semiconductors, *Phys. Scripta*, **50**, 507-513 (1994)
10. Nimje N., Yadav N. and Ghosh S., Effects of material parameter on interaction length to occur optical phase conjugation via stimulated Brillouin scattering in semiconductors, *Phys. Lett. A*, **376**, 850-853 (2012)
11. Nimje N. and Ghosh S., Optical phase conjugation reflectivity via stimulated Brillouin scattering in acousto-optic diffusive semiconductor plasma crystal, *J. Phys.: Conf. Ser.*, **365**, 012044 (2012)
12. Guha S., Sen P. K. and Ghosh S., Parametric instability of acoustic waves in transversely magnetised piezoelectric semiconductors, *Phys. Stat. Sol. (a)*, **52**, 407-414 (1979)
13. Adler R.B., Interaction between light and sound, *IEEE Spectrum*, **4**, 42-54 (1967)
14. Nimje N., Dubey S. and Ghosh S., Diffusion-induced modulational instability in magnetised semiconductor plasmas: effect of carrier heating, *Eur. Phys. J. D*, **59**, 223-231 (2010)
15. Libchaber A, Veilex R. Wave Propagation in a Gyromagnetic Solid Conductor: Helicon Waves, *Phys. Rev.*, **127**, 774-776 (1962)