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Variable control charts based on dagum distribution

P. Sricharani[1*] and B. Srinivasarao[2]
[1]Department of Basic Science, Shri Vishnu Engineering College for Women (Autonomous), Bhimavaram-534202, Andhra Pradesh, India
[2]Department of Statistics, Department of Mathematics & Humanities, R.V.R &J.C College of Engineering, Guntur-522 019, AP, India
charani.ravi657@gmail.com

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Abstract

In this paper, we are mainly interested in evaluated the percentiles of mean, median, range and mid-range, and also develop the control limits for those. Assuming that these statistic variable quality characteristic follows the well-known Dagum distribution. The power of the control limits and its adequacy is checked in comparison with those depend on the famous shewart limits using simulation and compares. The comparison between both the coverage probabilities approaches are calculated and compared.

Keywords: Dagum distribution (DD), mean, median, mid-range, range.

Introduction

The probability density function (pdf) of Dagum distribution is given by

$$f(x,\gamma,\beta,\delta)=\frac{\beta\delta}{x}\left(\frac{\left(\frac{x}{\gamma}\right)^{\beta\delta}}{\left(\left(\frac{x}{\gamma}\right)^{\beta}+1\right)^{\delta+1}}\right), x>0, \beta>0, \gamma>0, \delta>0 \quad (1)$$

Its cumulative distribution function (cdf)

$$F(x,\gamma,\beta,\delta)=\left(1+\left(\frac{x}{\gamma}\right)^{-\beta}\right)^{-\delta}, x>0, \beta>0, \gamma>0, \delta>0. \quad (2)$$

The Dagum distribution is skewed distribution and it is uni-model distribution. The mean, median, mode and variance of Dagum distribution are respectively.

$$Median=\gamma\left(-1+2^{\frac{1}{\delta}}\right)^{-\frac{1}{\beta}} \quad (3)$$

$$Mean=-\frac{\gamma}{\beta}\frac{\Gamma\left(-\frac{1}{\beta}\right)\Gamma\left(\frac{1}{\beta}+\delta\right)}{\Gamma\delta} \quad if \quad \beta>1 \quad (4)$$

$$Mode=\frac{1}{\gamma^2}\left(\frac{\beta\delta-1}{\beta+1}\right)^{\frac{1}{\beta}} \quad (5)$$

$$Variance=\begin{cases}-\frac{\gamma^2}{\beta^2}\left(2\beta\frac{\Gamma\left(-\frac{2}{\beta}\right)\Gamma\left(\frac{2}{\beta}+\delta\right)}{\Gamma\delta}+\left(\frac{\Gamma\left(-\frac{1}{\beta}\right)\Gamma\left(\frac{1}{\beta}+\delta\right)}{\Gamma\delta}\right)^2\right) & \text{if } \beta>2\\ Indeterminate & otherwise\end{cases} \quad (6)$$

In graph: i. $\beta=2,\gamma=3,\delta=0.25$ (ii) $\beta=2,\gamma=3,\delta=0.5$ (iii) $\beta=2,\gamma=3,\delta=1$

Many authors are develop the skewed distribution to statistical quality control (SQC) methods. In few of the authors are" Betul and Yaziki - Burr Distribution[1], Chan and Cui- have developed control chart constants for X- bar and R charts in a unified way for a Skewed distributions where the constants are dependent on the coefficient of skewness of the distribution[2], Edgeman - Inverse Gaussion Distribution[3], Gonzalez and Vilesy - Gamma Distribution[4], Kantam and Sriram - Gamma Distribution[5], Kantam etal - Log logistic Distribution[6], Rao et al - Inverse Rayleigh distribution[7], Rao and Babu - Linear failure rate distribution[8], Rosaiah et al - half logistic distribution[9] and references therein. This paper study to makes the offensive in differentiate manner. Dagum limits, shewart limits are based on normal distribution and the parametric values of Dagum distribution which tend to be meso-kurtic. Therefore, Let the life time data as quality data for preparing the control charts for the distribution is one of the probability models and it is useful for life testing and reliability studies .It is observed that the distribution is meso-kurtic for $\delta=0.5$, $\beta=2$ and $\gamma=3$. As the comparison between Dagum distribution and shewart use by subject followers. Here we built the best feasible solution to this problem and simplify its solution. The remaining of the work in this paper is coordinated in the following way. In section 2 the basic concept and the control chat constants over the percentiles of the sample statistics. In section-3 the comparison between the calculated Dagum distribution control limits and shewart control limits. In section 4 we are given the concise result.

