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An application of fuzzy game theory to industrial decision making

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Abstract

In the present article, an application of the fuzzy game is discussed for the best optimal strategy of sugar industries under the uncertain choice of action. Epsilon triangular fuzzy numbers are used as cell entries in fuzzy payoff matrix. Using 'Fuzzy Minimax-Maximin criterion' best optimal strategy is obtained.

Keywords: Epsilon triangular fuzzy number, Fuzzy payoff matrix, Fuzzy two person zero sum game, Optimal Strategy, Fuzzy Minimax-Maximin criterion.

Introduction

The fuzzy set theory in fuzzy decision making was first considered, in 1970 by Bellman and Zadeh[1]. The problem of industrial decision making is considered as an application of fuzzy game. The mathematical analysis of competitive Sugarcane Industry problems is based upon 'Minimax-Maximin Criterion' of J. Von Neuman. Two-person zero-sum game[2-4] is competitive situation which involves only two players called zero-sum game because benefit for one player is the loss for other player[5].

Fuzzy games are intended to model conflict situation with imprecise information, payoff strategies etc. In this paper, we have used 'Fuzzy Minimax-Maximin criterion' for obtaining optimal strategy[6] of sugarcane factory by MAX-MIN approximation[6,7].

In preliminaries we give basic notions which are used in the subsequent part of the paper. In problem definition, application of fuzzy game to Sugar Industry is discussed.

Preliminaries[5]

Basic notions of Two-Person Zero-Sum Game: Each players strategy and the payoff matrix characterizes a two-person zero-sum game. The entries in payoff matrix represent either positive or negative benefit for player I by each combination of strategies for the two players. Both players are rational and greedy.

The two-person zero-sum game needs two players. The sets $S_1=\{\gamma_1,\gamma_2,\gamma_3,\ldots,\gamma_n\}$ and $S_2=\{\beta_1,\beta_2,\beta_3,\ldots,\beta_n\}$ denotes pure strategies for two players I and II respectively. A payoff matrix for the player I and player II is respectively given by

$$A=(a_{ij})=\begin{array}{c|c} & \begin{array}{cccc}\beta_1 & \beta_2 & \cdots & \beta_n\end{array}\\ \begin{array}{c}\gamma_1\\ \gamma_2\\ \vdots\\ \gamma_m\end{array} & \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n}\\ a_{21} & a_{22} & \cdots & a_{2n}\\ \vdots & \vdots & \vdots & \vdots\\ a_{m1} & a_{m2} & \cdots & a_{mn}\end{pmatrix}\end{array} \text{ and } -A=(-a_{ij})=\begin{array}{c|c} & \begin{array}{cccc}\beta_1 & \beta_2 & \cdots & \beta_n\end{array}\\ \begin{array}{c}\gamma_1\\ \gamma_2\\ \vdots\\ \gamma_m\end{array} & \begin{pmatrix} -a_{11} & -a_{12} & \cdots & -a_{1n}\\ -a_{21} & -a_{22} & \cdots & -a_{2n}\\ \vdots & \vdots & \vdots & \vdots\\ -a_{m1} & -a_{m2} & \cdots & -a_{mn}\end{pmatrix}\end{array}$$

The optimal strategy of the players in two-person zero-sum game is based on a 'Minimax-Maximin criterion' principle.

Definition-1: A fuzzy number A is a subset of the set $\mathbb{R}$ with membership function $A:\mathbb{R}\to[0,1]$ such that A is normal, fuzzy convex and upper semi-continuous. Support of A is bounded. If left hand curve and right hand curve are straight lines then the fuzzy number is called triangular fuzzy number[8].

