# MYT decomposition and its invariant attribute 

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#### Abstract

One of the most popular scheme in monitoring multivariate statistical process control (MSPC) is the Hotelling's T2, which give a better result when compared with the simultaneous use of univariate counterpart, since the former captures the correlation among the multivariate observations. Despite the merits of this procedure, there are some demerits which includes variable(s) identification. That is, any signal by this scheme implies one or more variables in the process has gone out-of-control. Therefore, identification of such variable(s) correctly becomes serious challenge. The Mason, Young and Tracy developed diagnosis known as MYT hotelling's T2 decomposition, this procedure aid the decomposition of the hotelling's $T 2$ into diagonal components identifying the contribution of each and every variable(s) contributions. We will demonstrate the invariate attribute of this scheme using five process variables.


Keywords: Hotelling's T square, Invariance attribute, Matrix Permutation, Multivariate Statistical Process Control (MSPC), MYT decomposition.

## Introduction

Consumers are becoming much aware of quality and as such become very sensitive to the standard of the products they consumed. Manufacturers often give a listening hears to customer's complaint and certain quality are maintained. In that case, industrial process is monitored to achieve certain conformability of products. When there is outrageous measurement in the process, signal is observed and attention is needed. When the process variable is one, it is very easy to interpret univariate control chart but when the process variables are more than one, it becomes multivariate. And in a multivariate control chart, it is not easy to identify the process variable(s) influencing the out-of-control situation. But the advantages observed over the use of multivariate quality control charts when dealing with many variables has given it the desired attention over the use of univariate control charts.

Hotelling's $T^{2}$ control chart introduced by Hotelling H. ${ }^{1}$ is one the most used among the multivariate statistical process control tools. Many authors had reviewed the Hotelling's $T^{2}$ control chart for detecting mean shift in a process as seen in Sullivan and Woodall ${ }^{2}$, Mason and Young ${ }^{3}$, Tong, et al ${ }^{4}$, Vargas ${ }^{5}$ and so on. However, the identification of process variable(s) that contribute to the signal becomes the challenge.

Many approach has been provided by researcher as seen in the work of Doganasksoy et al ${ }^{6}$, Hay and Tsui ${ }^{7}$, Alt ${ }^{8}$, Jackson ${ }^{9}$, Chua and Montgomery ${ }^{10}$, Pignatiello and Runger ${ }^{11}$, Kourti and MacGregor ${ }^{12}$, Wasterhuis et al ${ }^{13}$, maravelakis et al ${ }^{14}$, Mason et al ${ }^{15}$, Mason et al ${ }^{16}$, Mason and Young ${ }^{17}$, Alfaro et al ${ }^{18}$, Verron et $a l^{19}$, Aparasi and Sanz ${ }^{20}$ to mention but few. The MYT
decomposition of the Hotelling's $T^{2}$ statistic is mostly adopted since it determine the variable(s) contributing to Hotelling's $T^{2}$ control chart alarm, by revealing individual contribution and the relative contribution among pairs and or more process variables in a multivariate process. In this paper, we intend to demonstrate the attribute of invariance of MYT Hotelling's $T^{2}$ decomposition.

## Hotelling's $\boldsymbol{T}^{\mathbf{2}}$

There exist two phases in Statistical Process Control (SPC), namely, phase 1 and phase 2 . Phase 1 is considered as a retrospective phase, and it constitutes set of observations obtained from an in-control process whereby control limits are determined, which involve estimation of the unknown statistic(s), with the aim of achieving observations from an incontrol process. Individual observation from such in-control process is then used as a reference data in phase 2 . The phase I test statistic and control limits for the Hotelling's $T^{2}$ control chart when $\mu_{o}$ and $\sum_{0}$ are known is given as
$T^{2}=n\left(\bar{x}-\mu_{0}\right)^{t} \sum_{0}^{-1}\left(\bar{x}-\mu_{0}\right)$
and the $\mathrm{UCL}=\chi_{1-\alpha(p)}^{2}$

But in a situation whereby the $\mu_{o}$ and $\sum_{0}$ are unknown. Two scenario are considered; 1. When handling individual observations (that is $n=1$ ) and 2 . When handling subgroup observations (that is $n>1$ ). The scenario are addressed below

The Phase I test statistic and control limit for the Hotelling's $T^{2}$ control chart on individual observations (when $n=1$ ) is given as $T^{2}=n\left(x_{j}-\bar{x}\right)^{t} S^{-1}\left(x_{j}-\bar{x}\right)$
and the $\mathrm{UCL}=\frac{(m-1)^{2}}{m} \beta_{\alpha, \frac{p}{2}, \frac{m-p-1}{2}}$
Also the phase II test statistic and control limit for the Hotelling's $T^{2}$ control chart on individual observations (when $n=1)$ is given as $T^{2}=n(x-\bar{x})^{t} S^{-1}(x-\bar{x})$
and the $\mathrm{UCL}=\frac{p(m+1)(m-1)}{m(m-p)} F_{\alpha, p, m-p}$
The Phase I test statistic and control limit for the Hotelling's $T^{2}$ control chart on subgroup observations (when $n>1$ ) is given $T^{2}=n\left(\bar{x}_{j}-\overline{\bar{x}}\right)^{t} S^{-1}\left(\bar{x}_{j}-\overline{\bar{x}}\right)$
and the $\mathrm{UCL}=\frac{(\mathrm{m}-1)(\mathrm{n}-1) \mathrm{p}}{m(n-1)+1-p} F_{1-\alpha(p, m(n-1)+1-p}$
The Phase II test statistic and control limit for the Hotelling's $T^{2}$ control chart on subgroup observations (when $n>1$ ) is given as $T^{2}=n(\bar{x}-\overline{\bar{x}})^{t} S^{-1}(\bar{x}-\overline{\bar{x}})$
and the $\mathrm{UCL}=\frac{(\mathrm{m}+1)(\mathrm{n}-1) \mathrm{p}}{m(n-1)+1-p} F_{1-\alpha(p, m(n-1)+1-p}$