Control chart constants through percentiles

Mean chart: Let us consider the sample size n, the samples are x_1, x_2,…x_n have been drawn from Dagum distribution with $\delta = 0.5$, β=2 and $\gamma = 3$. Whether it is studied as a subset of technology to process data with an aimed population mean, replicated sampling process, its mean gives Whenever the process mean to reach targeted mean or not, therefore we distinguish the most likely control limits inside which average falls. The concept of 3σ limits is taken as the 'most probable' limits. It is known that 3σ limits of normal distribution to include 99.73% of probability. Hence, we have to search two limits of the sampling is distribution of sample mean in Dagum distribution such that the probability content of those limits is 0.9973. Symbolically we have to find lower and upper limits. Therefore

$$P(LL \le \bar{x} \le UL) = 0.9973$$

We have to take the concept of equi- tailed values of lower and upper limit values.

But sampling distribution of $\bar{x}$ is not mathematically tractable- in the case of Dagum distribution. So, we prepare the sampling distribution of $\bar{x}$ through calculating simulation there by calculate its persentiles. These are given in the Table-1

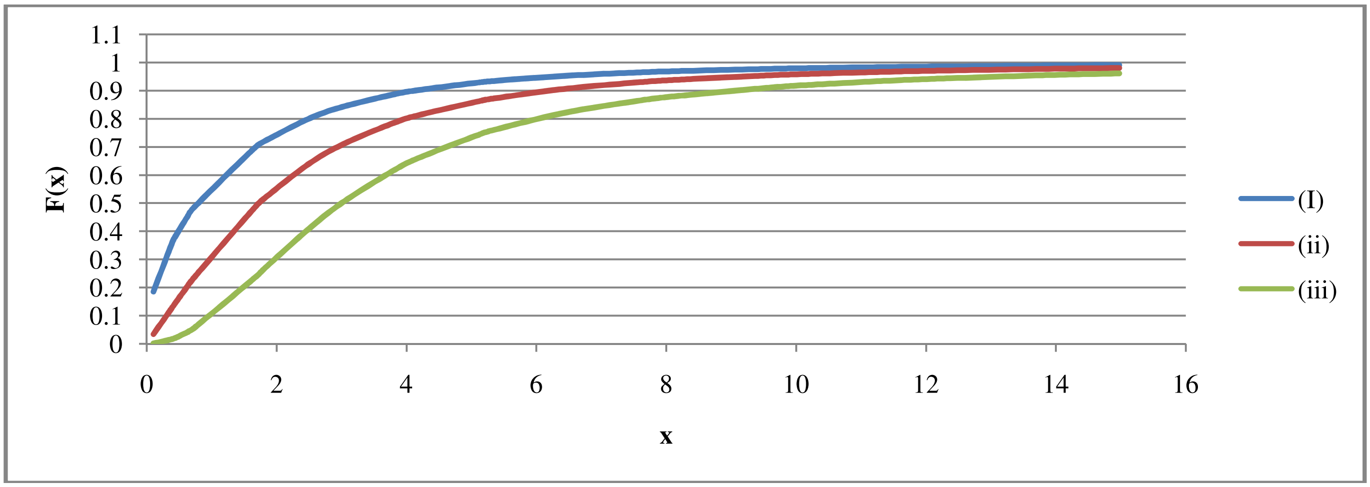

Figure-1: Plot of CDF for typical Parameters of Dagum distribution**.**

Table-1: In Dagum distribution the sample mean of percentiles in different limits.

