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Algorithm for Software Reliability Growth Model based on Order Statistics of Lehmann-type Laplace Distribution-Type I

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Abstract

In this paper, Software Reliability based on order statistics of Lehmann-Type Laplace Distribution-Type I is framed and an algorithm is generated for the model. The parameters of the distribution are estimated by using profile likelihood method and they are used to estimate the number of faults. A control mechanism is proposed using order statistics of Lehmann-Type Laplace distribution-Type I. The algorithm is tested for the software failure data. It is found that the algorithm works well for the data sets and the detection of failure is done early and frequently at many points. Thus the failures can be eradicated which will increase the life time of the software and in turn increases the software reliability.

Keywords: Software Reliability Growth Model, Order Statistics, Lehmann-Type Laplace distribution-Type I(LLD-I), Non-Homogeneous Poisson Process(NHPP), Profile likelihood method.

Introduction

Software Reliability[1] is an important research area because of its applications in many safety critical systems. Software reliability testing is being used as a tool to help assess the software engineering technologies. To perform software reliability testing, Software Reliability Growth Models (SRGM)[2] are used. Vamsidhar Y., Srinivas Y. and Achanta Brahmini used Pareto type III SRGM[3] to test the Software Reliability. Akilandeswari V.S., Poornima R. and Saavithri V.[4] used Lehmann-Type Laplace distribution Type I (LLD-I) SRGM to test Software Reliability which had a better fit for software failure data than the Weibull SRGM. Also a control mechanism based on LLD-I SRGM was framed to detect the software failure. Akilandeswari V.S., Poornima R. and Saavithri V.[4] used Lehmann-Type Laplace distribution Type II (LLD-II) SRGM to test Software Reliability which had a better fit for software failure data than Goel-okumoto, Weibull, Exponential Geometric, Pareto –Type III, Lehmann-Type Laplace distribution Type I (LLD-I) distributions. Also a control mechanism based on LLD-II SRGM was framed to detect the software failure. Ramchand H. Rao K., Satya Prasad R., and Kantham R.R.L. developed an SRGM based on half logistic distribution[5] by considering the mean value function as an order statistics. The grouping of data was done manually and the data was tested for the developed SRGM.

In this paper, one such SRGM is framed using order statistics of Lehmann-Type Laplace distribution-Type I (LLD-I)[6,7]. This SRGM is framed based on Non-Homogeneous Poisson Process (NHPP). The cumulative distribution function of order statistics of LLD-I is considered for forming the mean value function. Since the cumulative distribution function of order statistics of LLD-I is used, the data are grouped automatically. The parameters are estimated using profile likelihood method[8].

It is assumed here that the number of failures follow Poisson distribution. The expectation of the number of failures is the mean value function *m(x)*. The failure intensity function is proportional to the residual fault content.

To monitor software reliability process, Statistical process Control (SPC)[9,10] is used. The popular technique for maintaining process control is control charting. Mean value control chart is used here.

SRGM based on order statistics of LLD-I

Let $X_{r:n}$ be the r^{th} order statistics connected with a sample of size n from the LLD-I

(α,θ,ϕ) . Then the probability density function is

$$f_{r:n}(x)=\begin{cases} r\binom{n}{r}\dfrac{\alpha}{2^{\alpha}\phi}\left(\dfrac{1}{2^{\alpha}}e^{\alpha\left(\frac{x-\theta}{\phi}\right)}\right)^{(r-1)}\left(1-\dfrac{1}{2^{\alpha}}e^{\alpha\left(\frac{x-\theta}{\phi}\right)}\right)^{(n-r)}e^{\alpha\left(\frac{x-\theta}{\phi}\right)} & x\le\theta \\ r\binom{n}{r}\dfrac{\alpha}{2\phi}\left(1-\dfrac{1}{2}e^{-\left(\frac{x-\theta}{\phi}\right)}\right)^{(\alpha r-1)}\left(1-\left(1-\dfrac{1}{2}e^{-\left(\frac{x-\theta}{\phi}\right)}\right)^{\alpha}\right)^{(n-r)}e^{-\left(\frac{x-\theta}{\phi}\right)} & x\ge\theta \end{cases} \quad (1)$$

Similarly the cumulative distribution function is

$$F_{r:n}(x)=\begin{cases} \sum\limits_{i=r}^{n}\binom{n}{i}\dfrac{1}{2^{i\alpha}}\left(1-\dfrac{1}{2^{\alpha}}e^{\alpha\left(\frac{x-\theta}{\phi}\right)}\right)^{(n-i)}e^{i\alpha\left(\frac{x-\theta}{\phi}\right)} & x\le\theta \\ \sum\limits_{i=r}^{n}\binom{n}{i}\left(1-\dfrac{1}{2}e^{-\left(\frac{x-\theta}{\phi}\right)}\right)^{i\alpha}\left(1-\left(1-\dfrac{1}{2}e^{-\left(\frac{x-\theta}{\phi}\right)}\right)^{\alpha}\right)^{(n-i)} & x\ge\theta \end{cases} \quad (2)$$

Algorithm for order statistics of LLD-I based SRGM

3.1 Cumulative data: From the given data of time between failures, calculate the cumulative time between failures.

u←time between failures
n←length(u)
cum←u(first element) and sum←u(first element)
for i←2 to n
    sum←sum+u(ith element)
    concatenate cum and sum
end
cum is the cumulative data.