Definition-2: If r is a real number then *ε-δ fuzzy number* $r_{\varepsilon,\delta}$ is the triangular fuzzy number for some $\varepsilon,\delta\in\mathbb{R}$, $(\varepsilon,\delta>0)$ is a fuzzy set $r_{\varepsilon,\delta}:\mathbb{R}\to[0,1]$ defined by[9]

$$r_{\varepsilon,\delta}(x)=\begin{cases}\dfrac{x-(r-\varepsilon)}{\varepsilon}, & \text{if } r-\varepsilon<x\le r,\\ \dfrac{x-(r+\delta)}{-\delta}, & \text{if } r<x\le r+\delta,\\ 0, & \text{otherwise.}\end{cases}$$

Definition-3: If r is a real number then ε fuzzy number r_ε is called as *symmetric epsilon fizzy number* which is triangular fuzzy number for some $\varepsilon\in\mathbb{R}$, $(\varepsilon>0)$ is a fuzzy set $r_\varepsilon:\mathbb{R}\to[0,1]$ defined by[9]

$$r_{\varepsilon}(x)=\begin{cases}\dfrac{x-(r-\varepsilon)}{\varepsilon}, & \text{if } r-\varepsilon<x\le r,\\ \dfrac{x-(r+\varepsilon)}{-\varepsilon}, & \text{if } r<x\le r+\varepsilon,\\ 0, & \text{otherwise.}\end{cases}$$

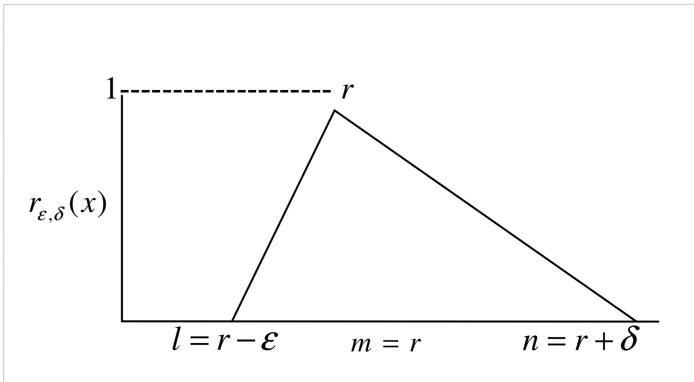

Figure-1: Membership functions of the fuzzy number $r_{\varepsilon,\delta}$ [7].

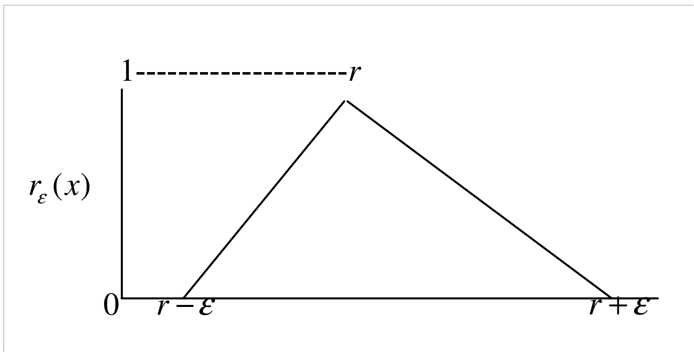

Figure-2: Membership functions of the fuzzy number r_{ε}, [7].

The above notation for triangular fuzzy number is simple and may be considered as a family of functions of three parameters.

Definition-4: Let $r_{\varepsilon_1,\delta_1}$ and $s_{\varepsilon_2,\delta_2}$ be any two epsilon-delta fuzzy numbers where $r \le s$. If $r-\varepsilon_1 \le s-\varepsilon_2$ and $r+\delta_1 \le s+\delta_2$ then we define[7]

$(r_{\varepsilon_1,\delta_1} \wedge s_{\varepsilon_2,\delta_2}) = r_{\varepsilon_1 \vee \varepsilon_2, \delta_1 \wedge \delta_2}$ and $r_{\varepsilon_1,\delta_1} \vee s_{\varepsilon_2,\delta_2} = s_{\varepsilon_1 \wedge \varepsilon_2,\, \delta_1 \vee \delta_2}$

If $s-\varepsilon_2 < r-\varepsilon_1$ and $r+\delta_1 \le s+\delta_2$ then we define

$(r_{\varepsilon_1,\delta_1} \wedge s_{\varepsilon_2,\delta_2}) = r_{\varepsilon_2-(s-r),\delta_1}$ and $(r_{\varepsilon_1,\delta_1} \vee s_{\varepsilon_2,\delta_2}) = s_{\varepsilon_1+(s-r),\delta_2}$.