n	0.99865	0.99	0.975	0.95	0.05	0.025	0.01	0.00135
2	24.5187	9.1919	6.0318	4.1152	0.4473	0.4029	0.3667	0.3265
3	23.3672	7.9278	5.4122	3.8473	0.5040	0.4609	0.4162	0.3624
4	20.1455	7.1094	4.9387	3.6889	0.5429	0.5047	0.4576	0.3970
5	16.5037	6.8046	4.6295	3.5204	0.5712	0.5334	0.4894	0.4333
6	14.1481	6.0280	4.3400	3.3268	0.5971	0.5582	0.5114	0.4503
7	12.9808	5.7843	4.1588	3.2156	0.6119	0.5728	0.5351	0.4746
8	12.0340	5.4248	3.9712	3.1484	0.6306	0.5884	0.5518	0.4853
9	11.2300	5.2639	3.8306	3.0770	0.6487	0.6073	0.5687	0.5083
10	11.0012	5.2287	3.8287	3.0509	0.6649	0.6253	0.5836	0.5212

By using the Table-1 are used in the successive way then we obtain the following mean limits.

Now we have to take

$$P(Z_{0.00135} \leq \bar{x} \leq Z_{0.99865}) = 0.9973 \tag{8}$$

From Equation (8) over repeated sampling, for the i[th] subgroup mean we can have

$$P(Z_{0.00135}\frac{\bar{\bar{x}}}{3} \leq \bar{x}_i \leq Z_{0.99865}\frac{\bar{\bar{x}}}{3}) = 0.9973 \tag{9}$$

It is also denoted as

$$P\left(A^d{}_{2p} \times \bar{\bar{x}} \leq \bar{x}_i \leq A^{dd}{}_{2p} \times \bar{\bar{x}}\right) = 0.9973 \tag{10}$$

Here $\bar{\bar{x}}$ is mean of the mean, $A^d{}_{2p} = \frac{Z_{0.00135}}{3}$, $A^{dd}{}_{2p} = \frac{Z_{0.99865}}{3}$. Thus $A^d{}_{2p}$, $A^{dd}{}_{2p}$ are the percentile constants of $\bar{x}$ chart for Dagum distribution process that is given in Table-2.

Median chart: Here the samples are taken from Dagum distribution. In this manner sample median covers the probability is 99.73%, then we calculate Lower and Upper values therefore

$$P(LL \leq m \leq UL) = 0.9973 \tag{11}$$

Here median is denoted by m and perform the n(sample size) simulation runs then the percentiles of these values are in the Table-3.

Table-2: Percentiles Constants of $\bar{x}$ -chart.

n	$A^d{}_{2P}$	$A^{dd}{}_{2P}$
2	0.1088	8.1720
3	0.1208	7.7890
4	0.1323	6.7151
5	0.1444	5.5012
6	0.1501	4.7160
7	0.1582	4.3299
8	0.1617	4.0113
9	0.1694	3.7433
10	0.1737	3.6670

Table-3: In Dagum distribution the Percentiles of sample Median in different limits.

n	0.99865	0.99	0.975	0.95	0.05	0.025	0.01	0.00135
2	24.5187	9.1916	6.0318	4.1152	0.4473	0.4029	0.3667	0.3265
3	7.2761	4.3270	3.1625	2.2789	0.4745	0.4238	0.3762	0.3336
4	5.6168	3.5047	2.6909	2.0515	0.4861	0.4402	0.4029	0.3551
5	4.8233	2.8870	2.0866	1.3920	0.4934	0.4535	0.4144	0.3606
6	3.7721	2.4206	1.8512	1.1594	0.5152	0.4763	0.4369	0.3727
7	3.2251	2.2244	1.4787	1.0397	0.5223	0.4824	0.4451	0.3757
8	2.7158	1.9730	1.5022	1.0282	0.5342	0.4953	0.4576	0.3934
9	2.6792	1.7585	1.2181	1.0180	0.5380	0.5097	0.4640	0.4003
10	2.5701	1.6141	1.2068	1.0167	0.5481	0.5129	0.4784	0.4227

By using the Table-3 we obtain the control limits for median, from the distribution of m,