3.2 Estimation of parameters α, θ, ϕ of rth order statistics of LLD-I: The parameters are estimated using profile likelihood method. Let $I_1 = \{i / x_i \le \theta\}$ and $I_2 = \{i / x_i > \theta\}$. Then $|I_1| = n_1, |I_2| = n_2 \; and \; n_1 + n_2 = n$.

The likelihood function of rth order statistics of LLD-I is

$$
\begin{aligned}
l = \prod_{i=1}^{n_1} & \left[r\binom{n}{r} \frac{\alpha}{2^{\alpha}\phi} \left(\frac{1}{2^{\alpha}} e^{\alpha\left(\frac{x_i-\theta}{\phi}\right)} \right)^{(r-1)} \left(1 - \frac{1}{2^{\alpha}} e^{\alpha\left(\frac{x_i-\theta}{\phi}\right)} \right)^{(n-r)} \right] \\
\prod_{i=1}^{n_2} & \left[r\binom{n}{r} \frac{\alpha}{2\phi} \left(1 - \frac{1}{2} e^{-\left(\frac{x_i-\theta}{\phi}\right)} \right)^{(\alpha r-1)} \left(1 - \left(1 - \frac{1}{2} e^{-\left(\frac{x_i-\theta}{\phi}\right)} \right)^{\alpha} \right)^{(n-r)} e^{-\left(\frac{x_i-\theta}{\phi}\right)} \right]
\end{aligned} \tag{3}
$$

Taking logarithms on both the sides of (3) and simplifying using properties of logarithms, that is, the log likelihood function is

$$
\begin{aligned}
\log l = & n\left[\log r + \log\binom{n}{r} + \log\alpha - \log\phi\right] - (n_2 + n_1\alpha r)\log 2 + \sum_{i=1}^{n_1} r\alpha\left(\frac{x_i-\theta}{\phi}\right) - \sum_{i=1}^{n_2}\left(\frac{x_i-\theta}{\phi}\right) + \\
& \sum_{i=1}^{n_1}(n-r)\log\left(1-\frac{1}{2^{\alpha}}e^{\alpha\left(\frac{x_i-\theta}{\phi}\right)}\right) + \sum_{i=1}^{n_2}(\alpha r-1)\log\left(1-\frac{1}{2}e^{-\left(\frac{x_i-\theta}{\phi}\right)}\right) + \sum_{i=1}^{n_2}(n-r)\log\left(1-\left(1-\frac{1}{2}e^{-\left(\frac{x_i-\theta}{\phi}\right)}\right)^{\alpha}\right)
\end{aligned} \tag{4}
$$

Using differentiation properties, partially differentiating (4) with respect to α and equating it to zero, (5) is obtained.

$$
\alpha = n\left\{
\begin{aligned}
& n_1 r\log 2 - \sum_{i=1}^{n_1}\frac{r}{\phi}(x_i-\theta) + (n-r)\sum_{i=1}^{n_1}\left[\frac{\frac{e^{\alpha\left(\frac{x_i-\theta}{\phi}\right)}}{2^{\alpha}}\left(\frac{x_i-\theta}{\phi}-\log 2\right)}{\left(1-\frac{1}{2^{\alpha}}e^{\alpha\left(\frac{x_i-\theta}{\phi}\right)}\right)}\right] \\
& -\sum_{i=1}^{n_2} r\log\left(1-\frac{1}{2}e^{-\left(\frac{x_i-\theta}{\phi}\right)}\right) + (n-r)\sum_{i=1}^{n_2}\frac{\left(1-\frac{1}{2}e^{-\left(\frac{x_i-\theta}{\phi}\right)}\right)^{\alpha}\log\left(1-\frac{1}{2}e^{-\left(\frac{x_i-\theta}{\phi}\right)}\right)}{\left(1-\left(1-\frac{1}{2}e^{-\left(\frac{x_i-\theta}{\phi}\right)}\right)^{\alpha}\right)}
\end{aligned}
\right\}^{-1} \tag{5}
$$

Similarly, partially differentiating (4) with respect to ϕ and equating it to zero, (6) is obtained.

$$
\phi = n\left\{
\begin{aligned}
& -\sum_{i=1}^{n_1}\frac{r\alpha}{\phi^2}(x_i-\theta) + \sum_{i=1}^{n_2}\frac{(x_i-\theta)}{\phi^2} + \sum_{i=1}^{n_1}\frac{\frac{(n-r)\alpha}{2^{\alpha}\phi^2}(x_i-\theta)e^{\alpha\left(\frac{x_i-\theta}{\phi}\right)}}{\left(1-\frac{1}{2^{\alpha}}e^{\alpha\left(\frac{x_i-\theta}{\phi}\right)}\right)} \\
& -\sum_{i=1}^{n_2}\frac{\frac{(\alpha r-1)}{2\phi^2}(x_i-\theta)e^{-\left(\frac{x_i-\theta}{\phi}\right)}}{\left(1-\frac{1}{2}e^{-\left(\frac{x_i-\theta}{\phi}\right)}\right)} + \sum_{i=1}^{n_2}\frac{\frac{(n-r)\alpha}{2\phi^2}(x_i-\theta)e^{-\left(\frac{x_i-\theta}{\phi}\right)}\left(1-\frac{1}{2}e^{-\left(\frac{x_i-\theta}{\phi}\right)}\right)^{(\alpha-1)}}{\left(1-\left(1-\frac{1}{2}e^{-\left(\frac{x_i-\theta}{\phi}\right)}\right)^{\alpha}\right)}
\end{aligned}
\right\}^{-1} \tag{6}
$$

Keeping θ fixed for different values, the equations (5) and (6) are solved using least square method for α and ϕ. For all possible values of θ, α and ϕ, the likelihood equation value is obtained from equation (3) out of which the maximum value is chosen. The values θ, α and ϕ for which the maximum value is obtained are the parameter values.

