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# Study of rural out migration through a probability model

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## Abstract

*Many studies have been carried out to estimate the rural-out migration at the household level through a probabilistic model. It has been found that inflated logarithmic series distribution is a fair approximation for the given data set. Data for the study has been taken from north eastern Bihar through a sample survey.*

**Keywords**: Inflated logarithmic distribution, Method of maximum likelihood, Co-variane.

## Introduction

Migration, traditionally, being an essential part of demography, is least studied as compared to fertility and mortality. Because of decreasing death and birth rate, migration (interstate or international) has become a more important concern for demographers and other social scientists. Most of the studies in past[1-5], depending upon the conceptualization of migration process and the scale of investigations, used macro approach by operating on highly aggregate data for countries, states and the nation as a whole. These types of studies are unable to provide sufficient justification for the remarkable regional and local heterogeneity. They also ignore the decision making process of migrating individuals[6]. Thus a study of atmosphere in which migration takes place and the migration decision process at the micro-level is more important. Studies of migration at the micro-level i.e. at the levels of individuals, families or households have important implications for housing policies and also for the development of other sociological models related to family and community[7,8].

It is seen that a migrant household (with single or more members involved in the practice of migration in relation to do some job outside the village) may have diverse social, cultural and economic characteristics through remittances besides ideas, awareness and environments than a non-migrant household. It is worth mentioning here that rural-urban migration in India is of chain type at least in the beginning of the migration process before finally they settled in the urban areas or returned back to the villages.

A useful way of addressing the diversity of migration system is to treat the major contemporary cases in a general comparative frame work[9]. There is need to put more emphasis on using models to analyse and explain important demographic issues and to predict demographic futures[10]. One of the important property of micro level models is that a researcher can introduce individual heterogeneity- measured (accounting different rates for individual with different characteristics such as age, sex, social class, cohort, etc.) and unmeasured (choosing for each person an adjusting factor).

Many Studies have been done to relate and/or frame probability models in basic and social sciences[11-14]. Since these models offer brief and flawless demonstration of widespread data sets in a superior way[15], in recent years increased attention has been paid to the proposition and derivation of probabilistic models to study the movement of human population at small and regional level[16]. Singh and yadava[6] have used the negative binomial distribution to observe the pattern of out migration at household level. Sharma[17] applied this model on a different data set and found that this model is not suitable in the case of total migrants (together with women and children) from a household. Since migrated females are more likely to affect the socio-cultural characteristics of households in comparison to other females of household, the study of the behavior of number of migrants is important.

Sharma[17] proposed a probability model under the assumption that: i. the number of 15 years and above male migrants follows negative binomial distribution and ii. the distribution of alive children to a couple be known. However, the distribution of alive children to a couple has not yet been derived theoretically; the prior knowledge about these two distributions is difficult.

Singh[18] assumed that there are two types of households – first in which only male aged more than 15 years migrate and second – in which the male migrate with their wives and children and proposed a probability model. Several authors have proposed models in the same line to explain the distribution of household based on the number of migrants including wife and children[19-23].

This paper aims to study the trends in rural-out migration at the household level through a probabilistic model and also to do some modification in the exiting estimation procedure.

## Data

The present study is based on a primary survey entitled "Migration and Related Characteristics- a Case Study of North Eastern Bihar" which was piloted during October 2009 to June 2010[24].

Data have been collected using a multistage random sampling procedure. The districts selected for the study are Katihar, Purnea and Bhagalpur. A sample of 664 households from 8 villages has been taken for this purpose. The selected area is situated in the flood prone area of Koshi and Ganges River. Due to flood, unavailability of employment opportunities, educational institutions migration rate is very high especially seasonal migration. Poor economic condition is one more cause of migration in this area.

## Methodology

A probabilistic model for explaining the variability in the number of rural out-migrant households has been derived based on the assumptions listed below: i. Let $\alpha$ be the probability of risk of migration and (1-$\alpha$) be the probability that there is no risk of migration at the time of survey. ii. The probability of migrating one male from a house is greater than the chance of migrating two males, and probability of two males migrating is greater than that of three males from a household and so on. Thus, the trend of migration from a family is a declining function following logarithmic distribution with parameter $\theta$.