Consider

$$P(Z_{0.00135} \leq m \leq Z_{0.99865}) = 0.9973 \quad (12)$$

From the above equation we can have to finished replicated sampling, for the ith subgroup median is

$$P(Z_{0.00135}\frac{\overline{m}}{1.7320} \leq m_i \leq Z_{0.99865}\frac{\overline{m}}{1.7320}) \quad (13)$$

This can be written as

$$P\left(A^{d}{}_{7p} \times \overline{m} \leq \overline{m_i} \leq A^{dd}{}_{7p} \times \overline{\overline{m}}\right) = 0.9973 \quad (14)$$

Let us denote the mean of subgroup of medians is $\overline{m}$. Thus A^{d}_{2P}, A^{dd}_{2p} are the percentile constants of median given inTable-4.

Mid-range chart: We have to search two limit of the sampling Mid–range in DD**.** Probability of these limit is 0.9973. We have to calculate Lower and Upper values so that

$$P(LL \leq M \leq UL) = 0.9973 \quad (15)$$

Let us denote the mean of subgroup of mid-range is M. By using simulation technique the percentiles of M are given in Table-5.

Table-4: Percentiles constants of Median–chart.

n	A^{d}_{2P}	A^{dd}_{2P}
2	0.1885	14.1562
3	0.1926	4.2009
4	0.2050	3.2429
5	0.2082	2.7842
6	0.2151	2.1778
7	0.2169	1.8620
8	0.2271	1.5680
9	0.2311	1.5468
10	0.2440	1.4838

Table-5: In Dagum distribution the percentiles of Mid-Range in different levels.

n	0.99865	0.99	0.975	0.95	0.05	0.025	0.01	0.00135
2	24.5187	9.1916	6.0318	4.1152	0.4473	0.4029	0.3667	0.3265
3	37.6812	10.9563	7.0416	4.8475	0.5101	0.4704	0.4193	0.3646
4	37.8812	12.3229	7.8501	5.5489	0.5582	0.5130	0.4683	0.3979
5	40.2452	13.8768	8.9501	6.2499	0.5971	0.5549	0.5124	0.4465
6	40.2773	14.9554	9.7007	6.8673	0.6234	0.5857	0.5415	0.4761
7	40.3894	15.3751	10.5090	7.3823	0.6390	0.6062	0.5694	0.4932
8	43.4547	16.6883	10.8245	7.7775	0.6534	0.6202	0.5834	0.5136
9	47.5686	17.8733	11.5795	8.2713	0.6375	0.6388	0.6002	0.5237
10	53.1730	19.5093	12.5933	8.8990	0.6808	0.6529	0.6227	0.5592

By using above Table-5 are used in the successive way then we obtain the control limits for Mid-range. From the distribution of M, therefore

$$P(Z_{0.00135} \leq M \leq Z_{0.99865}) = 0.9973 \tag{16}$$

From equation (16) for i^{th} subgroup we can have,

$$P\left(Z_{0.00135}\frac{\overline{M}}{\frac{\alpha_{(1)}+\alpha_{(2)}}{2}} \leq M_i \leq Z_{0.99865}\frac{\overline{M}}{\frac{\alpha_{(1)}+\alpha_{(2)}}{2}}\right) = 0.9973 \tag{17}$$

It is also denoted by

$$\mathrm{P}(\mathrm{A}^{d}{}_{4P}\overline{M} \leq M_i \leq \mathrm{A}^{dd}{}_{4P}\overline{M}) = 0.9973 \tag{18}$$

Where $\overline{M}$ mean of mid-ranges. Thus $A^{d}{}_{4P} = \frac{2Z_{0.00135}}{\alpha_{(1)}+\alpha_{(n)}}, A^{dd}{}_{4P} = \frac{2Z_{0.99865}}{\alpha_{(1)}+\alpha_{(n)}}$ are the percentile constants of mid-range chart for Dagum distribution Process data given in Table-6.

R-chart: Here the samples are taken from Dagum distribution. In such a way R covers three probability of the limits is 0.9973. So, we want to calculate Lower and Upper values so that

$$P(LL \leq R \leq UL) = 0.9973$$

Let us denote the mean of subgroup of range is R. Thus $A^{d}{}_{2P}$, $A^{dd}{}_{2p}$ are the percentile constants of range given in Table-7.

