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*Short Communication*

# Formulation of solutions of a special class of standard quadratic congruence of even composite power modulus

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## Abstract

*In this paper, a special class of standard quadratic congruence of even composite power modulus is formulated. Such congruence were not formulated by earlier mathematicians. Formulation made it possible to find the solutions orally. Thus the formulation of solutions of the said congruence is the merit of the paper. Also, the study of standard quadratic congruence becomes very interesting and easy. A large numbers of solutions can be calculated mentally. This is one more merit of the paper.*

**Keywords:** Composite-power modulus, Formulation, Quadratic congruence.

## Introduction

A standard quadratic congruence is a congruence of the type: $x^2 \equiv a^2 (mod\ m)$; m being a Prime or Composite integer. The solutions are the values of $x$ that satisfy the congruence. Such types of congruence are always solvable. If it is a standard quadratic congruence of prime modulus, then it has exactly two solutions[1]. *But if it is a standard quadratic congruence of composite modulus, then it may have more than two solutions*[2]*. Here, the author wishes to formulate the standard quadratic congruence of even composite power modulus of the type:* $x^2 \equiv a^2 (mod\ 2a^n)$; $n \geq 3$, *a is positive even composite integer.*

## Literature Review

It is found that in the literature of mathematics, a standard quadratic congruence of prime modulus is discussed prominently. A little discussion is found on quadratic congruence of prime-power modulus. Earlier mathematicians (it seems) were not much interested in it.

The author's formulation made it possible to find the solutions of the congruence directly. The author already formulated many standard quadratic congruence of prime and composite modulus[3-11].

## Need of Research

Though the author formulated many standard quadratic congruence of prime and composite modulus, even he found one more such special quadratic congruence of even composite modulus yet remained to formulate. Here in this paper, the author considered it for formulation and his efforts are presented here. This is the need of the paper.

## Problem statement

Here the problem is - "To formulate the solutions of the special class of standard quadratic congruence of even composite modulus:

$x^2 \equiv a^2 (mod\ 2a^n)$; $a$ is an even positive integer, $n \geq 3$".

## Results

Consider the congruence $x^2 \equiv a^2 (mod\ 2a^n)$; $a$ is an even positive integers, $n \geq 3$.

Let us consider that $x = a^{n-1}k \pm a$.
Then $x^2 = (a^{n-1}k \pm a)^2$
$= (a^{n-1}k)^2 \pm 2.a^{n-1}k.a + a^2$
$= a^2 + a^{2n-2}k^2 \pm 2a^n k$
$= a^2 + a^n(a^{n-2}k^2 \pm 2k)$, if a is even positive integer
$\equiv a^2\ (mod\ 2a^n)$.

Thus, $x \equiv a^{n-1}k \pm a\ (mod\ 2a^n)$ is a solution of the said congruence: $x^2 \equiv a^2 (mod\ 2a^n)$; $a\ is$ an even Positive integer, $n \geq 3$.

But, if we consider $k = 2a$, then $x \equiv a^{n-1}.2a \pm a\ (mod\ 2a^n)$
$\equiv 2a^n \pm a\ (mod\ 2a^n)$
$\equiv 0 \pm a \equiv \pm a\ (mod\ 2a^n)$
Which is the same solution as for $k = 0$.

Similarly, for higher values of k, the solutions repeats as for $k = 1, 2, 3, \ldots..$

Therefore, all the required solutions are given by
$x \equiv a^{n-1}k \pm a\ (mod\ 2a^n); k = 0, 1, 2, \ldots \ldots \ldots (2a - 1)$.

These are $4a$ incongruent solutions for all values of k. The congruence has two solutions for every value of k and k has a different values.

## Illustrations

A standard Consider the congruence $x^2 \equiv 36\ (mod\ 432)$.
It can be written as $x^2 \equiv 6^2(mod 2.6^3)$ with $a = 6$, an even integer and n = 3.

Such congruence always has $4a = 4.6 = 24$ solutions.
Those solutions are given by

$x \equiv a^{n-1}k \pm a\ (mod\ 2a^n); k = 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11.$

$i.e.\ x \equiv 6^{3-1}k \pm 6 \ \equiv 36k \pm 6\ (mod\ 2.6^3); k = 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11.$

$i.e. x \equiv 36k \pm 6\ (mod\ 432); k = 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11.$

$i.e.\ x \equiv 0 \pm 6; 36 \pm 6; 72 \pm 6; 108 \pm 6; 144 \pm 6; 180 \pm 6; 216 \pm 6; 252 \pm 6; 288 \pm 6; 324 \pm 6; 360 \pm 6; 396 \pm 6\ (mod\ 432)$

$i.e.\ x \equiv 6, 426; 30, 42; 66, 78; 102, 114; 138, 150; 174, 186; 210, 222; 246, 258; 282, 294; 318, 330; 354, 366; 390, 402\ (mod\ 432).$

These are the twenty four solutions of the congruence under consideration.

Consider the congruence$x^2 \equiv 16\ (mod\ 128)$.

It can be written as $x^2 \equiv 4^2(mod\ 2.4^3)$ with $a = 6$, an even integer and n = 3.

Such congruence always has $4a = 4.4 = 16$ solutions.

Those solutions are given by

$x \equiv a^{n-1}k \pm a\ (mod\ 2a^n); k = 0, 1, 2, 3, 4, 5, 6, 7.$

$i.e.\ x \equiv 4^{3-1}k \pm 4 \ \equiv 16k \pm 4\ (mod\ 2.4^3); k = 0, 1, 2, 3, 4, 5, 6, 7.$

$i.e.\ x \equiv 16k \pm 4\ (mod\ 2.64); k = 0, 1, 2, 3, 4, 5, 6, 7.$

$i.e.\ x \equiv 0 \pm 4; 16 \pm 4; 32 \pm 4; 48 \pm 4; 64 \pm 4; 80 \pm 4; 96 \pm 4; 112 \pm 4;\ (mod\ 128)$

$i.e. x \equiv 4, 124; 12, 20; 28, 36; 44, 52; 60, 68; 76, 84; 90, 100; 108, 116\ (mod\ 128).$

These are the sixteen solutions of the congruence under consideration.

## Conclusion

Thus, it can be concluded that the congruence under consideration $x^2 \equiv a^2(mod\ 2a^n);\ n \geq 3$ is formulated successfully and has $4a$ solutions which can be given by

$x \equiv a^{n-1}k \pm a\ (mod\ 2a^n); k = 0, 1, 2, \ldots \ldots \ldots (2a - 1).$

**Merit of the paper:** Formulation is the merit of the paper. It made the finding of solutions very easy. This formulation is time-saving and very simple. Sometimes the solutions can be obtained orally. It is one more merit of the paper.

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