Suppose the number of out-migrants from a rural household be x, then from the above assumptions, the stochastic function of x is given by

$$\left.\begin{aligned} P(\mathrm{x}=\mathrm{a}) &= 1-\alpha, \qquad \text{for a} = 0 \\ &= \alpha\left[\frac{-\theta^{\mathrm{a}}}{\mathrm{a}\ln(1-\theta)}\right] \text{for a} = 1,2,3,\ldots;\ 0<\theta<1; 0<\alpha<1 \end{aligned}\right\} \qquad (1)$$

The nature of the long tail of log-series distribution is same as the nature of the tail of geometric distribution for big values of *a*. Alternatively the logarithmic-series distribution has single parameter.

## Estimation

Suppose $x_1, x_2, \ldots, x_n$ be a random sample and the probability distribution function of the sample is written by equation (1). Assume that $n_a$ ($a$ = 0, 1, 2,…, m) are the observation of the $a$'th cell and $\sum_{a=0}^{m} n_a = n.$ The mle for the given sample ($x_1, x_2, \ldots, x_n$) can be given as :

$$L[\alpha,\theta \mid (x_1, x_2, \ldots, x_n)] = (1-\alpha)^{n_0} \prod_{a=1}^{m}\left[\alpha\left(\frac{-\theta^{a}}{a\log(1-\theta)}\right)\right]^{n_a} \qquad (2)$$

$$= \frac{(1-\alpha)^{n_0}(-\alpha)^{n-n_0}\,\theta^{\sum_{a=1}^{m} n_a x_a}}{\left(\prod_{a=1}^{m} x_a^{n_a}\right)\left[\log(1-\theta)\right]^{n-n_0}} \qquad (3)$$

Where: $x_a$ represents the value of $a$.

After taking logarithms and differentiation of (3) we get the following equations:

$$\frac{\partial \log L}{\partial \alpha} = -\frac{n_o}{1-\alpha} + \frac{n-n_o}{\alpha} = 0 \qquad (4)$$

$$\frac{\partial \log L}{\partial \theta} = \frac{\sum_{a=1}^{m} n_a x_a}{\theta} + \frac{n-n_0}{(1-\theta)\log(1-\theta)} = 0 \qquad (5)$$

The estimator of α from equation (4) is given as

$$\hat{\alpha} = \frac{n-n_0}{n}$$

Equation (5) gives the estimating equation for $\theta$ as:

$$(1-\theta)\log(1-\theta)\sum_{a=1}^{m} n_a x_a + (n-n_0)\theta = 0 \qquad (6)$$

The mle of $\theta$ is obtained by numerical solution of (6)

We know that $E(n_0) = n(1-\alpha)$, $E(n-n_0) = n\alpha$

$$E(x_a) = \frac{-\alpha\theta}{(1-\theta)\log(1-\theta)} \text{ and } E\left(\sum_{a=1}^{m} n_a x_a\right) = -\frac{n\alpha\theta}{(1-\theta)\log(1-\theta)}$$

We have the expected values as

$$-E\left(\frac{\partial^2 \log L}{\partial \alpha^2}\right) = -\frac{E(n_0)}{(1-\alpha)^2} - \frac{E(n-n_0)}{\alpha^2} = \frac{n}{\alpha(1-\alpha)} = \varphi_{11} \text{ (say)} \qquad (7)$$

$$-E\left(\frac{\partial^2 \log L}{\partial \theta^2}\right) = -\frac{E\left(\sum_{a=1}^{m} n_a x_a\right)}{\theta^2} + \frac{[1+\log(1-\theta)]E(n-n_0)}{[(1-\theta)\log(1-\theta)]^2}$$

$$= -n\alpha\left[\frac{1}{\theta(1-\theta)\log(1-\theta)} + \frac{1+\log(1-\theta)}{[(1-\theta)\log(1-\theta)]^2}\right] = \phi_{22} \text{ (say} \qquad (8)$$

