Research Journal of Mathematical and Statistical Sciences _____ISSN 2320-6047
Vol. **6(8),** 1-6, December **(2018)** *Res. J. Mathematical and Statistical Sci.*

Case Study

Statistical analysis of melody lengths with a new formula for computing melodic weight of a note in a song: a case study

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Available online at: www.isca.in, www.isca.me
Received 30th September 2018, revised 12th November 2018, accepted 10th December 2018

Abstract

Melody is that part of music which is pleasant while rhythm is the part that is periodic. For research purpose, we may take melody more appropriately as a sequence of notes which is complete and hence can be taken as a single entity. For example, each line in a song is a melody. A sequential subset of a melody is called a segment. Length of a melody (or a segment) refers to the number of notes in it. Aye Mere Watan Ke Logo (O people of my country) is a Hindi patriotic song written by the lyricist Pradeep, the tune being composed by C. Ramchandra that was sung by the legendary playback singer Lata Mangeshkar. The present paper provides a statistical analysis of melody lengths of this popular patriotic song. A Chi-Square goodness of fit test reveals that the distribution of the melody lengths over the lines of the song is not uniform, a feature surprisingly found to be common with India's national anthem Jana Gana Mana. A set of descriptive statistical summary measures are next calculated to compare these two songs. It may be noted that analysis of melody lengths is a neglected area in music research. From that angle, the paper provides some food for thought. Additionally, the paper also provides a new formula for calculation of melody weights in a song. The experimental results are encouraging.

Keywords: Melody length, chi-square goodness of fit test, melodic weight, descriptive statistics.

Introduction

Melody is a sequence of notes which is complete and hence can be taken as a single entity. For example, each line in a song is a melody. A sequential subset of a melody is called a segment. Length of a melody (or a segment) refers to the number of notes in it. *Aye Mere Watan Ke Logo* (O people of my country) is a Hindi patriotic song written by the lyricist Pradeep, the tune being composed by C. Ramchandra that was sung by the legendary playback singer Lata Mangeshkar. The present paper provides a statistical analysis of melody lengths of this popular patriotic song. A Chi-Square goodness of fit test reveals that the distribution of the melody lengths over the lines of the song is not uniform, a feature surprisingly found to be common with Indian national anthem Jana Gana Mana.

A set of descriptive statistical summary measures of the melody lengths are next calculated to compare these two songs. Finally, the melodic weights of the notes in this song are calculated using our novel yet simplistic formula and a comparison is made with those in Indian national anthem.

State of the art

There is a wide range of literature on the scientific studies of music, primarily in western music. See for example Beran[1], Meredith[2], Muller[3], Mazzola et. al[4] and the references cited therein. Some literature on scientific studies in Indian music is also available. See for example Chakraborty et. al[5], Rao and Rao[6] and Datta et. al[7] and the references cited therein. Temperley[8] has connected music and probability using a Bayesian approach. However, the analysis of melody lengths is generally overlooked in music research. This is the issue addressed mainly in this paper. Additionally, a new formula for calculation of melodic weights is provided. For the note sequences of the song Aye Mere Watan Ke Logo and Indian national anthem, we refer the reader to the website[9] and Chakraborty et. al[10] respectively.

Experimental results

In the first study we apply Chi-Square goodness of fit test to verify whether the melody lengths are distributed uniformly over the lines of the song or not. The square of a standard normal variate is a Chi-Square variate with one degree of freedom. The formula for calculating Chi-Square in goodness of fit test is as follows:-

Chi-Square = $\sum(O - E)^2/E$ where O = observed class frequency; E= expected class frequency.

Which follows ChiSquare distribution with k-1 degrees of freedom where k is the number of classes. In case any class frequency is less than 5, adjacent classes will be pooled resulting in reduction in degrees of freedom. The summation is over all the k classes.

Our null hypothesis is:

H_0: The melody lengths of the song are uniformly distributed over the lines in the song which is to be tested against the alternative hypothesis:

H_1: The distribution of melody lengths is non-uniform over the lines in the song.

The tests would be carried out al 5% level of significance.

As there are 8 lines in the song and 197 notes in all, the expected frequency, assuming uniform model under H_0, should be 197/8 for each line of the song. The observed frequencies in the 8 lines are obviously different and we need to test if the difference between the observed and the expected frequencies are statistically significant. Our results are summarized in Tables-1 and 2.

Table-3 gives a comparative analysis of the descriptive features of melody lengths between Aye Mre Watan Ke Logo and Indian National anthem. Figure-1 and 2 depict the bar graphs of melody lengths of the two songs.