Set the value of r, Initialize the values of l,m and n, for θ←l to n incremented by m, x←store the values of cum which are less than or equal to θ, y← store the values of cum which are greater than θ, n_1←number of values of x, n_2←number of values of y, n←add n_1 and n_2, for i←1 to n_1

sum1←sum$\left(\frac{r}{\phi}\left(x(i)-\theta\right)\right)$

sum2←sum$\left[\frac{\frac{e^{\alpha\left(\frac{x(i)-\theta}{\phi}\right)}}{2^{\alpha}}\left(\frac{x(i)-\theta}{\phi}-\log 2\right)}{\left(1-\frac{1}{2^{\alpha}}e^{\alpha\left(\frac{x(i)-\theta}{\phi}\right)}\right)}\right]$

sum5← sum$\left(\frac{r\alpha}{\phi^2}(x(i)-\theta)\right)$

sum6←sum$\left(\frac{\frac{(n-r)\alpha}{2^{\alpha}\phi^2}(x(i)-\theta)e^{\alpha\left(\frac{x(i)-\theta}{\phi}\right)}}{\left(1-\frac{1}{2^{\alpha}}e^{\alpha\left(\frac{x(i)-\theta}{\phi}\right)}\right)}\right)$

s1←sum $\left(r\alpha\left(\frac{x(i)-\theta}{\phi}\right)\right)$

s2←sum$\left((n-r)\log\left(1-\frac{1}{2^{\alpha}}e^{\alpha\left(\frac{x(i)-\theta}{\phi}\right)}\right)\right)$

end
for i←1 to n_2

$$\text{sum3} \leftarrow \text{sum}\left(r\log\left(1-\frac{1}{2}e^{-\left(\frac{y(i)-\theta}{\phi}\right)}\right)\right)$$

$$\text{sum4} \leftarrow \text{sum}\left(\frac{\left(1-\frac{1}{2}e^{-\left(\frac{y(i)-\theta}{\phi}\right)}\right)^{\alpha}\log\left(1-\frac{1}{2}e^{-\left(\frac{y(i)-\theta}{\phi}\right)}\right)}{\left(1-\left(1-\frac{1}{2}e^{-\left(\frac{y(i)-\theta}{\phi}\right)}\right)^{\alpha}\right)}\right)$$

$$\text{sum7} \leftarrow \text{sum}\left(\frac{(y(i)-\theta)}{\phi^2}\right)$$

$$\text{sum8} \leftarrow \text{sum}\left(\frac{\frac{(\alpha r-1)}{2\phi^2}(y(i)-\theta)e^{-\left(\frac{y(i)-\theta}{\phi}\right)}}{\left(1-\frac{1}{2}e^{-\left(\frac{y(i)-\theta}{\phi}\right)}\right)}\right)$$

$$\text{sum9} \leftarrow \text{sum}\left(\frac{\frac{(n-r)\alpha}{2\phi^2}(y(i)-\theta)e^{-\left(\frac{y(i)-\theta}{\phi}\right)}\left(1-\frac{1}{2}e^{-\left(\frac{y(i)-\theta}{\phi}\right)}\right)^{(\alpha-1)}}{\left(1-\left(1-\frac{1}{2}e^{-\left(\frac{y(i)-\theta}{\phi}\right)}\right)^{\alpha}\right)}\right)$$

$$\text{s3} \leftarrow \text{sum}\left(\frac{y(i)-\theta}{\phi}\right)$$

$$\text{s4} \leftarrow \text{sum}\left((\alpha r-1)\log\left(1-\frac{1}{2}e^{-\left(\frac{y(i)-\theta}{\phi}\right)}\right)\right)$$

$$\text{s5} \leftarrow \text{sum}\left((n-r)\log\left(1-\left(1-\frac{1}{2}e^{-\left(\frac{y(i)-\theta}{\phi}\right)}\right)^{\alpha}\right)\right)$$

end
F1← α -n(n_1*r*log(2)-sum1+(n-r)*sum2-sum3+(n-r)*sum4)$^{-1}$
F2← ϕ -n*(-sum5+sum7+sum6-sum8+sum9)$^{-1}$

L=n*(log(r)+log$\binom{n}{r}$+log(α)-log(ϕ))-(n_2+n_1* α *r)*log(2)+s1-s3+s2+s4+s5

end

Solving the 2 equations using lsqnonlin function in MATLAB 2013b for α and ϕ and substituting L gives all values of L for all possible values of θ, α and ϕ. The parameter values are chosen for which L is maximum.