If $r-\varepsilon_1 \le s-\varepsilon_2$ and $s+\delta_2 < r+\delta_1$ then

$(r_{\varepsilon_1,\delta_1} \wedge s_{\varepsilon_2,\delta_2}) = r_{\varepsilon_1,\delta_2+(s-r)}$ and $(r_{\varepsilon_1,\delta_1} \vee s_{\varepsilon_2,\delta_2}) = s_{\varepsilon_2,\delta_1-(s-r)}$.

If $s-\varepsilon_2 < r-\varepsilon_1$ and $s+\delta_2 < r+\delta_1$ then

$r_{\varepsilon_1,\delta_1} \wedge s_{\varepsilon_2,\delta_2} = r_{\varepsilon_2-(s\text{-}r),\, \delta_2+(s\text{-}r)}$ and
$(r_{\varepsilon_1,\delta_1} \vee s_{\varepsilon_2,\delta_2}) = s_{\varepsilon_1+(s-r),\delta_1-(s-r)}$.

The above evaluations are triangular approximations of max and min operations on fuzzy numbers. Many researchers have used Dubois and Prade approximation of max and min operations given below

$r_{\varepsilon_1,\delta_1} \wedge s_{\varepsilon_2,\chi_2} = (r \wedge s)_{\varepsilon_1 \vee \varepsilon_2,\, \delta_1 \vee \delta_2}$ and
$r_{\varepsilon_1,\delta_1} \vee s_{\varepsilon_2,\chi_2} = (r \vee s)_{\varepsilon_1 \wedge \varepsilon_2,\, \delta_1 \vee \delta_2}$.

Addition or subtraction of the term $|r-s|$ may be considered as a correcting factor and thereby gives better approximation.

Definition-5: A quantitative measure of satisfaction of a person expressed in terms of epsilon-delta fuzzy numbers is called *epsilon-delta payoff* [6]

Suppose player A has m activities and player B has n activities. Then a fuzzy Payoff matrix can be formed by adopting following rules.

Table-1: The player A's fuzzy payoff matrix

	Player B						
Player A	Strategies	1	2	…	j	…	n
	1	$r^{11}_{\varepsilon_{11},\delta_{11}}$	$r^{12}_{\varepsilon_{12},\delta_{12}}$	….	$r^{1j}_{\varepsilon_{1j},\delta_{1j}}$	…..	$r^{1n}_{\varepsilon_{1n},\delta_{1n}}$
	2	$r^{21}_{\varepsilon_{21},\delta_{21}}$	$r^{22}_{\varepsilon_{22},\delta_{22}}$	…..	$r^{2j}_{\varepsilon_{2j},\delta_{2j}}$	……	$r^{2n}_{\varepsilon_{2n},\delta_{2n}}$
	: :	: :	: :	…..	: :	……	: :
	i	$r^{i1}_{\varepsilon_{i1},\delta_{i1}}$	$r^{i2}_{\varepsilon_{i2},\delta_{i2}}$	……	$r^{ij}_{\varepsilon_{ij},\delta_{ij}}$	…….	$r^{in}_{\varepsilon_{in},\delta n}$
	: :	: :	: :	…..	: :	……	: :
	m	$r^{m1}_{\varepsilon_{m1},\delta_{m1}}$	$r^{m2}_{\varepsilon_{m2},\delta m2}$	……	$r^{mj}_{\varepsilon_{mj},\delta_{mj}}$	…….	$r^{mn}_{\varepsilon_{mn},\delta_{mn}}$

The cell entries in player B's fuzzy payoff matrix is exactly negative of as A's fuzzy payoff matrix as sum of payoff matrices of player A and player B is zero.