Table-1: Chi-square test for melody lengths for Aye Mere Watan Ke Logo.

Melody line	O = Observed Frequency	E = Expected Frequency	O-E	$(O-E)^2/E$
Line 1	20	197/8	-37/8	0.8686
Line 2	21	197/8	-29/8	0.5336
Line 3	19	197/8	-45/8	1.2848
Line 4	21	197/8	-29/8	0.5336
Line 5	46	197/8	171/8	18.5539
Line 6	39	197/8	115/8	8.3914
Line 7	22	197/8	-21/8	0.2798
Line 8	09	197/8	-125/8	9.9143

Calculated Chi-Square = $\sum((O-E)^2/E)$ = 40.36, Table Chi-Square at 7 d.f. & 5% level=14.067. Result: Calculated Chi-Square exceeds table Chi-Square; hence H_0 is rejected at 5% level.

Table-2: Chi-square test for melody lengths for National Anthem.

Melody line	O = Observed Frequency	E= Expected Frequency	O-E	$(O-E)^2/E$
Line 1	21	131/6	-5/6	0.0318
Line 2	23	131/6	-7/6	0.0623
Line 3	39	131/6	103/6	13.4976
Line 4	09	131/6	-65/6	5.3753
Line 5	21	131/6	-5/6	0.0318
Line 6	18	131/6	-23/6	0.6730

Chi-Square = $\sum((O-E)^2/E)$ = 19.6716, Table Chi-Square at 5 d.f. & 5% level =11.07. Result: Calculated Chi-Square exceeds table Chi-Square; hence H_0 is rejected at 5% level.

Table-3: Descriptive features of melody lengths between the two songs.

Descriptive Features	Aye Mere Watan Ke Logo	Indian national Anthem
Mean	24.63	21.83
Median	21	21
Mode	21	21
SE Mean	4.21	3.99
First Quartile Q1	19.25	15.75
Third Quartile Q3	34.75	27
Maximum	46	39
Minimum	9	9
Range = Max - Min	37	30
N for mode (No. of repetitions of the modal length in the song)	2	2
Standard Deviation	11.92	9.77
Variance	141.98	95.37
Inter Quartile Range	15.5	11.25
Skewness	0.96	0.93
Kurtosis	0.42	2.54