3.3 NHPP Model: The mean value function $m(x)$ and intensity function $\lambda(x)$ using cumulative distribution function F(x) and probability density function f(x) are given by

$$m(x) = aF(x) \tag{7}$$

$$\lambda(x) = af(x) \tag{8}$$

Where: 'a' is the number of faults. To estimate 'a', the likelihood function for NHPP model can be written as

$$L = e^{-m(x_n)}\prod_{i=1}^{n}\lambda(x_i) \tag{9}$$

$$L = e^{-aF(x_n)}\prod_{i=1}^{n}af(x_i) \tag{10}$$

The corresponding log likelihood function is

$$\log L = -aF(x_n) + n\log a + \sum_{i=1}^{n}\log f(x_i) \tag{11}$$

Partially differentiating (11) with respect to 'a' and equating to zero

$$a = \frac{n}{F(x_n)} \tag{12}$$

That is, the value of a for order statistics of LLD-I based SRGM is

$$a=\frac{n}{\sum_{i=r}^{n}\binom{n}{i}\frac{1}{2^{i\alpha}}\left(1-\frac{1}{2^{\alpha}}\mathrm{e}^{\alpha\left(\frac{x_{n_1}-\theta}{\phi}\right)}\right)^{(n-i)}\mathrm{e}^{i\alpha\left(\frac{x_{n_1}-\theta}{\phi}\right)}+\sum_{i=r}^{n}\binom{n}{i}\left(1-\frac{1}{2}\mathrm{e}^{-\left(\frac{x_{n_2}-\theta}{\phi}\right)}\right)^{i\alpha}\left(1-\left(1-\frac{1}{2}\mathrm{e}^{-\left(\frac{x_{n_2}-\theta}{\phi}\right)}\right)^{\alpha}\right)^{(n-i)}} \tag{13}$$

for i←r to n

if n_1>0

$$\text{Fn1} \leftarrow \text{sum}\left(\binom{n}{i}\frac{1}{2^{i\alpha}}\left(1-\frac{1}{2^{\alpha}}\mathrm{e}^{\alpha\left(\frac{x(n_1)-\theta}{\phi}\right)}\right)^{(n-i)}\mathrm{e}^{i\alpha\left(\frac{x(n_1)-\theta}{\phi}\right)}\right)$$

end

if n_2>0

$$Fn2 \leftarrow \text{sum}\left(\binom{n}{i}\left(1-\frac{1}{2}e^{-\left(\frac{y(n_2)-\theta}{\phi}\right)}\right)^{i\alpha}\left(1-\left(1-\frac{1}{2}e^{-\left(\frac{y(n_2)-\theta}{\phi}\right)}\right)^{\alpha}\right)^{(n-i)}\right)$$

end
end

$$a = \frac{n}{Fn1 + Fn2}$$

3.4 Control mechanism: The control limits Upper Control Limit (UCL), Control limit (CL) and Lower Control Limit(LCL) are respectively the solutions of the following equations taking equitailed probabilities

$$F(x) = 0.99865$$
$$F(x) = 0.5$$
$$F(x) = 0.00135$$

Let X_u, X_c, X_l be respectively the solutions of these equations in standard form. Then

$$X_u = F^{-1}(0.99865)$$
$$X_c = F^{-1}(0.5)$$
$$X_l = F^{-1}(0.00135)$$

Equating the mean value function (7) to equitailed probabilities, the control limits for order statistics of LLD-I are calculated.
UCL (Upper Control Limit) = m(X_u)
CL (Control Limit) = m(X_c)
LCL (Lower Control Limit) = m(X_l)

A point above the UCL is an alarm signal. A point below the LCL is an indication of better quality software. A point within the control limits indicates stable process.
UCL←0.99865*a
LCL←0.00135*a
CL←0.5*a

3.5 Detection of Software failure: The mean value function is calculated for all the 'n' failure data. Then successive differences of mean value function are calculated. If successive differences of mean value function are less than LCL value then failure of software is detected at that failure number. Early detection of failure of the software can be used to identify and rectify the error which will improve software quality and in turn increases software reliability.

for j←1 to n
for i←r to n
if cum(j)<= θ

$$F1 \leftarrow \text{sum}\left(\binom{n}{i}\frac{1}{2^{i\alpha}}\left(1-\frac{1}{2^{\alpha}}e^{\alpha\left(\frac{cum(j)-\theta}{\phi}\right)}\right)^{(n-i)}e^{i\alpha\left(\frac{cum(j)-\theta}{\phi}\right)}\right)$$

else

$$F2 \leftarrow \text{sum}\left(\binom{n}{i}\left(1-\frac{1}{2}e^{-\left(\frac{cum(j)-\theta}{\phi}\right)}\right)^{i\alpha}\left(1-\left(1-\frac{1}{2}e^{-\left(\frac{cum(j)-\theta}{\phi}\right)}\right)^{\alpha}\right)^{(n-i)}\right)$$

end
end
if cum(j)<= θ
m←a*F1
else
m←a*F2
end

end

Software failure data analysis

Dataset 1: Cumulative data: Using the algorithm in section 3.1, Table-1. gives the time between failures and its cumulative time between failures of a software failure data.

Estimation of parameters α, θ, ϕ of order statistics of LLD-I: Using the algorithm in section 3.2, Table-2. The parameters θ, α and ϕ are calculated for the cumulative time between failures of software failure data set 1 given in Table-1 for different values of 'r'.

NHPP Model: Using the algorithm in section 3.3, Table-3 gives the number of faults 'a' for the software failure data set 1 for different values of 'r'.

Control mechanism: Using the algorithm in section 3.4, Table-4 gives the control limits UCL, LCL and CL for the software failure data set 1.

Software failure detection: Using the algorithm in section 3.5, Table-5 gives the successive differences of mean value function. Figures 1 to 4 gives the mean value chart for different values of 'r'. All the successive differences from 1 to 28 lies below their respective LCL values. Hence failure is detected at all these points and those can be eradicated in turn increasing the software reliability.