We denote above fuzzy payoff matrix as follows

$$A_{E,D} = \begin{bmatrix} \overset{11}{r}_{\varepsilon_{11},\delta_{11}} & \overset{12}{r}_{\varepsilon_{12},\delta_{12}} & \cdots & \overset{1n}{r}_{\varepsilon_{1n},\delta_{1n}} \\ \overset{21}{r}_{\varepsilon_{21},\delta_{21}} & \overset{22}{r}_{\varepsilon_{22},\delta_{22}} & \cdots & \overset{2n}{r}_{\varepsilon_{2n},\delta_{2n}} \\ \vdots & \vdots & \vdots & \vdots \\ \overset{m1}{r}_{\varepsilon_{m1},\delta_{m1}} & \overset{m2}{r}_{\varepsilon_{m2},\delta_{m2}} & \cdots & \overset{mn}{r}_{\varepsilon_{mn},\delta_{mn}} \end{bmatrix}_{m\times n},$$

where $$[A] = \begin{bmatrix} r^{11} & r^{12} & \cdots & r^{1n} \\ r^{21} & r^{22} & \cdots & r^{2n} \\ \vdots & \vdots & \vdots & \vdots \\ r^{m1} & r^{m2} & \cdots & r^{mn} \end{bmatrix}_{m\times n},$$

$$E = [\varepsilon_{ij}] = \begin{bmatrix} \varepsilon_{11} & \varepsilon_{12} & \cdots & \varepsilon_{1n} \\ \varepsilon_{21} & \varepsilon_{22} & \cdots & \varepsilon_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ \varepsilon_{m1} & \varepsilon_{m2} & \cdots & \varepsilon_{mn} \end{bmatrix}_{m\times n} \text{ and}$$

$$D = [\delta_{ij}] = \begin{bmatrix} \delta_{11} & \delta_{12} & \cdots & \delta_{1n} \\ \delta_{21} & \delta_{22} & \cdots & \delta_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ \delta_{m1} & \delta_{m2} & \cdots & \delta_{mn} \end{bmatrix}_{m\times n}$$

Fuzzy Minimax-Maximin Criterion[6]

Let, P be the payoff matrix for a two person zero sum game where $P : X \to [0,1]$.

If $\vee_i \left(\wedge_j \overset{ij}{r}_{\varepsilon_{ij},\delta_{ij}} \right) = \underline{r}$ and $\wedge_j \left(\vee_i \overset{ij}{r}_{\varepsilon_{ij},\delta_{ij}} \right) = \overline{r}$ then $\overline{r} \geq \underline{r}$

i.e. $\wedge_j \left(\vee_i \overset{ij}{r}_{\varepsilon_{ij},\delta_{ij}} \right) \geq \vee_i \left(\wedge_j \overset{ij}{r}_{\varepsilon_{ij},\delta_{ij}} \right)$

Problem Definition

The largest economic activity providing direct and indirect opportunities is agriculture sector in India. The present study has been made in Sangli district of Maharashtra state. The primary data obtained from the decision maker is not deterministic. Hence it is necessary to process this non deterministic data by fuzzy set theory. Sugar industry needs to attract more sugarcane producers in order to optimize the benefit against various constraints such as near distance, maximum rate, more reliability, good recovery, less deduction, more availability of sugarcane in own zone etc. In present problem we develop payoff matrix by considering only four constraints.

Development of Fuzzy decision making model: Let *A* and *B* are two different sugar factories in Sangli district. We consider near distance, maximum rate, more reliability and less deduction as constraints for obtaining optimal strategy.

Table-2: Primary Data for the season 2014-2015.

Factory	Distance (Km)	Reliability	Rate in Rs.	Deduction in Rs.
A	5 km	0.8	Rs. 2650	Rs. 159
B	14 km	0.9	Rs. 2715	Rs. 109

Table-3: Normalization of data.

Factory	Distance	Reliability	Rate	Deduction
A	0.26	0.47	0.49	0.59
B	0.74	0.53	0.51	0.41

We define payoff matrix considering the following rules.