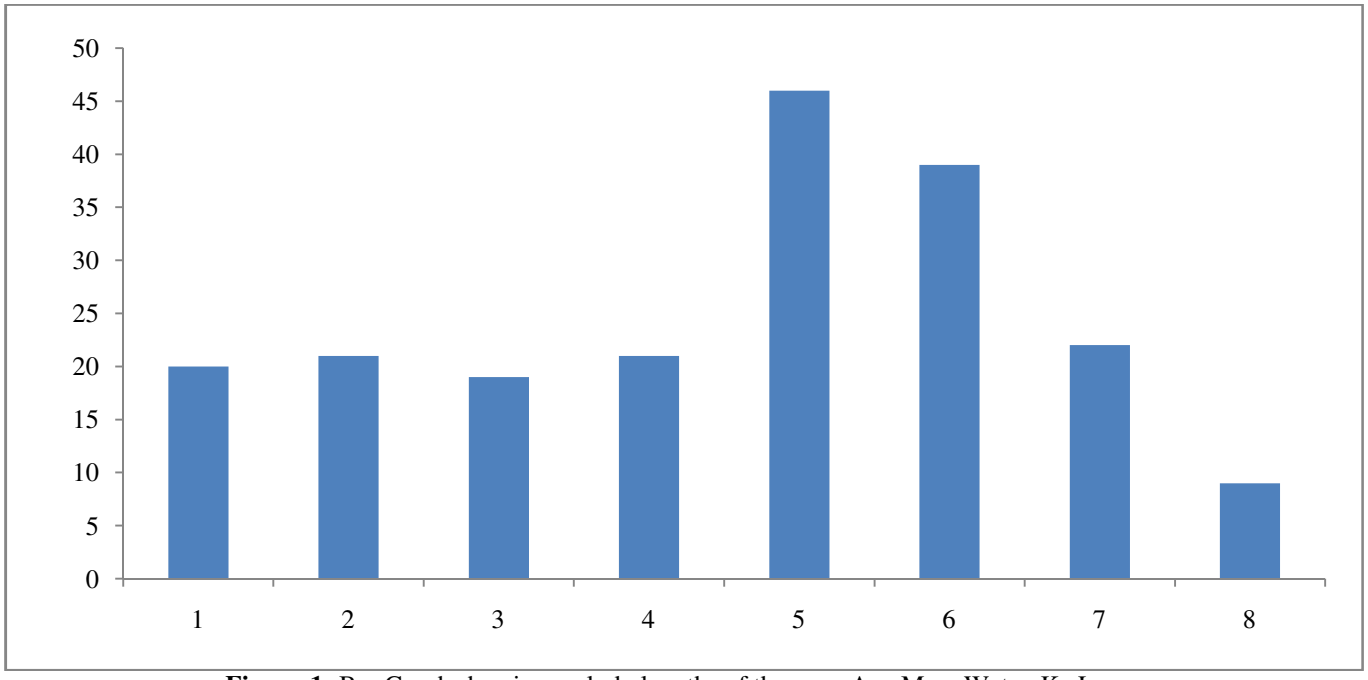

Figure-1: Bar Graph showing melody lengths of the song Aye Mere Watan Ke Logo.

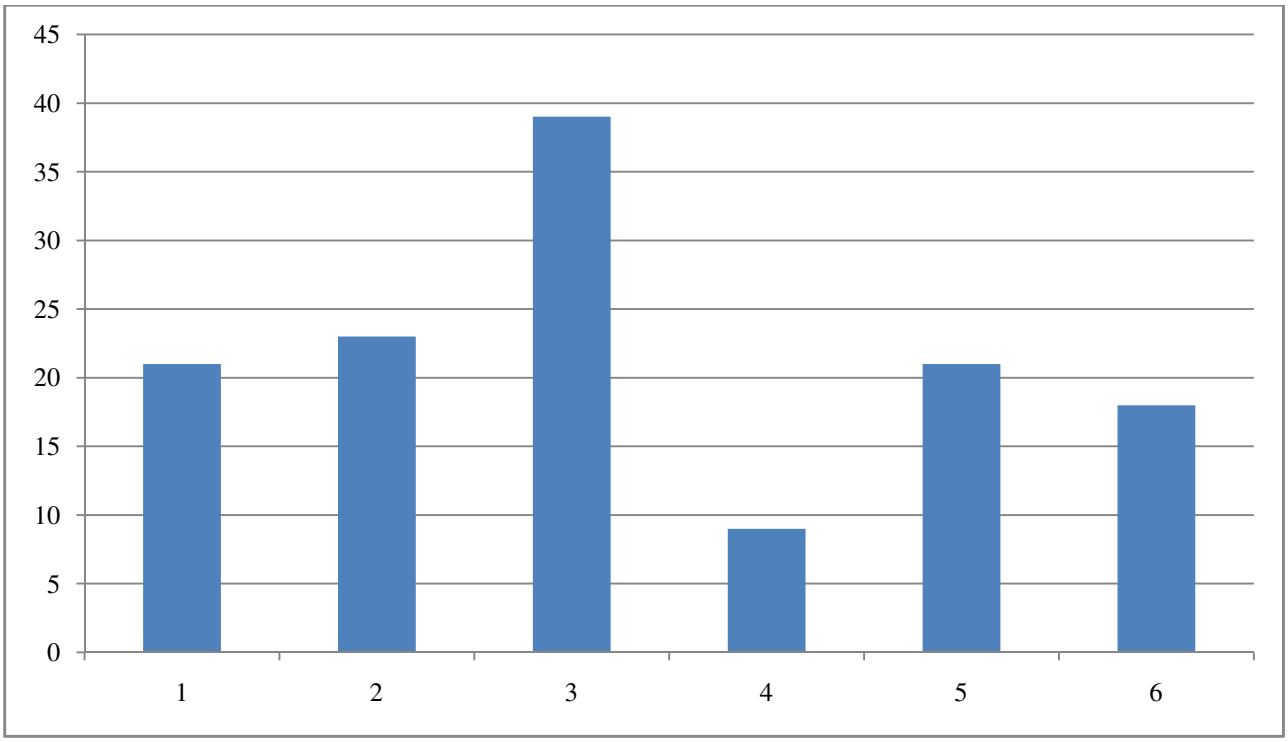

Figure-2: Bar Graph showing melody lengths of India's National Anthem.

In the next study we shall provide a new formula for calculation of melodic weight of a note in a song.

We define melodic weight of a particular note = A x B

Where A is the number of lines in which that particular note occurs and B is the average (mean) number of occurrences of that particular note in the lines of the song. It is to be noted that in doing these calculations the octave in which the note occurs is to be ignored. Our computations are summarized for the two songs in Tables-4 and 5.