Dataset 2: Cumulative data: Using the algorithm in section 3.1, Table-6 gives the time between failures and its cumulative time between failures of a software failure data[6].

Estimation of parameters α, θ, ϕ of order statistics of LLD-I: Using the algorithm in section 3.2, in Table-7, the parameters θ, α and ϕ are calculated for the cumulative time between failures of software failure data set 1 given in Table-6 for different values of 'r'.

NHPP Model: Using the algorithm in section 3.3, Table-8 gives the number of faults 'a' for the software failure data set 2 for different values of 'r'.

Table-1
Time and Cumulative time between failures for a software failure data

Failure Number	Time Between Failures (hrs)	Time Between Failures (hrs) (Cumulative)
1	30.02	30.02
2	1.44	31.46
3	22.47	53.93
4	1.36	55.29
5	3.43	58.72
6	13.2	71.92
7	5.15	77.07
8	3.83	80.9
9	21	101.9
10	12.97	114.87
11	0.47	115.34
12	6.23	121.57
13	3.39	124.97
14	9.11	134.07
15	2.18	136.25
16	15.53	151.78
17	25.72	177.5
18	2.79	180.29
19	1.92	182.21
20	4.13	186.34
21	70.47	256.81
22	17.07	273.88
23	3.99	277.87
24	176.06	453.93
25	81.07	535
26	2.27	537.27
27	15.63	552.9
28	120.78	673.68
29	30.81	704.49
30	34.19	738.68

Table-2
Parameter values for IV, VII, XVI and XXV order statistics of LLD-I

R	θ	α	ϕ
4	739	$2.0517X10^{-4}$	$6.9959X10^{-4}$
7	739	$2.0514X10^{-4}$	$6.9959X10^{-4}$
16	739	$2.0512X10^{-4}$	$6.9959X10^{-4}$
25	739	$2.0511X10^{-4}$	$6.9959X10^{-4}$

Table-3
Number of faults for IV, VII, XVI and XXV order statistics of LLD-I

r	a
4	30
7	30
16	30
25	31.4918

Table-4
Control limits for IV, VII, XVI and XXV order statistics of LLD-I

r	UCL	CL	LCL
4	29.9595	15	0.0405
7	29.9595	15	0.0405
16	29.9595	15	0.0405
25	31.4493	15.7459	0.0425

Control mechanism: Using the algorithm in section 3.4, Table 9 gives the control limits UCL, LCL and CL for the software failure data set 2.

Software failure detection: Using the algorithm in section 3.5, Table 10 gives the successive differences of mean value function. Figures 5 to 8 gives the mean value chart for different values of 'r'. Here LCL value is 0.0513. All the values of successive differences from 1 to 36 lies below LCL value. Hence failure is detected at all these points and those can be eradicated. Reliability of any software increases if the failures of the software are identified and corrected soon. Thus SRGM based on order statistics of LLD-I will improve software reliability.

Table-5
Successive differences of mean value function for IV, VII, XVI and XXV order statistics of LLD-I

Failure Number	Successive differences of *m(x)*			
	r=4	r=7	r=16	r=25
1	0	0	0	0
2	0	0	0	0
3	0	0	0	0
4	0	0	0	0
5	0	0	0	0
6	0	0	0	0
7	0	0	0	0
8	0	0	0	0
9	$9.7421X10^{-313}$	0	0	0
10	$7.1649X10^{-313}$	0	0	0
11	$2.5193X10^{-309}$	0	0	0
12	$1.3347X10^{-307}$	0	0	0
13	$5.8742X10^{-303}$	0	0	0
14	$6.9883X10^{-302}$	0	0	0
15	$6.1697X10^{-294}$	0	0	0
16	$7.7946X10^{-281}$	0	0	0
17	$1.9782X10^{-279}$	0	0	0
18	$1.7488X10^{-278}$	0	0	0
19	$2.4625X10^{-276}$	0	0	0
20	$1.9569X10^{-240}$	0	0	0
21	$9.7023X10^{-232}$	0	0	0
22	$1.0358X10^{-229}$	0	0	0
23	$5.0429X10^{-140}$	$4.6173X10^{-247}$	0	0
24	$9.9768X10^{-99}$	$8.5689X10^{-175}$	0	0
25	$1.3301X10^{-97}$	$8.9608X10^{-173}$	0	0
26	$1.3094X10^{-89}$	$7.7551X10^{-159}$	0	0
27	$4.3782X10^{-28}$	$3.6055X10^{-51}$	$3.6151X10^{-124}$	$5.2738X10^{-202}$
28	$2.1640X10^{-12}$	$1.0512X10^{-23}$	$2.1335X10^{-61}$	$6.2698X10^{-104}$
29	30	30	30	30

Table-6
Time and Cumulative time between failures for a software failure data

Failure Number	Time Between Failures (hrs)	Time Between Failures (hrs) (Cumulative)
1	81	81
2	48	129
3	9	138
4	4.5	142.5
5	4.5	147
6	60	207
7	24	231
8	21	252
9	21	273
10	12.6	285.6
11	12	297.6
12	3	300.6
13	90	390.6
14	6	396.6
15	24	420.6
16	6	426.6
17	150.6	577.2
18	1.2	578.4
19	3.6	582
20	12	594
21	3	597
22	12	609
23	12	621
24	33	654
25	198	852
26	30	882
27	6	888
28	96	984
29	84	1068
30	81	1149
31	156	1305
32	18	1323
33	54	1377
34	39	1416
35	24	1440
36	12	1452
37	795	2247
38	90	2337