Table-6: Percentiles constants of Mid-range chart.

n	$A^{d}{}_{4P}$	$A^{dd}{}_{4P}$
2	0.2243	16.8438
3	0.2098	21.6857
4	0.2015	19.1895
5	0.2009	18.1088
6	0.1946	16.4682
7	0.1844	15.1025
8	0.1779	15.0570
9	0.1698	15.4255
10	0.1699	16.1512

Table-7: In Dagum distribution the percentiles of Range in different levels

n	0.99865	0.99	0.975	0.95	0.05	0.025	0.01	0.00135
2	46.6285	15.0007	9.4976	6.2331	0.0332	0.0176	0.0069	0.0011
3	74.4184	20.8062	12.7476	8.3693	0.1481	0.1052	0.0713	0.0236
4	74.5714	23.6588	14.5571	10.0390	0.2623	0.2056	0.1518	0.0618
5	79.2355	26.7721	16.7852	11.4735	0.3634	0.3008	0.2300	0.1323
6	79.6965	29.0791	18.6873	12.7686	0.4378	0.3747	0.3047	0.2024
7	79.7825	29.8817	20.0890	13.9097	0.5052	0.4414	0.3748	0.2537
8	86.2206	32.6782	20.8038	14.7679	0.5554	0.4819	0.4171	0.2849
9	94.3770	34.8712	22.2130	15.7619	0.5999	0.5340	0.4571	0.3436
10	105.6102	38.3361	24.4404	16.9862	0.6425	0.5787	0.5102	0.4162

From the above Table-7 the percentiles of range in DD are used in the following manner to get the control limits for sample range. From distribution of R, Consider

$$P(Z_{0.00135} \le R \le Z_{0.99865}) = 0.9973 \qquad (19)$$

From the above equation, we obtain the following

$$P(Z_{0.00135} \frac{\overline{R}}{\alpha_{(n)-}\alpha_{(1)}} \le R_i \le Z_{0.99865} \frac{\overline{R}}{\alpha_{(n)-}\alpha_{(1)}}) = 0.9973 \qquad (20)$$

This can be written as

$$P(D^d{}_{3P}\overline{R} \le R_i \le D^{dd}{}_{4p}\overline{R}) = 0.9973 \qquad (21)$$

where $\overline{R}$ is mean of ranges, $R_{i,}$ is i[th] subgroup range .Thus $D^d{}_{3P} = \frac{Z_{0.00135}}{\alpha_{(n)} - \alpha_{(1)}}$, $D^{dd}{}_{4P} = \frac{Z_{0.00135}}{\alpha_{(n)} - \alpha_{(1)}}$ are the percentile constants of R Chart for Dagum distribution process data given in Table-8.

Comparative Study

In section-2 the statistics mean, median, mid range, and range control chart constants are developed and the samples are taken from the population based on Dagum distribution. If we apply this for real datasets, the data must follow the Dagum distribution. To evaluate to achieve their application for a true Dagum distribution data in relation to the application of shewart limits for the help of the power of the control limits. Overcome this type of problem, the samples are taken from the Dagum distribution and use the simulation technique for different n values i.e., for n=2, n=3 … and n=10 and done its comparative study, and find the boundaries for mean, median, mid-range, and range in this continuation. Dagum distribution coverage probabilities are calculated that is the number of calculated these values that fall inside the boundary values respectively. In the statistical tables available the Shewart constants, using these constants to calculate in the previous manner. These calculate values are known as Shewart coverage probabilities. The Dagum distribution and shewart limits of the coverage probabilities are presented in the following Tables -9, 10, 11 and 12.

Table-8: Percentiles constants of Range chart.

n	$D^d{}_{3P}$	$D^{dd}{}_{4P}$
2	0.0007	29.6977
3	0.0098	31.0542
4	0.0207	25.0449
5	0.0374	22.4107
6	0.0502	16.7772
7	0.0561	17.6513
8	0.0573	17.3524
9	0.0638	17.5281
10	0.0715	18.1548

Table-9: Coverage probabilities within the limits of mean-chart.