The covariance of α and $\theta$ is zero since $E\left(\frac{\partial^2 \log L}{\partial\alpha\partial\theta}\right) = 0$ and hence the variance of α and λ can be obtained as

$$v(\hat{\alpha}) = \frac{1}{\phi_{11}} \qquad \text{and} \qquad v(\hat{\theta}) = \frac{1}{\phi_{22}}$$

**Table-1:** Distribution of Number of Households

| No of Migrants | observed | Expected(Model I) | $\chi^2_{0.05}$ |
|---|---|---|---|
| 0 | 401 | 401 | |
| 1 | 147 | 157.50 | |
| 2 | 57 | 53.33 | |
| 3 | 29 | 24.08 | |
| 4 | 16 | 12.23 | 4.67 |
| 5 | 8 | 6.63 | |
| 6 | 5 | | |
| 7 | 1 | 9.22 | |
| 8 | 0 | | |
| | 664 | 664 | |
| α | 0.396084 | | |
| p | 0.537832 | | |
| λ | 0.677246 | | |
| var(α) | 0.00036 | | |
| Var(λ) | 0.000756 | | |

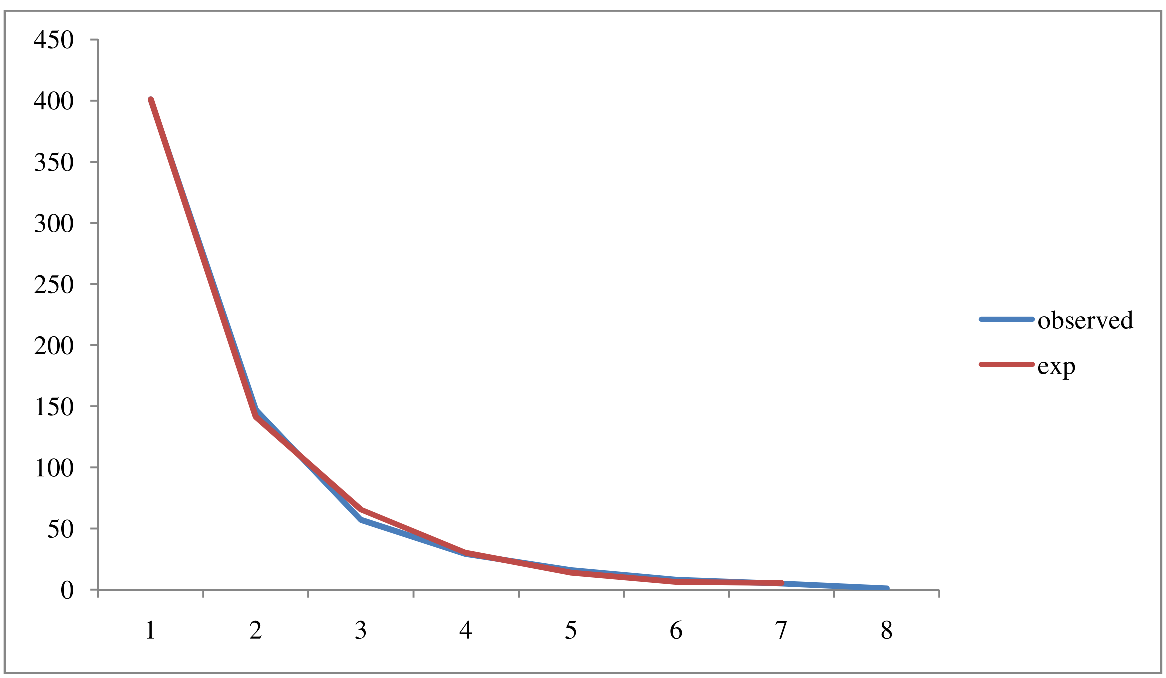


**Figure-1:** Expexted and Observed Frequency of Number of Migrants

## Conclusion

The model discussed above has been applied to the collected data from north eastern Bihar. The Study points out that applied model is a sensible approximation to explain the distribution of households for the rural out migrants and at least at the micro-level. The variance and covariance of the estimator for the model has also been computed. Although every study has some limitations and this is not an exception. In spite of some limitations we thought the study will be helpful for future study.

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