## Decomposition of hotelling's $\mathbf{T}^{\mathbf{2}}$

The form of the MYT model of 5 dimensional vector can be written as
$T^{2}=T_{1}^{2}+\left(T_{2.1}^{2}+T_{3.1,2}^{2}+\ldots+T_{5.1,2,3,4}^{2}\right)=T_{1}^{2}+\sum_{j=2}^{5} T_{j .1,2, \ldots, j-1}^{2}(1)$
The decomposition first term, $T_{1}^{2}$ is known as the unconditional Hotelling's $T^{2}$ and the decomposition second term, $\sum_{j=2}^{5} T_{j .1,2, \ldots, j-1}^{2}$ is known as the conditional Hotelling's $T^{2}$
$T_{j}^{2}=\frac{\left(x_{j}-\bar{x}_{j}\right)^{2}}{s_{j}^{2}} \sim \frac{n+1}{n} F_{\alpha, 1, n-1}(2) \quad j=1,2,3 \ldots p$
where $\bar{x}_{j}$ and $s_{j}^{2}$ can be used to estimate the mean and standard deviation respectively for variable $x_{j}$. The unconditional term can be viewed as a univariate Shewhart control chart. It estimates the squared standardized variance of $j^{\text {th }}$ variable. A signal in this regards simply means that the $j^{\text {th }}$ variable deviate greatly from the sample mean. $T_{j}$ will follow an $F$ distribution which can be used as critical value as shown above

$$
\begin{equation*}
T_{j .1, \ldots j-1}^{2}=\frac{\left(x_{j}-\bar{x}_{j .1,2, \ldots j-1}\right)}{s_{j .1,2, \ldots j-1}^{2}} \sim \frac{(n+1)(n-1)}{n(n-k-1)} F_{\alpha, 1, n-k-1} \tag{3}
\end{equation*}
$$

The conditional term is a standardized observation of the $j^{\text {th }}$ variable adjusted by estimates of the mean and variance from the conditional distribution of $x_{j}$ given $x_{1}, x_{2}, \cdots x_{j-1}$, and it follows $F$ distribution which is also used as critical value as shown above, Thus, this statistic is used to ascertain whether the $j^{\text {th }}$ variable comply with the relationship with other variables as established in the phase I of the process, since the adjusted observation is more sensitive to changes in the covariance structure.

In order to generate the invariant property of the five variables, we start by choosing any one of the $p=5$ variables. Then we choose any of the $(p-1)$ remaining variables to condition on the first chosen variable. Next we choose any of the remaining ( $p-$ 2) variables to condition on the first two chosen variables, then choose any of the remaining $(p-3)$ variables to condition on the first three chosen variables. Then we choose any of the remaining $(p-4)$ variables to condition on the first four chosen variables. Iterating the same procedure will generate all the decomposition equations which contain the same overall $T^{2}$ statistic, and this is known as the invariant property of the decomposition. This decomposition is shown below.

Illustration: Five process individual variables were used to construct hotelling's $\mathrm{T}^{2}$ control chart and the control chart below is obtained. It is noticed that two points were outside the upper control limit (point 2 and point 18). We will use the MYT decomposition method to illustrate how to identify the variable(s) that contribute to this out of control situation using point 2 for illustration.

The hotelling's $\mathrm{T}^{2}$ control chart obtained for 20 observations containing five variables


Figure-1: Hotelling T ${ }^{2}$ Control Chart
From the control chart above, we will derive the decomposition terms and also obtained the invariant attribute of the Decomposition.

