Research Journal of Mathematical and Statistical Sciences E-ISSN 2320-6047
Vol. **4(8),** 10-14, September **(2016)** *Res. J. Mathematical and Statistical Sci.*

Ant Colony Strategy and Desirability Function Approach for Continuous Correlated Multiple Response Optimization Problems

Goutam Barman[1*] and Subrata Mukhopadhyay[2]
[1]Department of Statistics, Krishnagar Government College, West Bengal, India
[2]Department of Mathematics, Krishnagar Government College, West Bengal, India
goutambrmn@gmail.com

Available online at: www.isca.in, www.isca.me
Received 3rd July 2016, revised 5th August 2016, accepted 8th September 2016

Abstract

In Quality Engineering and Management Field, multiple response optimization problems is critical and important area of research. These problems may be correlated or uncorrelated. For various multimodal single and multi-response optimization problems, meta-heuristics iterative search strategies provide satisfactory global solution. In this paper, ant colony optimization strategy with desirability function approach are used to optimize correlated multi-response problems. The multi-response optimization problems are modeling by using ordinary least-square regression.

Keywords: Desirability function, Ant colony optimization (ACO), Response surface methodology, Multi-response optimization (MRO) problems, Ordinary least-square (OLS).

Introduction

Heuristics is a search technique design to solve optimization problems. In operation research and computer science, various heuristics are widely used although this technique may not give the actual or correct results but usually produces a satisfactory good solution for difficult problems. In last few years, researchers are tries to combine and modified the basic heuristic methods for efficiently and effectively exploring the search space. These methods are nowadays commonly called meta-heuristics. Recently several meta-heuristic algorithms such as ant colony optimization (ACO), honey bee optimization (HBO), evolutionary computation (EC) including genetic algorithms (GA), iterated local search (ILS), simulated annealing (SA), and tabu search (TS) are becoming increasingly popular.

Dorigo[1] first proposed ant colony optimization (ACO) for solving the Traveling Salesman Problem (TSP). The main focus of ant colony optimization is that 'how ants manage to establish the shortest path from their nest to food sources'. By now, the idea of ACO has been used in a large number of intractable combinatorial problems such as quadratic assignment problems (QAP), graph coloring, scheduling problems etc. Although, to solve the continuous variable optimization problems, ant colony optimization algorithm could not applied directly.

Later Bilchev and Parmee[2] improved ant colony optimization and first introduced an ACO metaphor for continuous problems, but the mechanism of ACO was focused on the local search procedure only. This algorithm was further modified by Wodrich and Bilchev[3]. In this modified algorithm global search optimization procedure is introduced. The ants are distributed in locally and globally. The local ants and global ants both search and explore the regions repeatedly and move to destinations of better fitness. Mathur et al.[4] further extend this algorithm by introducing a random walk and trail diffusion component in the global search, in 2000 and the performance were significantly improved. Later many researchers[5-7] improved and used ant colony strategy in various fields of operation research.

Search Strategy

Ant colony strategy are used in this study to find out the optimal solutions. The multi-response optimization problem is formulated based on regression analysis, and desirability functions.

Dorigo[1] first proposed the concept of an ant colony optimization (ACO) method to resolve discrete optimization problem. A pheromone deposition concept is used in ACO to probabilistically direct the search in feasible solution space. Socha and Dorigo[8] proposed a variant of ACO which is suitable for continuous problem.

To apply ant colony optimization in continuous space, the domain of the solution space randomly divided into a specific number (say, R) of regions. These regions are treated as the trail solutions for the problem. Then the fitness of these trial solutions are evaluated and sorted the solutions in the order of ascending fitness. Now update these regions by local search and global search mechanisms respectively. These local search and global search done by local ants (say, L) and global ants (say, G). The ratio between global and local ants is an important parameter for this methodology.

In global search methodology, firstly identify the G weak regions in terms of fitness and replace them by new regions (solutions) crated by all global ants. By applying the crossover, mutation and trail diffusion processes, the global ants create these new regions from the $(R\text{-}G)$ fittest regions. The global ants do not have any pheromone activity.