Table-4: Melodic weights of notes in India's National Anthem.

Notes	A	B	Weight of the note
C	5	1.5	7.5
D	6	3.67	22.02
E	6	7.17	43.02
F	5	2.83	14.15
F#	2	0.33	0.66
G	4	4.5	18
A	3	0.83	2.49
B	4	1	4

Figure-3 based on Table-4 gives the bar graph for melodic weights of the notes of Indian national anthem.

Figure-4 based on Table-5 gives the bar graph for melodic weights of the notes in the song Aye Mere Watan Ke Logo.

Table-5: Melodic weights of notes in Aye Mere WatanKe Logo.

Notes	A	B	Weight of the note
C	2	1.5	3
D	7	3.125	21.875
E	2	0.375	0.75
F	8	4	32
G	8	8.25	66
A	7	5.375	37.625
Bb	4	0.75	3
B	4	1.2	4.8

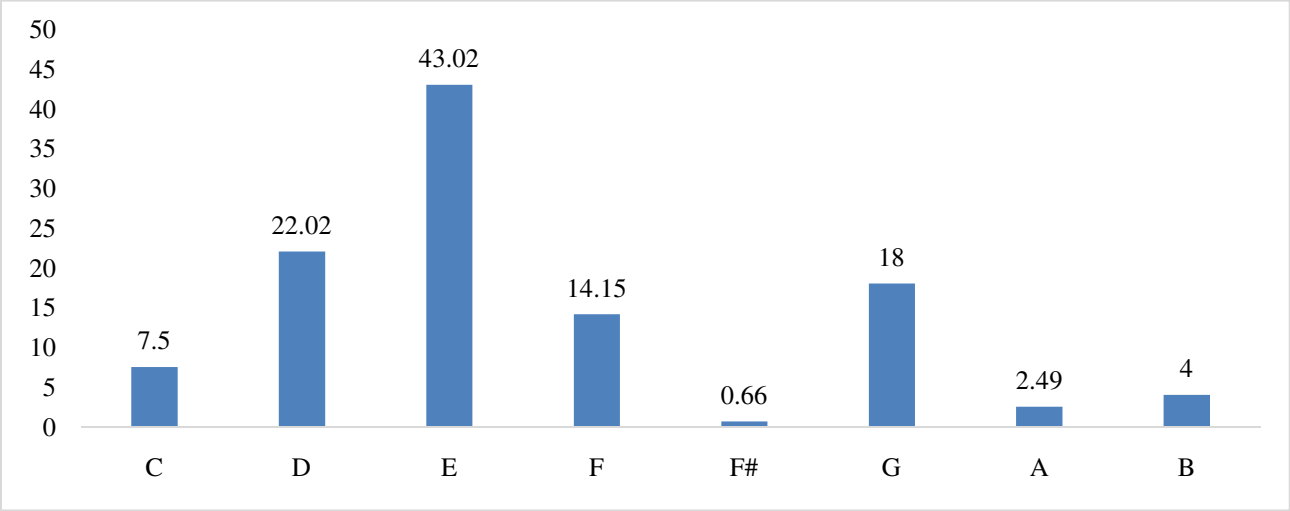

Figure-3: Bar graph showing melodic weight of each individual note in Indian national anthem.