Table-1: MYT decomposition of the point 2

| Components | Point 2 values | Critical value |
| :---: | :---: | :---: |
| $\mathrm{T}_{1}$ | *5.9768 | 4.0071 |
| $\mathrm{T}_{2}$ | 1.5344 | 4.0071 |
| $\mathrm{T}_{3}$ | 0.2062 | 4.0071 |
| $\mathrm{T}_{4}$ | 0.1244 | 4.0071 |
| $\mathrm{T}_{5}$ | 3.1971 | 4.0071 |
| $\mathrm{T}_{1.2}$ | 5.4128 | 6.3761 |
| $\mathrm{T}_{1.3}$ | 5.8630 | 6.3761 |
| $\mathrm{T}_{1.4}$ | 6.3213 | 6.3761 |
| $\mathrm{T}_{1.5}$ | *6.4333 | 6.3761 |
| $\mathrm{T}_{2.1}$ | 0.9704 | 6.3761 |
| $\mathrm{T}_{2.3}$ | 1.7095 | 6.3761 |
| $\mathrm{T}_{2.4}$ | 1.4355 | 6.3761 |
| $\mathrm{T}_{2.5}$ | 0.9552 | 6.3761 |
| $\mathrm{T}_{3.1}$ | 0.0924 | 6.3761 |
| $\mathrm{T}_{3.2}$ | 0.3813 | 6.3761 |
| $\mathrm{T}_{3.4}$ | 0.1647 | 6.3761 |
| $\mathrm{T}_{3.5}$ | 0.0924 | 6.3761 |
| $\mathrm{T}_{4.1}$ | 0.4689 | 6.3761 |
| $\mathrm{T}_{4.2}$ | 0.0255 | 6.3761 |
| $\mathrm{T}_{4.3}$ | 0.0829 | 6.3761 |
| $\mathrm{T}_{4.5}$ | 0.0107 | 6.3761 |
| $\mathrm{T}_{5.1}$ | 3.6536 | 6.3761 |
| $\mathrm{T}_{5.2}$ | 2.6179 | 6.3761 |
| $\mathrm{T}_{5.3}$ | 3.0833 | 6.3761 |
| $\mathrm{T}_{5.4}$ | 3.0834 | 6.3761 |
| $\mathrm{T}_{1.2,3}$ | 5.3567 | 8.4694 |
| $\mathrm{T}_{1.2,4}$ | 5.6592 | 8.4694 |
| $\mathrm{T}_{1.2,5}$ | 5.9544 | 8.4694 |
| $\mathrm{T}_{1.3,4}$ | 6.1954 | 8.4694 |
| $\mathrm{T}_{1.3,5}$ | 6.6262 | 8.4694 |
| $\mathrm{T}_{1.4,5}$ | 6.6086 | 8.4694 |
| $\mathrm{T}_{2.1,3}$ | 1.2032 | 8.4694 |
| $\mathrm{T}_{2.1,4}$ | 0.7734 | 8.4694 |
| $\mathrm{T}_{2.1,5}$ | 0.4763 | 8.4694 |
| $\mathrm{T}_{2,3,4}$ | 1.6642 | 8.4694 |
| $\mathrm{T}_{2.3,5}$ | 3.0107 | 8.4694 |
| $\mathrm{T}_{2.4,5}$ | 0.9455 | 8.4694 |
| $\mathrm{T}_{3.1,2}$ | 0.3252 | 8.4694 |
| $\mathrm{T}_{3.1,4}$ | 0.0388 | 8.4694 |
| $\mathrm{T}_{3.1,5}$ | 0.2853 | 8.4694 |


| Components | Point 2 values | Critical value |
| :---: | :---: | :---: |
| $\mathrm{T}_{3.2,4}$ | 0.3934 | 8.4694 |
| $\mathrm{T}_{3.2,5}$ | 2.1479 | 8.4694 |
| $\mathrm{T}_{3.4,5}$ | 0.1000 | 8.4694 |
| $\mathrm{T}_{4.1,2}$ | 0.2719 | 8.4694 |
| $\mathrm{T}_{4.1,3}$ | 0.4153 | 8.4694 |
| $\mathrm{T}_{4.1,5}$ | 0.1860 | 8.4694 |
| $\mathrm{T}_{4.2,3}$ | 0.0376 | 8.4694 |
| $\mathrm{T}_{4.2,5}$ | 0.0010 | 8.4694 |
| $\mathrm{T}_{4.3,5}$ | 0.0183 | 8.4694 |
| $\mathrm{T}_{5.1,2}$ | 3.1595 | 8.4694 |
| $\mathrm{T}_{5.1,3}$ | 3.8465 | 8.4694 |
| $\mathrm{T}_{5.1,4}$ | 3.3707 | 8.4694 |
| $\mathrm{T}_{5.2,3}$ | 4.3845 | 8.4694 |
| $\mathrm{T}_{5.2,4}$ | 2.5934 | 8.4694 |
| $\mathrm{T}_{5.3,4}$ | 3.0187 | 8.4694 |
| $\mathrm{T}_{1.2,3,4}$ | 5.6235 | 10.4645 |
| $\mathrm{T}_{1.2,3,5}$ | 6.0242 | 10.4645 |
| $\mathrm{T}_{1.2,4,5}$ | 6.0608 | 10.4645 |
| $\mathrm{T}_{1.3,4,5}$ | 6.8541 | 10.4645 |
| $\mathrm{T}_{2.1,3,4}$ | 1.0923 | 10.4645 |
| $\mathrm{T}_{2.1,3,5}$ | 2.4087 | 10.4645 |
| $\mathrm{T}_{2.1,4,5}$ | 0.3977 | 10.4645 |
| $\mathrm{T}_{2.3,4,5}$ | 2.9932 | 10.4645 |
| $\mathrm{T}_{3.1,2,4}$ | 0.3577 | 10.4645 |
| $\mathrm{T}_{3.1,2,5}$ | 2.2177 | 10.4645 |
| $\mathrm{T}_{3.1,4,5}$ | 0.3455 | 10.4645 |
| $\mathrm{T}_{3.2,4,5}$ | 2.1477 | 10.4645 |
| $\mathrm{T}_{4.1,2,3}$ | 0.3044 | 10.4645 |
| $\mathrm{T}_{4.1,2,5}$ | 0.1074 | 10.4645 |
| $\mathrm{T}_{4.1,3,5}$ | 0.2462 | 10.4645 |
| $\mathrm{T}_{4.2,3,5}$ | 0.0008 | 10.4645 |
| $\mathrm{T}_{5.1,2,3}$ | 5.0520 | 10.4645 |
| $\mathrm{T}_{5.1,2,4}$ | 2.9950 | 10.4645 |
| $\mathrm{T}_{5.1,3,4}$ | 3.6774 | 10.4645 |
| $\mathrm{T}_{5.2,3,4}$ | 4.3477 | 10.4645 |
| $\mathrm{T}_{1.2,3,4,5}$ | 6.1352 | 12.4223 |
| $\mathrm{T}_{2.1,3,4,5}$ | 2.2743 | 12.4223 |
| $\mathrm{T}_{3.1,2,4,5}$ | 2.2221 | 12.4223 |
| $\mathrm{T}_{4.1,2,3,5}$ | 0.1118 | 12.4223 |
| $\mathrm{T}_{5.1,2,3,4}$ | 4.8594 | 12.4223 |