	DD percentage values			Shewart values		
n	$A^d{}_{2p}\overline{x}$	$A^{dd}{}_{2p}\overline{x}$	count	$\overline{\overline{x}} - A_2\overline{R}$	$\overline{\overline{x}} + A_2\overline{R}$	count
2	0.3313	24.8824	9970	0	8.8341	9617
3	0.3694	23.8236	9982	0	7.8413	9588
4	0.4061	20.6095	9994	0	7.3283	9581
5	0.4467	17.0158	9984	0	7.0572	9596
6	0.4652	14.6164	9976	0	6.8312	9615
7	0.4904	13.4227	9988	0	6.6438	9636
8	0.5016	12.4399	9992	0	6.4907	9641
9	0.5267	11.6365	9999	0	6.3876	9635
10	0.5402	11.4034	9994	0	6.2845	9664

Table-10: Coverage probabilities within the limits of Median Chart.

	DD percentage values			Shewart values		
n	$A^{d}{}_{7p}\overline{m}$	$A^{dd}{}_{7p}\overline{m}$	count	$\overline{m} - A_2\overline{R}$	$\overline{m} + A_2\overline{R}$	count
2	0.5739	43.0988	9765	0	8.8341	9617
3	0.4507	9.8315	9820	0	7.3287	9832
4	0.4782	7.5654	9890	0	6.9836	9886
5	0.4530	6.0590	9849	0	6.7106	9925
6	0.4680	4.7371	9772	0	6.6564	9967
7	0.4565	3.9192	9526	0	6.5113	9970
8	0.4775	3.2964	9147	0	6.4369	9987
9	0.4792	3.2079	9115	0	6.3939	9988
10	0.5048	3.0694	9037	0	6.3874	9996

Table-11: Coverage probabilities within the limits of Mid-range chart.

	DD percentage values			Shewart values		
n	$A^{d}{}_{4p}\overline{mr}$	$A^{dd}{}_{4p}\overline{mr}$	count	$\overline{mr} - A_4\overline{R}$	$\overline{mr} + A_4\overline{R}$	count
2	0.6828	51.2809	9764	0	9.8904	9711
3	0.7171	74.1176	9849	0	9.3366	9549
4	0.7669	73.0202	9928	0	8.6428	9381
5	0.8442	76.0917	9954	0	9.0935	9304
6	0.8857	74.9306	9973	0.1999	8.9000	9151
7	0.8958	73.3619	9990	0.4510	9.2641	9085
8	0.9117	77.1373	9994	1.0973	9.1486	8917
9	0.9190	83.4785	9997	1.3250	9.4983	8840
10	0.9632	91.5886	9997	1.8465	9.4948	8447

Table-12: Coverage probabilities within the limits of Range chart.

	DD percentage values			Shewart values		
n	$D^{d}{}_{3p}\overline{R}$	$D^{dd}{}_{4p}\overline{m}$	count	$D_3\overline{R}$	$D_4\overline{R}$	Count
2	0.0021	91.45739	9984	0	10.0610	9453
3	0.0460	145.1848	9995	0	12.0386	9317
4	0.1212	146.3277	10000	0	13.3328	9209
5	0.2570	153.9686	10000	0	14.5306	9138
6	0.3880	152.8091	10000	0	15.4839	9072
7	0.4747	149.2936	10000	0.6428	16.2730	9005
8	0.5210	157.6864	10000	1.2358	16.9387	8916
9	0.6209	170.5509	10000	1.7903	17.6698	8816
10	0.7374	187.1362	10000	2.2986	18.3169	8676

Conclusion

From the Tables-9,10,11,12 gives that for a true Dagum distribution, In generally the process variation for mean ,median, mid-range and range charts are shows the result as less confidence coefficient by using the Shewart limits. So that we interpret the data must be follow the Dagum distribution. The Shewart constants for statistics mean, median, mid-range and range charts method is not preferable. Hence we conclude that exclusionary evaluation applications of Dagum distribution constants is better in statistical quality control.

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