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Short Communication

Some results on double sequence theorems in metrizable spaces

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Abstract

In this paper, we have discussed some generalized results in double sequence theorems on metrizable spaces and also some new concepts of generalized metric spaces.

Keywords: Metric space, locally-finite, covering space, vector space.

Introduction

The discussion of some equivalence metrization theorems, modified single sequence theorem and modified double sequence theorems has been studied by Nigata[1]. Here we also defined metric topologies, before that, however, we want to give a name to those topological spaces whose topologies are metric topologies. For the sake of compactness we have studied and the proof given by Mattin[2]. Some of authors studied various types of theorems in metric spaces. In this paper, we have establish and generalized results which is observed in results given by Bing [3]. Further we have added some illustrative examples and results.

Definition of T_1 spaces: A T_1 - space is a topological space in which given any pair of disjoint points, each has a neighbourhood which does not contain the other.

It is obvious that any subspace of T_1 space is also a T_1 space.

Main results

In the present section, clearly, it validates the equivalences of metrization conditions of Bing's Theorem, Nagata Theorem, and Double sequence Theorems. Symmetrically which follows only from Urysohn's Theorem. Hence therefore we studied the following theorems based on T_1-spaces.

Theorem 1 : If a topological space τ then i. Is a T_1-space, ii. is a regular space, iii. has a $\sigma-$discrete base then τ is a metrizablespace.

Theorem-2: If a topological space τ then iv. is a T_1-space, v. is a regular space, vi. has a $\sigma-$locally finite base then τ is a metrizablespace.

Theorem-3: If a topological space τ then vii. is a T_1-space, viii. there exists a sequence u_1, u_2, u_3............................of open covering such that,

ix. $u_1 > u_2^* > u_2 > u_3^* >$.......................
can be repaced as $u_1 > u_1^\Delta > u_2 > u_2^\Delta$....

x. $\{s(p,u_n) : n = 1,2,3.....\}$ is a neighbourhood basis at each point of "p" of R then R is a metrizable.

Proof:
Theorem (1) $\Rightarrow$ Theorem (2) is obvious thus implies.
Theorem (3) $\Rightarrow$ Theorem (2) have been proved by Nagata[1].
It is enough to show that conditions of theorem 3 imply the conditions of theorem 1. Which we do now.
We assume theorem 3.
(vii) $\Rightarrow$ (i) (1)

We now prove (viii)
Let $N(p)$ be neighbourhood of a point p, There exists from (x) and $n \in N$ such that

$$S(p,u_n) \subset N(p) \quad (2)$$

Now consider $S(p,u_{n+1})$ then $S(p,u_n)$ is closed neighbourhood of p.

Let $b \in \overline{S(p,u_{n+1})}$. (3)

Every G_{n+1} such that $b \in G_{n+1}$. $G_{n+1} \in u_{n+1}$ call them as $G_{n+1}(b)$ which meets $S(p,u_{n+1})$, i.e. meets to G_{n+1} which contains "p". Call it as $G_{n+1}(p)$.

$$G_{n+1}(p) \cup G_{n+1}(b) \subset u_{n+1}^* \subset u_n \quad (4)$$

There exists a set u_n which contains "p" for every b.

Now $S(p,u_n)$ examples contains p and b.

$$b \in S(p,u_n) \tag{5}$$

$$\overline{S(p,U_{n+1})} \in S(p,u_n) \subset N(p). \tag{6}$$

Which established (viii).
We prove (ix) in a series of steps.

Step 1: The following results are obvious

$$\{q \in S_n(p)\} \Rightarrow \{p \in S_n(q)\} \tag{7}$$

If $q \in S(p,u_{n+1})$, then from (viii)(ix)(x)

$$S(p,u_{n+1}) \cup S(q,u_{n+1}) \subset S(p,u_n) \tag{8}$$

Step 2: Let u be an open cover in R such that $U \in u$.
Let

$$U_n = \{x \in U : x \neq S(p,u_n), p \in (R-U)\} \tag{9}$$

We assert that if $q \notin U_{n+1}$ (10)

And $x \notin U_{n+1}$ (11)

Further

$$x \notin S_{n+1}(q) \tag{12}$$

For otherwise if $x \in S_{n+1}(q) = S(q,u_{n+1})$.

Then for some value of "p" in (R-U), $q \in S(p,u_{n+1})$ and $S(p,u_{n+1}) \cap U_{n+1} = \phi$. Which contradiction, Hence the proof.

Conclusion

In the present paper, we gave generalized concepts of Nagata and Bing, further the sequence theorems and some double sequence theorems will play an important rule on T_1-spaces.

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