The Invariant Attributes of the MYT Decomposition

$$
\begin{aligned}
& T^{2}=T_{1}^{2}+T_{2.1}^{2}+T_{3.1,2}^{2}+T_{4.1,2,3}^{2}+T_{5.1,2,3,4}^{2} \\
& T^{2}=T_{1}^{2}+T_{3.1}^{2}+T_{4.1,3}^{2}+T_{5.1,3,4}^{2}+T_{2.1,3,4,5}^{2} \\
& T^{2}=T_{1}^{2}+T_{4.1}^{2}+T_{5.1,4}^{2}+T_{2.1,4,5}^{2}+T_{3.1,2,4,5}^{2} \\
& T^{2}=T_{1}^{2}+T_{5.1}^{2}+T_{2.1,5}^{2}+T_{3.1,2,5}^{2}+T_{4.1,2,3,5}^{2} \\
& T^{2}=T_{1}^{2}+T_{2.1}^{2}+T_{4.2,1}^{2}+T_{5.1,2,4}^{2}+T_{3.1,2,4,5}^{2} \\
& T^{2}=T_{1}^{2}+T_{3.1}^{2}+T_{2.1,3}^{2}+T_{4.1,2,3}^{2}+T_{5.1,2,3,4}^{2} \\
& T^{2}=T_{1}^{2}+T_{4.1}^{2}+T_{3.1,4}^{2}+T_{5.1,3,4}^{2}+T_{2.1,3,4,5}^{2} \\
& T^{2}=T_{1}^{2}+T_{5.1}^{2}+T_{3.1,5}^{2}+T_{4.1,2,5}^{2}+T_{2.1,3,4,5}^{2} \\
& T^{2}=T_{1}^{2}+T_{2.1}^{2}+T_{3.1,2}^{2}+T_{5.1,2,3}^{2}+T_{4.1,2,3,4}^{2} \\
& T^{2}=T_{1}^{2}+T_{3.1}^{2}+T_{4.1,3}^{2}+T_{2.1,3,4}^{2}+T_{5.1,2,3,4}^{2} \\
& T^{2}=T_{1}^{2}+T_{4.1}^{2}+T_{5.1,4}^{2}+T_{3.1,4,5}^{2}+T_{2.1,3,4,5}^{2} \\
& T^{2}=T_{1}^{2}+T_{5.1}^{2}+T_{2.1,5}^{2}+T_{4.1,2,5}^{2}+T_{3.1,2,4,5}^{2} \\
& T^{2}=T_{1}^{2}+T_{4.1,4}^{2}+T_{3.1,2,4}^{2}+T_{5.1,2,3,4}^{2} \\
& T^{2}=T_{1}^{2}+T_{5.1}^{2}+T_{4.1,5}^{2}+T_{2.1,4,5}^{2}+T_{3.1,2,4,5}^{2} \\
& T^{2}+T_{2.1}^{2}+T_{4.1,2}^{2}+T_{5.1,2,4}^{2}+T_{3.1,2,4,5}^{2} \\
& T^{2}=T_{1.1}^{2}+T_{2.1}^{2}+T_{1.1}^{2}+T_{3.1}^{2}+T_{2.1,3}^{2}+T_{5.1,2,3}^{2}+T_{4.1,2,3,5}^{2} \\
& T^{2}=T_{1}^{2}+T_{5.1}^{2}+T_{4.1,5}^{2}+T_{3.1,4,5}^{2}+T_{2.1,3,4,5}^{2} \\
& T^{2}=T_{1}^{2}+T_{4.1}^{2}+T_{3.1,4}^{2}+T_{2.1,3,4}^{2}+T_{5.1,2,3,4}^{2} \\
& T^{2}=T_{1.1,3}^{2}+T_{5.1,3}^{2}+T_{4.1,5}^{2}+T_{4.1,2,3,5}^{2} \\
& T^{2}=T_{1}^{2}+T_{2.1,2,5}^{2}+T_{3.1,2,4,5}^{2} \\
& T^{2}=T_{1}^{2}+T_{3.1}^{2}+T_{5.1,3}^{2}+T_{4.1,2,5}^{2}+T_{2.1,3,4,5}^{2} \\
& 2
\end{aligned}
$$

