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Short Communication

Sized biased migration model

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Abstract

The population migration and its effects on social economic and cultural aspect has recently occupied a considerable place in social science research. In the present paper we have developed a Probability model for the Human migration pattern. The parameters of the proposed model estimated by MLE estimation technique. The validity of our model checked through observed set of migration data.

Keywords: Probability model, Parameters, Estimation technique, Migration process.

Introduction

In modern society migration has play an important role in the development of rural and urban area of any region. The studies in relation to nature and behavior of these components have always been handled with great interest, because the development of a country, to a great extent, depends upon them[1-3].

Model

If $w(x,\beta) = X\,; x^w = X$ is called a sized-biased x and its distribution is called a sized-biased distribution with probability function as

$$f^*(x,\theta) = \frac{x\,f(x,\theta)}{\mu}$$

Where: $\mu = E(x)$, $f^*$is the size-biased form of f.

Let x denote the random number of rural out-migration from a household. Probability model for describing the variation in the number of single male migrants has been obtained on the basis of the following assumptions: i. Due to establishment by migrants in the village at the origin. Let α be the probability that a household is exposed to risk of migration at the time of survey and $(1-\alpha)$ be the Probability that a household is not exposed to risk of migration. ii. X is male migrating from a household follows a sized-biased geometric distribution type with parameter θ, i.e

$$f^*(x,\theta) = x\theta^2(1-\theta)^{x-1} \;; x = 1,2,\dots\dots$$

From assumption (1) and (2) the Model given by the probability function p_x as follows,

$$P(X=x) = \begin{cases} 1-\alpha, x = 0 \\ \alpha x\theta^2(1-\theta)^{x-1}; x = 1,2,\dots\dots \end{cases} \quad (1)$$

Estimation

In model (1), consider a sample consisting of f observations of the random variable X, in which f_0 designates the number of zero observation, f_1 the number of one observation and f the total number of observations. With the help of MLE technique in proposed model (1) Likelihood function takes the following form:

$$L = (1-\alpha)^{f_0}(\alpha\theta^2)^{f_1}[1-(1-\alpha+\alpha\theta^2)]^{f-f_0-f_1} \quad (2)$$

Taking logarithmic on both sides we get

$$\log L == f_0\log(1-\alpha) + f_1\log(\alpha\theta^2) + (f-f_0-f_1)[\alpha-\alpha\theta^2]$$

$$\log L = f_0\log(1-\alpha) + f_1\log(\alpha\theta^2) + (f-f_0-f_1)[\alpha(1-\theta^2)] \quad (3)$$

Differentiating partiality w.r, to α and θ and equating it to be zero.

$$\frac{\partial logL}{\partial logL} = -\frac{f_0}{(1-\alpha)} + \frac{f-f_0}{\alpha} \quad (4)$$

$$\frac{\partial logL}{\partial\theta} = \frac{2f_1}{\theta} - \frac{2(f-f_1-f_0)}{(1-\theta^2)} = 0 \quad (5)$$

The required Maximum Likelihood estimates α and θ can be obtained by simultaneously solving (4) and (5) to facilitate their solutions, the above equations are required to:

$$\hat{\alpha} = \frac{f-f_0}{f} \quad (6)$$

$$\widehat{\theta^2} = \frac{f_1}{f-f_0} \quad (7)$$

The second derivatives of the L follows from (4) and (5) as:

$$\frac{\partial^2 logL}{\partial\alpha^2} = -\frac{f_0}{(1-\alpha)^2} + \frac{f-f_0}{\alpha^2} \tag{8}$$

$$\frac{\partial^2 logL}{\partial\theta^2} = \frac{2f_1}{\theta^2} - \frac{2(f-f_1-f_0)(1+\theta^2)}{(1-\theta^2)^2} \tag{9}$$

$$\frac{\partial^2 logL}{\partial\alpha\,\partial\theta} = \frac{\partial^2 logL}{\partial\alpha\,\partial\theta} = 0 \tag{10}$$

Now using the fact:

$$E(f_0) = f(1-\alpha)$$

$$E(f_1) = f\alpha\theta^2$$

$$E(f - f_1 - f_0) = f\alpha(1-\theta^2)$$

Where: E denote for the expectation. The elements of information Matrix follows from (8), (9) and (10) as:

$$\emptyset_{11} = E^{\left(\frac{\partial^2 logL}{\partial\alpha^2}\right)}\Big/_f = \left[-\frac{1}{\alpha(1-\alpha)}\right]$$

$$\emptyset_{22} = E\left(\frac{\partial^2 logL}{\partial\theta^2}\Big/_f\right) = \left[-\frac{4\alpha}{1-\theta^2}\right]$$

$$\emptyset_{12} = E^{\left(\frac{\partial^2 logL}{\partial\alpha\,\partial\theta}\right)}\Big/_f = \emptyset_{21} = E^{\left(\frac{\partial^2 logL}{\partial\alpha\,\partial\theta}\right)}\Big/_f = 0$$

On inverting the information Matrix, the asymptotic variances as:

$$V(\hat{\alpha}) = \left[\frac{\emptyset_{22}}{\emptyset_{11}\emptyset_{22}-\emptyset_{12}{}^2}\right]\Big/_f = \frac{\alpha(\alpha-1)}{f} \tag{11}$$

$$V(\hat{\theta}) = \left[\frac{\emptyset_{11}}{\emptyset_{11}\emptyset_{22}-\emptyset_{21}{}^2}\right]\Big/_f = \frac{(\theta^2-1)}{4\alpha f} \tag{12}$$

Application

The proposed sized biased migration model is applied to an observed set of migration data from the two different survey[4-6] which given in the Table-1 and 2.

Conclusion

It is observed that the value of x^2 is insignificant at 5% level of significant which shows that, the proposed model satisfied the given migration pattern in the two different survey. The proposed model may be utilized for estimation and prediction of the vital events. So that the present model may be taken as useful tool for calculating the various probabilities of migrants which provide better prediction for the future planning of rural an urban society.

Table-1: Observed and Expected number of Households according to the number of migrants for different household size group (1969 survey).

Number of Migrants	Household size – group			
	(4-6)		(7-9)	
	Observed	Expected	observed	Expected
0	651	651	362	362
1	120	119.93	114	113.696
2	28	26.77	39	39.63
3	3		12	10.40
4	1		2	
5	0	5.3	0	3.01
6	0		0	
7	0		0	
8 & over	0		0	
Total	803		529	
$\hat{\alpha}$	0.1892		0.3156	
$\hat{\theta}$	0.8884		0.8261	
$V(\widehat{\alpha})$	0.00019		0.00040	
$V(\widehat{\theta})$	0.00034		0.00047	
χ^2	0.3753		0.595	
$d.f.$	3		1	

Table-2: Observed and Expected number of household according to the number of migration for different household size group (1978 survey).

Number of migrants	Household size – group			
	(4-6)		(7-9)	
	Observed	Expected	Observed	Expected
0	388	388.02	188	188.001
1	61	60.97	61	60.99
2	12	10.47	17	16.78
3	0		3	
4	0		1	
5	0	1.54	0	4.23
6	0		0	
7	0		0	
8 & over	0		0	
Total	461		270	
$\hat{\alpha}$	0.15835		0.3037	
$\hat{\theta}$	0.9141		0.8624	
$V(\widehat{\alpha})$	0.000289		0.000783	
$V(\widehat{\theta})$	0.00286		0.0002750	
χ^2	1.2192		0.1332	
$d.f.$	1		1	

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