Table-7
Parameter values for II, VIII, XIV and XXXIII order statistics of LLD-I

r	θ	α	ϕ
2	2338	$2.0512X10^{-4}$	$6.9959X10^{-4}$
8	2338	$2.0511X10^{-4}$	$6.9959X10^{-4}$
14	2338	$2.0511X10^{-4}$	$6.9959X10^{-4}$
33	2337	$2.0510X10^{-4}$	$6.9959X10^{-4}$

Table-8
Number of faults for II, VIII, XIV and XXXIII order statistics of LLD-I

R	a
4	38
7	38
16	38
25	38

Table-9
Control limits for II, VIII, XIV and XXXIII order statistics of LLD-I

r	UCL	CL	LCL
4	37.9487	19	0.0513
7	37.9487	19	0.0513
16	37.9487	19	0.0513
25	37.9487	19	0.0513

Table-10
Successive differences of mean value function for II, VIII, XIV and XXXIII order statistics of LLD-I

Failure Number	**Successive differences of *m(x)***			
	r=2	**r=8**	**r=14**	**r=33**
1	0	0	0	0
2	0	0	0	0
3	0	0	0	0
4	0	0	0	0
5	0	0	0	0
6	0	0	0	0
7	0	0	0	0
8	0	0	0	0
9	0	0	0	0
10	0	0	0	0
11	0	0	0	0
12	0	0	0	0
13	0	0	0	0
14	0	0	0	0
15	0	0	0	0
16	0	0	0	0
17	0	0	0	0
18	0	0	0	0
19	0	0	0	0
20	0	0	0	0
21	0	0	0	0
22	0	0	0	0
23	0	0	0	0
24	0	0	0	0
25	0	0	0	0
26	0	0	0	0
27	0	0	0	0
28	1.3198X10^{-319}	0	0	0
29	4.2008X10^{-299}	0	0	0
30	2.2487X10^{-259}	0	0	0
31	8.6298X10^{-255}	0	0	0
32	4.8777X10^{-241}	0	0	0
33	4.1723X10^{-231}	0	0	0
34	5.4009X10^{-225}	0	0	0
35	6.1394X10^{-222}	0	0	0
36	1.7848X10^{-19}	3.7417X10^{-84}	2.2238X10^{-151}	0
37	38	38	38	38