Determination of diagonal elements:

$$P_{ii} = B_i - A_i, \quad i = 1,2,3$$

Determination of non diagonal elements:

$$P_{ij} = A_i \times B_j, \quad i \neq j$$

Table-4: Payoff Matrix

		Factory *B*			
		Near distance	More Reliability	Maximum Rate	Less Deduction
Factory *A*	Near distance	0.47	0.14	0.13	0.11
	More Reliability	0.35	–0.06	–0.24	–0.19
	Maximum Rate	0.36	–0.26	–0.02	–0.20
	Less Deduction	0.44	0.31	0.30	–0.02

Calculations:

$$P_{11} = B_1 - A_{1,} \quad (i = 1)$$

$\therefore P_{11} = 0.74 - 0.26 = 0.47$ and the rest.

And

$P_{12} = A_1 \times B_2, \quad i \neq j$

$\therefore P_{12} = 0.26 \times 0.53 = 0.14$ and the rest.

Table-5: Fuzzy payoff matrix

Factory A \ Factory B	Near distance	More Reliability	Maximum Rate	Less Deduction
Near distance	$47_{10,10}$	$14_{10,10}$	$13_{10,10}$	$11_{10,10}$
More Reliability	$35_{10,10}$	$-6_{10,10}$	$-24_{10,10}$	$-19_{10,10}$
Maximum Rate	$36_{10,10}$	$-26_{10,10}$	$-2_{10,10}$	$-80_{10,10}$
Less Deduction	$44_{10,10}$	$-31_{10,10}$	$-30_{10,10}$	$-2_{10,10}$

MAX MIN Approximation: Row Minimum:

$$\text{Row I} : 47_{10,10} \wedge 14_{10,10} \wedge 13_{10,10} \wedge 11_{10,10} = 11_{10,10}$$

$$\text{Row II} : 35_{10,10} \wedge -6_{10,10} \wedge -24_{10,10} \wedge -19_{10,10} = -24_{10,10}$$

$$\text{Row III} : 36_{10,10} \wedge -26_{10,10} \wedge -2_{10,10} \wedge -80_{10,10} = -80_{10,10}$$

$$\text{Row IV} : 44_{10,10} \wedge -31_{10,10} \wedge -30_{10,10} \wedge -2_{10,10} = -31_{10,10}$$

Fuzzy Maximin

$$11_{10,10} \vee -24_{10,10} \vee -80_{10,10} \vee -31_{10,10} = 11_{10,10}$$

Column Maximum:

$$\text{Column I} : 47_{10,10} \wedge 35_{10,10} \wedge 36_{10,10} \wedge 44_{10,10} = 47_{10,10}$$

$$\text{Column II} : 14_{10,10} \wedge -6_{10,10} \wedge -26_{10,10} \wedge -31_{10,10} = 14_{10,10}$$

$$\text{Column III} : 13_{10,10} \wedge -24_{10,10} \wedge -2_{10,10} \wedge -30_{10,10} = 13_{10,10}$$

$$\text{Column IV} : 11_{10,10} \wedge -19_{10,10} \wedge -80_{10,10} \wedge -2_{10,10} = 11_{10,10}$$

Fuzzy Minimax

$$47_{10,10} \vee 14_{10,10} \vee 13_{10,10} \vee 11_{10,10} = 11_{10,10}$$

Thus, $\wedge_j \left(\vee_i r^{ij}_{\varepsilon_{ij},\varepsilon_{ij}} \right) = 11_{10,10} = \vee_i \left(\wedge_j r^{ij}_{\varepsilon_{ij},\varepsilon_{ij}} \right)$.

Conclusion

In the present article we developed the application of fuzzy game theory to industrial decision making. The information given by farmers is non deterministic and is modeled in terms of fuzzy sets. The 'Fuzzy Minimax-Maximin criterion' is used for obtaining best optimal strategy for sugar factory A and B .

Thus, the fuzzy optimal strategy for Factory A is I (near distance) and the fuzzy optimal strategy for factory B is IV (less deduction).

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