For crossover operations, about 90% of the G global ants take part. In this operation, combines the fittest regions (which are called existing parent) and produce new regions (which are called children). It is done as follows: the first element of the randomly chosen parent from the fittest regions is set as the first element of the child's position vector. The second element of a randomly chosen another parent from the fittest region with a probability cp, called crossover probability, is set as the second element of the child's position vector. By the same procedure, the elements of the position vector of child's are chosen.

After crossover operations the newly created regions (solutions) are further improved by mutation operation. This operation is carried out by randomly adding or subtracting a value to each and every variable of the newly created region with a probability proportional to a suitable predefined mutation probability. The mutation step size is given by the relation:

$$\triangle x = MS.(1 - r^{(1-T)^b})$$

where MS: maximum step size, r: random number from [0,1], T: ratio of the current iteration no. to the, total no. of iterations, b: a positive constant (between 0 and 10)

A random number is generated. If this random number is less than mutation probability then $\triangle x$ is added with the old value otherwise it is subtracted from the old value.

The remaining last 10% of weakest solutions are improved by the trail diffusion technique. In this technique two distinct parents are selected at random from the fittest region. A random number α is generated between 0 and 1 for every regions. The elements of the child's position vector depends upon the random number α.

If α is greater than 0.8 then the corresponding element of child from the second parent.

If α is lies between 0.6 to 0.8 then the corresponding element of child from the first parent.

If α is less than 0.6 (mutation probability)
then: $x_i(\text{child}) = (\alpha).x_i(\text{parent}_1) + (1-\alpha).x_i(\text{parent}_2)$

The same techniques are followed to improve the other solutions.

Thus at the end of global search procedure $'G'$ new regions (solutions) are created. Then G weaker regions are replaced by these newly regions. So in these search processes, we see that, in each iteration global ant's move away from the weaker solutions to fitter solutions produced by recombination of fitter parents. The age and pheromone values for the newly created solutions now have to be assigned. This is done by taking average values of the parent solutions from which the new solution is created

In the local search strategy, the L local ants select L regions from the $(R-G)$ fittest regions as per the fitness of the regions and move over these regions in search of better fitness. These local ants have pheromone activity. The following steps explain the local search method:

a. Initially the pheromone value for every regions is set to 1.0 and the age for every regions is taken as 0.

b. Now the limiting step is calculated for the regions by using the following equation
$\text{ls}(i) = k_1 - \text{age}(i).k_2$ Where $\text{age}(i)$ is the age for region i and k_1, k_2 are user defined constants. Here k_1 is always greater than k_2. This formula indicates that the limiting step of the local search is reduced linearly with increasing age. Here we take the values of k_1 and k_2 as k_1=0.1 and k_2=0.001.

c. we calculate $P_i(t) = \frac{\tau_i(t)}{\sum_k \tau_k(t)}$ for selected regions by the local ants. where i is the region index and $\tau_i(t)$ is the pheromone trail on region i at time t.

d. Then a random number is generated from 0 to 1.

e. If the random number is greater than $P_i(t)$ for the region i at time t then the ant moves through a distance and the new position of the ant is given by:

$$x_{(i)\text{new}} = x_{(i)\text{old}} + \text{ls}(i)$$

Otherwise the local ants remains same position.

f. Then we calculate fitness of these new regions.

g. If the fitness is improved in this process, then: i. age will be same corresponding to their. ii. Parents. iii. region is update. iv. pheromone trail is also update and is given by

$$\text{new\_ph}(i) = \frac{F(x_{\text{new}}(i)) - F(x_{\text{old}}(i))}{F(x_{\text{old}}(i))} + \text{old\_ph}(i)$$

where i indicates i^{th} region.

If the fitness is not improved, then
1. age is increased by 1. The increase in age can be compared to lack of interest and capabilities of older ants in searching places far away from the nest.
2. pheromone trail will be same corresponding to their parents.
3. again generate new regions randomly in the search space.