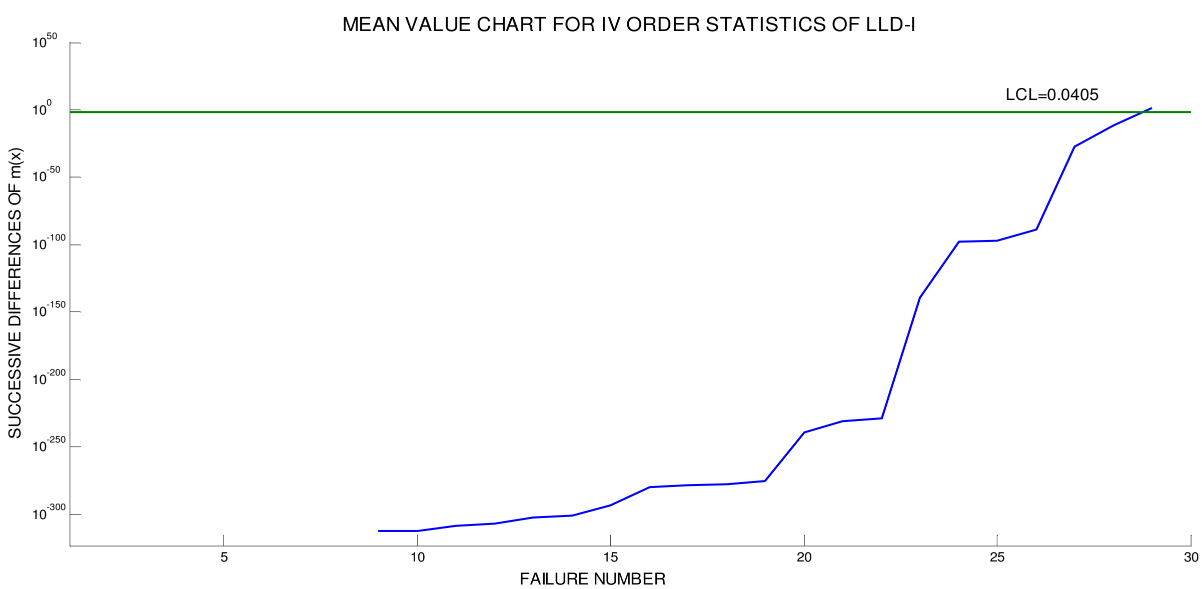

Figure-1
Mean Value Chart for IV order statistics of LLD-I

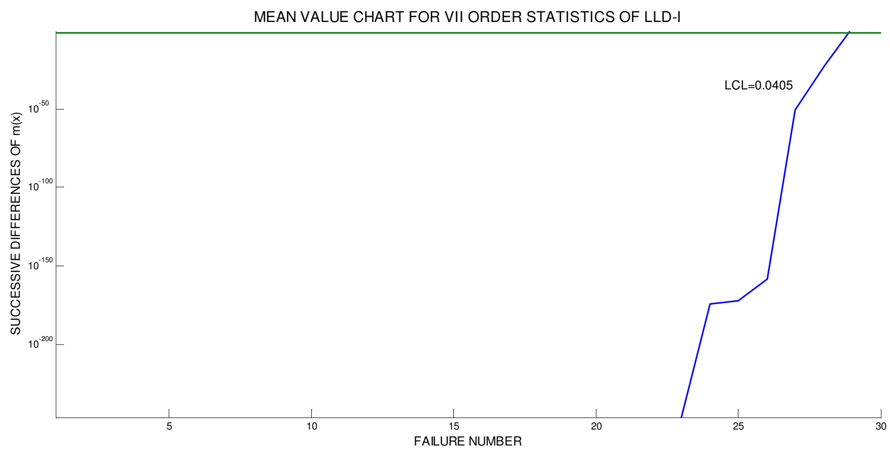

Figure-2
Mean Value Chart for VII order statistics of LLD-I

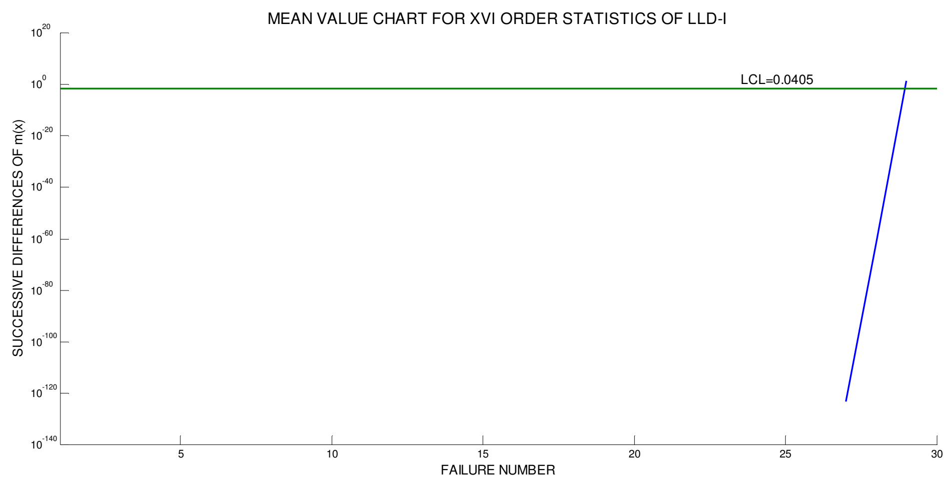

Figure-3
Mean Value Chart for XVI order statistics of LLD-I

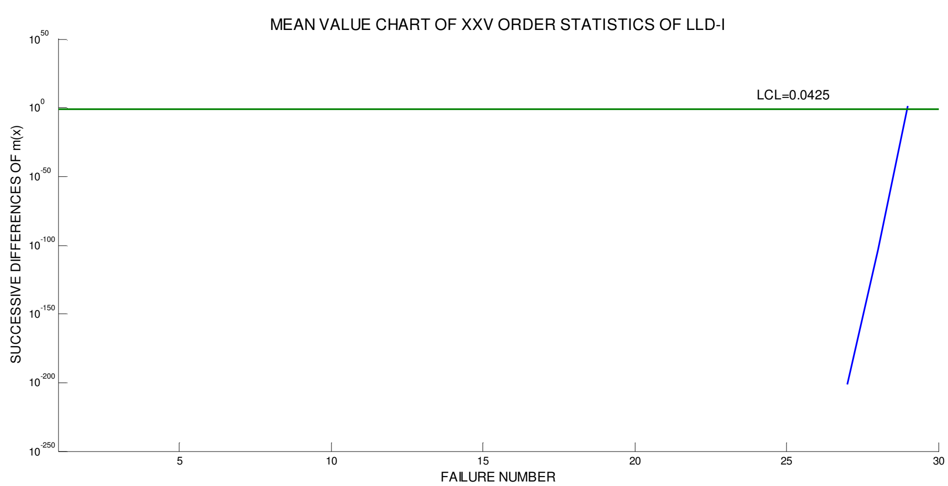

Figure-4
Mean Value Chart for XXV order statistics of LLD-I

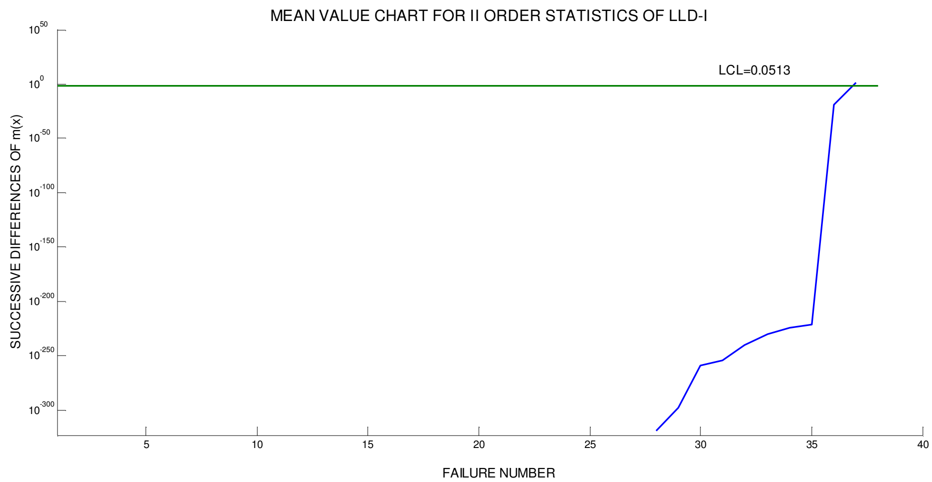

Figure-5
Mean Value Chart for II order statistics of LLD-I

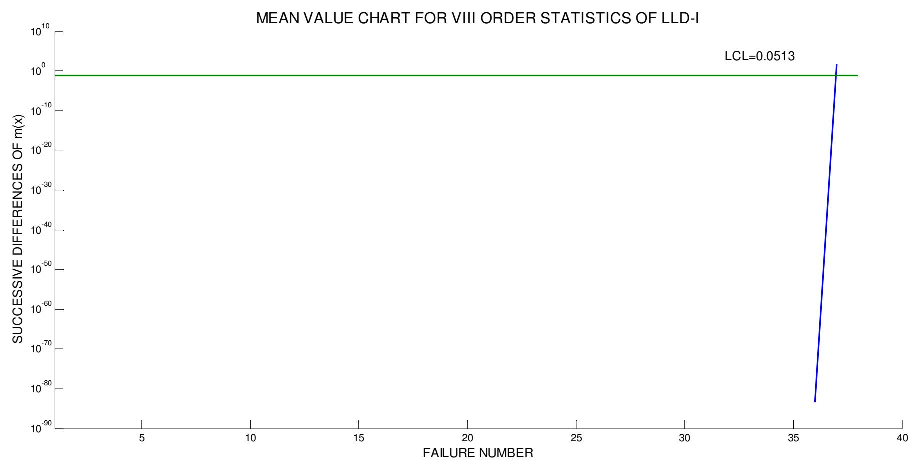

Figure-6
Mean Value Chart for VIII order statistics of LLD-I

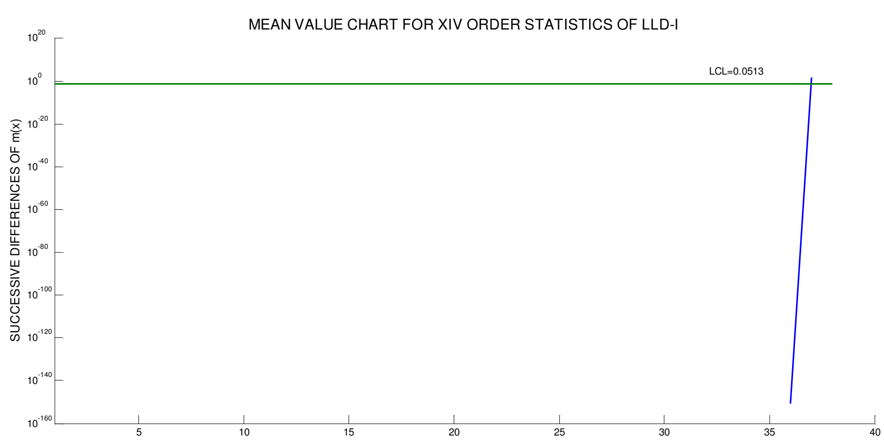