$12.4362=5.9768+0.9704+0.3252+0.3044+4.8594$
$12.4362=5.9768+0.0924+0.4153+3.6774+2.2743$
$12.4362=5.9768+0.4689+3.3707+0.3977+2.2221$
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$$
\begin{aligned}
& T^{2}=T_{2}^{2}+T_{1.2}^{2}+T_{3.1,2}^{2}+T_{4.1,2,3}^{2}+T_{5.1,2,3,4}^{2} \\
& T^{2}=T_{2}^{2}+T_{3.2}^{2}+T_{4.2,3}^{2}+T_{5.2,3,4}^{2}+T_{1.2,3,4,5}^{2} \\
& T^{2}=T_{2}^{2}+T_{4.2}^{2}+T_{1.1,4}^{2}+T_{3.1,2,4}^{2}+T_{5.1,2,3,4}^{2} \\
& T^{2}=T_{2}^{2}+T_{5.2}^{2}+T_{3.2,5}^{2}+T_{4.2,3,5}^{2}+T_{1.2,3,4,5}^{2} \\
& T^{2}=T_{2}^{2}+T_{1.2}^{2}+T_{3.1,2}^{2}+T_{5.1,2,3}^{2}+T_{4.1,2,3,5}^{2} \\
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& T^{2}=T_{2}^{2}+T_{4.2}^{2}+T_{5.2,4}^{2}+T_{1.2,4,5}^{2}+T_{3.1,2,4,5}^{2} \\
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& T^{2}=T_{2}^{2}+T_{1.2}^{2}+T_{5.1,2}^{2}+T_{4.1,2,3}^{2}+T_{3.1,2,4,5}^{2} \\
& T^{2}=T_{2}^{2}+T_{3.2}^{2}+T_{1.2,3}^{2}+T_{4.1,2,3}^{2}+T_{5.1,2,3,4}^{2} \\
& T^{2}=T_{2}^{2}+T_{4.2}^{2}+T_{3.2,4}^{2}+T_{1.2,3,4}^{2}+T_{5.1,2,3,4}^{2} \\
& T^{2}=T_{2}^{2}+T_{5.2}^{2}+T_{1.2,5}^{2}+T_{4.1,2,5}^{2}+T_{3.1,2,5,4}^{2} \\
& T^{2}=T_{2}^{2}+T_{1.2}^{2}+T_{4.1,2}^{2}+T_{3.1,2,4}^{2}+T_{5.1,2,3,4}^{2} \\
& T^{2}=T_{2}^{2}+T_{4.2}^{2}+T_{1.2,4}^{2}+T_{5.1,2,4}^{2}+T_{3.1,2,4,5}^{2} \\
& T^{2}=T_{2}^{2}+T_{5.2}^{2}+T_{4.2,5}^{2}+T_{3.2,4,5}^{2}+T_{1.2,3,4,5}^{2} \\
& T^{2}=T_{2}^{2}+T_{3.2}^{2}+T_{4.2,3}^{2}+T_{1.2,3,4}^{2}+T_{5.1,2,3,4}^{2} \\
& T^{2}=T_{2}^{2}+T_{4.2}^{2}+T_{3.2,4}^{2}+T_{5.2,3,4}^{2}+T_{1.2,3,4,5}^{2} \\
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& T^{2}=T_{2}^{2}+T_{1.2}^{2}+T_{4.1,2}^{2}+T_{5.1,2,4}^{2}+T_{3.1,2,4,5}^{2} \\
& T^{2}=T_{2}^{2}+T_{3.2}^{2}+T_{1.2,3}^{2}+T_{5.1,2,3}^{2}+T_{4.1,2,3,5}^{2} \\
& T^{2}=T_{2}^{2}+T_{4.2}^{2}+T_{5.2,4}^{2}+T_{3.2,4,5}^{2}+T_{1.2,3,4,5}^{2} \\
& 2
\end{aligned}
$$