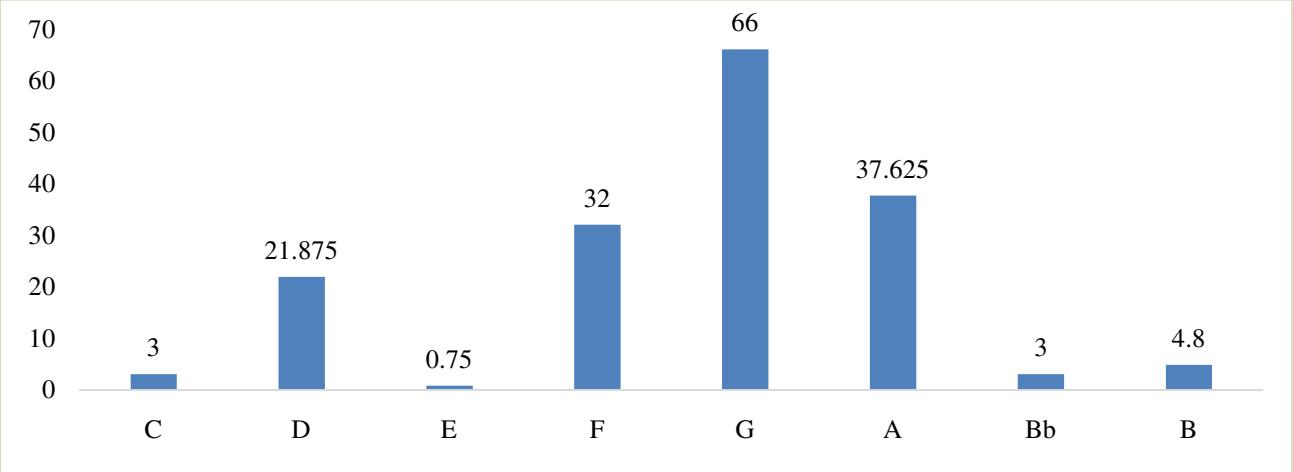

Figure-4: Bar graph showing melodic weight of each individual note in Aye Mere WatanKe Logo.

Remarks: i. Minitab version 16 has been used to generate the numerical results while the graphs have been generated using MS-Excel. ii. Advanced techniques for weight calculation notes exist using mathematical music theory. The idea is to describe each musical note by four features: note onset (point of time of its arrival), pitch (which tells which note it is), note duration and amplitude of the note (which determines its loudness). The weights are calculated based on the information obtained from these features. The interested reader is referred to chapter 3 of Chakraborty et. al[4]. iii. In all the four figures, X-axis is used for the notes. Y-axis is used for melody length (Figure- 1-2) or melodic weights (Figure-3-4), the exact values being mentioned on top of the bars.

Discussion

From the first study on Chi-Square goodness of fit test, we conclude that the melody lengths are varying considerably over the lines in both the songs and hence their distribution is non uniform.

The second study on descriptive features reveals several interesting findings. First, the average length of the melodies in both the songs can be taken to be same (observe that the two songs have identical median and mode and the mean is also very close). This is quite surprising as Indian National Anthem was composed by a different composer (the Nobel laureate Rabindranath Tagore). Also we notice that skewness is positive in both the songs [as mean > mode; Pearson's measure of coefficient of skewness = (mean – mode) / std. dev.] and that the degree of skewness is also quite close! Moreover, the frequency curves of both the songs are flat (platykurtic) as kurtosis < 3. Another similarity is the identical minimum melody length in both which is 9. However, there are differences too. For example, the dispersion of melody lengths is more in Aye Mere Watan Ke Logo as compared to Indian national anthem. Also the maximum melody length is 46 in the former as compared to 39 in the latter. The range is therefore more in the former.

In the third study we observe that the note E has the maximum melodic weight in Indian national anthem whereas the note G

has the maximum melodic weight in Aye Mere WatanKe logo. In fact, the reader can verify that the average (mean) melodic weight of a note in Indian national anthem is 13.98 (with a standard deviation 14.02) while that in Aye Mere Watan Ke Logo is 21.13 (with standard deviation 23.13). Although the average melodic weight is more in the latter, a striking similarity is that both the songs have used 8 distinct notes (out of a possible 12, namely, C, Db, D, Eb, E, F, F#, G, Ab, A, Bb, B; the symbols b and # indicate flat and sharp respectively; a natural note will not be succeeded by these symbols; the tonic is taken at natural C) and that 7 of these 8 notes are identical! The note F# used in Indian National anthem is not used in the other song which has used a note Bb that is not used in Indian national anthem.

Conclusion

Analysis of melody lengths is a neglected issue in music research. Seen from that angle, our paper which compares the melody lengths of two patriotic Indian songs will definitely provide the scientific researchers in music some food for thought. Additionally we have provided a fairly simple formula for calculation of melodic weights in a song which could be useful. The fact that the average melodic weight of the song Aye Mere WatanKe Logo is more than that of Indian national anthem is interesting. While Indian national anthem lifts our spirits for the nation, Aye Mere WatanKe Logo makes us sad as it takes our memory to Indo-China war in 1962. It is a tribute to the Indian soldiers who lost their lives in that war. Is this the melancholy nature which makes the notes of this song melodically heavier? The question is worth a debate. If the answer is in the affirmative, then it leads to an interesting question. We know that in a sad song the note duration is generally more and the tempo is slow. But our study did not compute these features, nor did it consider intensity (loudness). It worked on the note assemblies only. So the debate boils down to the question: in what way the notes should be arranged to create a pattern that can be connected with sadness?

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