Figure-7
Mean Value Chart for XIV order statistics of LLD-I

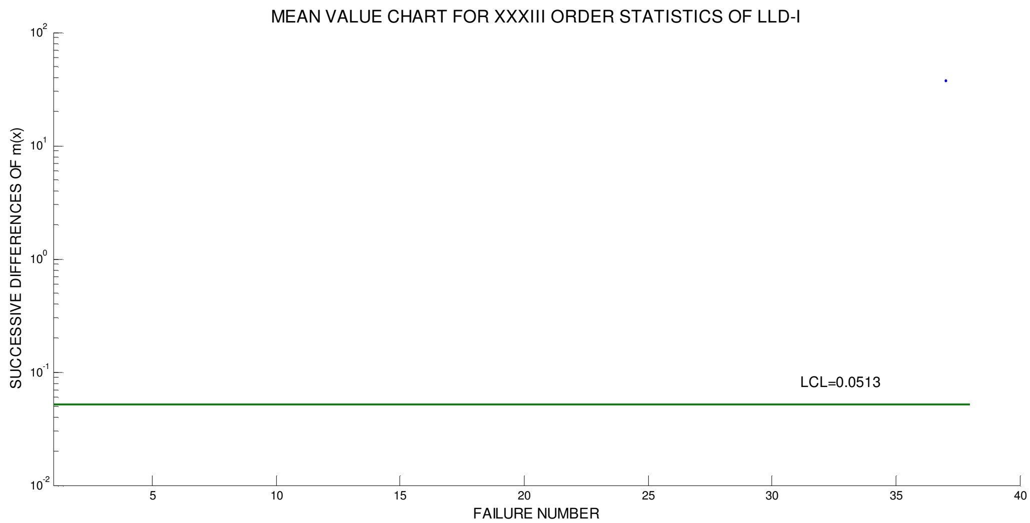

Figure-8
Mean Value Chart for XXXIII order statistics of LLD-I

Conclusion

SRGM based on order statistics of LLD-I is used to test two sets of software failure data. An algorithm is developed to show the working of SRGM based on order statistics of LLD-I. Software failure data analysis is performed for the developed SRGM using the proposed algorithm and it is proved software failure is detected for both the datasets early and frequently. This will improve software reliability.

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