$T^{2}=T_{3}^{2}+T_{1.3}^{2}+T_{2.1,3}^{2}+T_{4.1,2,3}^{2}+T_{5.1,2,3,4}^{2}$
$T^{2}=T_{3}^{2}+T_{2.3}^{2}+T_{1.2,3}^{2}+T_{4.1,2,3}^{2}+T_{5.1,2,3,4}^{2}$
$T^{2}=T_{3}^{2}+T_{4.3}^{2}+T_{1.3,4}^{2}+T_{2.1,3,4}^{2}+T_{5.1,2,3,4}^{2}$
$T^{2}=T_{3}^{2}+T_{5.3}^{2}+T_{4.3,5}^{2}+T_{1.3,4,5}^{2}+T_{2.1,3,4,5}^{2}$
$T^{2}=T_{3}^{2}+T_{1.3}^{2}+T_{2.1,3}^{2}+T_{5.1,2,3}^{2}+T_{4.1,2,3,5}^{2}$
$T^{2}=T_{3}^{2}+T_{2.3}^{2}+T_{1.2,3}^{2}+T_{5.1,2,3}^{2}+T_{4.1,2,3,5}^{2}$
$T^{2}=T_{3}^{2}+T_{4.3}^{2}+T_{1.3,4}^{2}+T_{5.1,2,4}^{2}+T_{2.1,3,4,5}^{2}$
$T^{2}=T_{3}^{2}+T_{5.3}^{2}+T_{4.3,5}^{2}+T_{2.3,4,5}^{2}+T_{1.2,3,4,5}^{2}$
$T^{2}=T_{3}^{2}+T_{1.3}^{2}+T_{4.1,3}^{2}+T_{5.1,2,4}^{2}+T_{2.1,3,4,5}^{2}$
$T^{2}=T_{3}^{2}+T_{1.3}^{2}+T_{2.1,3}^{2}+T_{4.1,2,3}^{2}+T_{5.1,2,3,4}^{2}$
$T^{2}=T_{3}^{2}+T_{4.3}^{2}+T_{2.3,4}^{2}+T_{1.2,3,4}^{2}+T_{5.1,2,3,4}^{2}$
$T^{2}=T_{3}^{2}+T_{5.3}^{2}+T_{1.3,5}^{2}+T_{2.1,2,5}^{2}+T_{4.1,2,3,5}^{2}$
$T^{2}=T_{3}^{2}+T_{1.2}^{2}+T_{4.1,3}^{2}+T_{2.1,3,4}^{2}+T_{5.1,2,3,4}^{2}$
$T^{2}=T_{3}^{2}+T_{2.3}^{2}+T_{4.2,3}^{2}+T_{5.2,3,4}^{2}+T_{1.2,3,4,5}^{2}$
$T^{2}=T_{3}^{2}+T_{4.3}^{2}+T_{2.3,4}^{2}+T_{5.2,3,4}^{2}+T_{1.2,3,4,5}^{2}$
$T^{2}=T_{3}^{2}+T_{5.3}^{2}+T_{1.3,5}^{2}+T_{4.1,3,5}^{2}+T_{2.1,3,4,5}^{2}$
$T^{2}=T_{3}^{2}+T_{1.3}^{2}+T_{5.1,3}^{2}+T_{2.1,3,5}^{2}+T_{4.1,2,3,5}^{2}$
$T^{2}=T_{3}^{2}+T_{2.3}^{2}+T_{5.2,3}^{2}+T_{4.2,3,5}^{2}+T_{1.2,3,4,5}^{2}$
$T^{2}=T_{3}^{2}+T_{4.3}^{2}+T_{5.3,4}^{2}+T_{1.3,4,5}^{2}+T_{2.1,3,4,5}^{2}$
$T^{2}=T_{3}^{2}+T_{5.3}^{2}+T_{2.3,5}^{2}+T_{1.2,3,5}^{2}+T_{4.1,2,3,5}^{2}$
$T^{2}=T_{3}^{2}+T_{1.3}^{2}+T_{5.1,3}^{2}+T_{4.1,3,5}^{2}+T_{2.1,3,4,5}^{2}$
$T^{2}=T_{3}^{2}+T_{2.3}^{2}+T_{5.2,3}^{2}+T_{1.2,3,5}^{2}+T_{4.1,2,3,5}^{2}$
$T^{2}=T_{3}^{2}+T_{4.3}^{2}+T_{5.3,4}^{2}+T_{2.3,4,5}^{2}+T_{1.2,3,4,5}^{2}$
$T^{2}=T_{3}^{2}+T_{5.3}^{2}+T_{2.3,5}^{2}+T_{4.2,3,5}^{2}+T_{1.2,3,4,5}^{2}$
$12.4362=0.2062+5.8630+1.2032+0.3044+4.8594$
$12.4362=0.2062+1.7095+5.3567+0.3044+4.8594$
$12.4362=0.2062+0.0829+6.1954+1.0923+4.8594$
$12.4362=0.2062+3.0833+0.0183+6.8541+2.2743$
$12.4362=0.2062+5.8630+1.2032+5.0520+0.1118$
$12.4362=0.2062+1.7095+5.3567+5.0520+0.1118$
$12.4362=0.2062+0.0829+6.1954+3.6774+2.2743$
$12.4362=0.2062+3.0833+0.0183+2.9932+6.1352$
$12.4362=0.2062+5.8630+0.4153+3.6774+2.2743$
$12.4362=0.2062+1.7095+0.0376+5.6235+4.8594$
$12.4362=0.2062+0.0829+1.6642+5.6235+4.8594$
$12.4362=0.2062+3.0833+6.6262+2.4087+0.1118$
$12.4362=0.2062+5.8630+0.4153+1.0923+4.8594$
$12.4362=0.2062+1.7095+0.0376+4.3477+6.1352$
$12.4362=0.2062+0.0829+1.6642+4.3477+6.1352$
$12.4362=0.2062+3.0833+6.6262+0.2462+2.2743$
$12.4362=0.2062+5.8630+3.8465+2.4087+0.1118$
$12.4362=0.2062+1.7095+4.3845+0.0008+6.1352$
$12.4362=0.2062+0.0829+3.0187+6.8541+2.2743$
$12.4362=0.2062+3.0833+3.0107+6.0242+0.1118$
$12.4362=0.2062+5.8630+3.8465+0.2462+2.2743$
$12.4362=0.2062+1.7095+4.3845+6.0242+0.1118$
$12.4362=0.2062+0.0829+3.0187+2.9932+6.1352$
$12.4362=0.2062+3.0833+3.0107+0.0008+6.1352$

$$
\begin{aligned}
& T^{2}=T_{4}^{2}+T_{1.4}^{2}+T_{2.1,4}^{2}+T_{3.1,2,4}^{2}+T_{5.1,2,3,4}^{2} \\
& T^{2}=T_{4}^{2}+T_{2.4}^{2}+T_{2.1,4}^{2}+T_{3.1,2,4}^{2}+T_{5.1,2,3,4}^{2} \\
& T^{2}=T_{4}^{2}+T_{3.4}^{2}+T_{1.3,4}^{2}+T_{2.1,3,4}^{2}+T_{5.1,2,3,4}^{2} \\
& T^{2}=T_{4}^{2}+T_{5.4}^{2}+T_{1.4,5}^{2}+T_{2.1,4,5}^{2}+T_{3.1,2,4,5}^{2} \\
& T^{2}=T_{4}^{2}+T_{1.4}^{2}+T_{2.1,4}^{2}+T_{5.1,2,4}^{2}+T_{3.1,2,4,5}^{2} \\
& T^{2}=T_{4}^{2}+T_{2.4}^{2}+T_{1.2,4}^{2}+T_{5.1,2,4}^{2}+T_{3.1,2,4,5}^{2} \\
& T^{2}=T_{4}^{2}+T_{3.4}^{2}+T_{1.3,4}^{2}+T_{5.1,3,4}^{2}+T_{2.1,3,4,5}^{2} \\
& T^{2}=T_{4}^{2}+T_{5.4}^{2}+T_{1.4,5}^{2}+T_{3.1,4,5}^{2}+T_{2.1,3,4,5}^{2} \\
& T^{2}=T_{4}^{2}+T_{1.4}^{2}+T_{3.1,4}^{2}+T_{2.1,3,4}^{2}+T_{5.1,2,3,4}^{2} \\
& T^{2}=T_{4}^{2}+T_{2.4}^{2}+T_{3.2,4}^{2}+T_{1.2,3,4}^{2}+T_{5.1,2,3,4}^{2} \\
& T^{2}=T_{4}^{2}+T_{3.4}^{2}+T_{2.3,4}^{2}+T_{1.2,3,4}^{2}+T_{5.1,2,3,4}^{2} \\
& T^{2}=T_{4}^{2}+T_{5.4}^{2}+T_{2.4,5}^{2}+T_{1.2,4,5}^{2}+T_{3.1,2,3,4}^{2} \\
& T^{2}=T_{4}^{2}+T_{1.4}^{2}+T_{3.1,4}^{2}+T_{5.1,3,4}^{2}+T_{2.1,3,4,5}^{2} \\
& T^{2}=T_{4}^{2}+T_{3.4}^{2}+T_{5.3,4}^{2}+T_{1.3,4,5}^{2}+T_{2.1,3,4,5}^{2} \\
& T^{2}=T_{4}^{2}+T_{5.4}^{2}+T_{3.4,5}^{2}+T_{2.3,4,5}^{2}+T_{1.2,3,4,5}^{2} \\
& T^{2}=T_{4}^{2}+T_{2.4}^{2}+T_{3.2,4}^{2}+T_{5.2,3,4}^{2}+T_{1.2,3,4,5}^{2} \\
& T^{2}=T_{4}^{2}+T_{3.4}^{2}+T_{2.3,4}^{2}+T_{5.2,3,4}^{2}+T_{1.2,3,4,5}^{2} \\
& T^{2}=T_{4}^{2}+T_{5.4}^{2}+T_{2.4,5}^{2}+T_{3.2,4,5}^{2}+T_{1.2,3,4,5}^{2} \\
& T^{2}=T_{4}^{2}+T_{1.4}^{2}+T_{5.1,4}^{2}+T_{2.1,4,5}^{2}+T_{3.1,2,4,5}^{2} \\
& T^{2}=T_{4}^{2}+T_{2.4}^{2}+T_{5.2,4}^{2}+T_{3.2,4,5}^{2}+T_{1.2,3,4,5}^{2} \\
& T^{2}=T_{4}^{2}+T_{3.4}^{2}+T_{5.3,4}^{2}+T_{2.3,4,5}^{2}+T_{1.2,3,4,5}^{2} \\
& T^{2}=T_{4}^{2}+T_{5.4}^{2}+T_{3.4,5}^{2}+T_{1.3,4,5}^{2}+T_{2.1,3,4,5}^{2} \\
& 2
\end{aligned}
$$

$12.4362=0.1244+6.3213+0.7734+0.3577+4.8594$
$12.4362=0.1244+1.4355+5.6592+0.3577+4.8594$
$12.4362=0.1244+0.1647+6.1954+1.0923+4.8594$
$12.4362=0.1244+3.0834+6.6086+0.3977+2.2221$
$12.4362=0.1244+6.3213+0.7734+2.9950+2.2221$
$12.4362=0.1244+1.4355+5.6592+2.9950+2.2221$
$12.4362=0.1244+0.1647+6.1954+3.6774+2.2743$
$12.4362=0.1244+3.0834+6.6086+0.3455+2.2743$
$12.4362=0.1244+6.3213+0.0388+1.0923+4.8594$
$12.4362=0.1244+1.4355+0.3934+5.6235+4.8594$
$12.4362=0.1244+0.1647+1.6642+5.6235+4.8594$
$12.4362=0.1244+3.0834+0.9455+6.0608+2.2221$
$12.4362=0.1244+6.3213+0.0388+3.6774+2.2743$
$12.4362=0.1244+1.4355+0.3934+4.3477+6.1352$
$12.4362=0.1244+0.1647+1.6642+4.3477+6.1352$
$12.4362=0.1244+3.0834+0.9455+2.1477+6.1352$
$12.4362=0.1244+6.3213+3.3707+0.3977+2.2221$
$12.4362=0.1244+1.4355+2.5934+2.1477+6.1352$
$12.4362=0.1244+0.1647+3.0187+2.9932+6.1352$
$12.4362=0.1244+3.0834+0.1000+6.8541+2.2743$
$12.4362=0.1244+6.3213+3.3707+0.3455+2.2743$
$12.4362=0.1244+1.4355+2.5934+6.0608+2.2221$
$12.4362=0.1244+0.1647+3.0187+6.8541+2.2743$
$12.4362=0.1244+3.0834+0.1000+2.9932+6.1352$

$$
\begin{array}{ll}
T^{2}=T_{5}^{2}+T_{1.5}^{2}+T_{2.1,5}^{2}+T_{3.1,2,5}^{2}+T_{4.1,2,3,5}^{2} & 12.4362=3.1971+6.4333+0.4763+2.2177+0.1118 \\
T^{2}=T_{5}^{2}+T_{2.5}^{2}+T_{1.2,5}^{2}+T_{3.1,2,5}^{2}+T_{4.1,2,3,5}^{2} & 12.4362=3.1971+0.9552+5.9544+2.2177+0.1118 \\
T^{2}=T_{5}^{2}+T_{3.5}^{2}+T_{1.3,5}^{2}+T_{2.1,3,5}^{2}+T_{4.1,2,3,5}^{2} & 12.4362=3.1971+0.0924+6.6262+2.4087+0.1118 \\
T^{2}=T_{5}^{2}+T_{4.5}^{2}+T_{1.4,5}^{2}+T_{2.1,4,5}^{2}+T_{3.1,2,4,5}^{2} & 12.4362=3.1971+0.0107+6.6086+0.3977+2.2221 \\
T^{2}=T_{5}^{2}+T_{1.5}^{2}+T_{2.1,5}^{2}+T_{4.1,2,5}^{2}+T_{3.1,2,4,5}^{2} & 12.4362=3.1971+6.4333+0.4763+0.1074+2.2221 \\
T^{2}=T_{5}^{2}+T_{2.5}^{2}+T_{1.2,5}^{2}+T_{4.1,2,5}^{2}+T_{3.1,2,4,5}^{2} & 12.4362=3.1971+0.9552+5.9544+0.1074+2.2221 \\
T^{2}=T_{5}^{2}+T_{3.5}^{2}+T_{1.3,5}^{2}+T_{4.1,3,5}^{2}+T_{2.1,3,4,5}^{2} & 12.4362=3.1971+0.0924+6.6262+0.2462+2.2743 \\
T^{2}=T_{5}^{2}+T_{4.5}^{2}+T_{1.4,5}^{2}+T_{3.1,4,5}^{2}+T_{2.1,3,4,5}^{2} & 12.4362=3.1971+0.0107+6.6086+0.3455+2.2743 \\
T^{2}=T_{5}^{2}+T_{1.5}^{2}+T_{3.1,5}^{2}+T_{2.1,3,5}^{2}+T_{4.1,2,3,5}^{2} & 12.4362=3.1971+6.4333+0.2853+2.4087+0.1118 \\
T^{2}=T_{5}^{2}+T_{2.5}^{2}+T_{3.2,5}^{2}+T_{1.2,3,5}^{2}+T_{4.1,2,3,5}^{2} & 12.4362=3.1971+0.9552+2.1479+6.0242+0.1118 \\
T^{2}=T_{5}^{2}+T_{3.5}^{2}+T_{2.3,5}^{2}+T_{1.2,3,5}^{2}+T_{4.1,2,3,5}^{2} & 12.4362=3.1971+0.9552+0.0010+2.1477+6.1352 \\
T^{2}=T_{5}^{2}+T_{4.5}^{2}+T_{2.4,5}^{2}+T_{1.2,4,5}^{2}+T_{3.1,2,4,5}^{2} & 12.4362=3.1971+0.0924+3.0107+6.0242+0.1118 \\
T^{2}=T_{5}^{2}+T_{3.5}^{2}+T_{4.3,5}^{2}+T_{2.3,4,5}^{2}+T_{1.2,3,4,5}^{2}+T_{3.4,5}^{2}+T_{2.3,4,5}^{2}+T_{1.2,3,4,5}^{2} & 12.4362=3.1971+0.0107+0.9455+6.0608+2.2221 \\
T^{2}=T_{5}^{2}+T_{1.5}^{2}+T_{3.1,5}^{2}+T_{4.1,3,5}^{2}+T_{2.1,3,4,5}^{2} & 12.4362=3.1971+6.4333+0.2853+0.2462+2.2743 \\
T^{2}=T_{5}^{2}+T_{2.5}^{2}+T_{3.2,5}^{2}+T_{4.2,3,5}^{2}+T_{1.2,3,4,5}^{2} & 12.1971+0.0107+0.1000+2.9932+6.1352
\end{array}
$$

## Conclusion

We were able to reveal the invariate attribute of the MYT decomposition using five variables for illustration and this usually aid correctness and proper identification of the process variable contributing to signal of any process.

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