The pseudo code for ant colony optimization is given below:
1. Randomly generate R solutions in the search space.
2. According to the fitness value, sort the solutions.
3. Set age and pheromone value equal to 0 and 1 respectively for all solutions.
4. While termination condition is not met: i. Send G global ants and replace G weakest solutions by global search. ii. Send L local ants to selected solutions. iii. If the fitness is improved, move the local ants to the new solutions an update pheromone value. iv. Update global maximum. v. Evaporate trail for all solutions
5. End while.

Ordinary Least Square Regression Modeling

Let us consider a multiple response experiment with M response variables namely $y_1, y_2, y_3, \ldots\ldots, y_M$. Total n observations are collected for each response variable. The model for responses are given in matrix notation as follows:

$$y = X\beta + \varepsilon$$

Where: each y is an $n\times 1$ vector of observations, each X is an $n\times p$ matrix of the design variables, each β is a $p\times 1$ vector of unknown parameters, and each ε is respective random error vector of the responses.

The least-square functions for each response from the above equations are

$$s(\beta_i) = \varepsilon_i'\varepsilon_i = (y_i - X_i\beta_i)'(y_i - X_i\beta_i)$$

$$= y_i'y_i - \beta_i'X_i'y_i - y_i'X_i\beta_i + \beta_i'X_i'X_i\beta_i$$

$$= y_i'y_i - 2\beta_i'X_i'y_i + \beta_i'X_i'X_i\beta_i$$

Therefore, the least-squares normal equations are given by $\frac{\partial s(\beta_i)}{\partial \beta_i} = -2X_i'y_i + 2X_i'X_i\beta_i = 0$

That is., $X_i'X_i\beta_i = X_i'y_i$

and the solution to theses equations are,

$$\hat{\beta}_i = (X_i'X_i)^{-1}X_i'y_i$$

Desirability Function Approach

By this approach, multi-response optimization problems reduce to a single objective optimization problem. Firstly, each predicted response variables are transformed to an individual scale free desirability functions d_i where $0 \le d_i \le 1$. This individual desirability transformation function varied according to its desired target of the response viz., nominal-the-best (NTB), larger-the-best (LTB), and smaller-the-best (STB). The value of d_i increases as the "desirability" of the corresponding response increases. $d_i = 0$ represent a completely undesirable value of the i^{th} response and $d_i = 1$ represents a completely desirable or ideal response value. Then an overall composite or overall desirability measure is obtained by aggregating individual desirability d_i's.

Depending on whether a particular response Y_i is to be maximized, minimized or assigned a target value, different desirability function can be used. Let d_i be the i^{th} individual desirability function, L_i, U_i and T_i be the lower, upper and target values respectively that are desired for that response with $L_i \le T_i \le U_i$.

If a response is LTB (larger the better) type that is., the response is to be maximized then the individual desirability is defined as,

$$d_i = \begin{cases} 0 & \text{, if } \hat{Y}_i \le L_i \\ \left(\frac{\hat{Y}_i - L_i}{U_i - L_i}\right)^r & \text{, if } L_i < \hat{Y}_i < U_i \\ 1.0 & \text{, if } \hat{Y}_i \ge U_i \end{cases}$$

If the response is STB (smaller the better) type that is., the response is to be minimized then

$$d_i = \begin{cases} 1.0 & \text{, if } \hat{Y}_i \le L_i \\ \left(\frac{\hat{Y}_i - U_i}{L_i - U_i}\right)^s & \text{, if } L_i < \hat{Y}_i < U_i \\ 0 & \text{, if } \hat{Y}_i \ge U_i \end{cases}$$

Again if the response is NTB (nominal the best) type that is., for this response target is the best, then

$$d_i = \begin{cases} 0 & \text{, if } \hat{Y}_i < L_i \text{ or } \hat{Y}_i > U_i \\ \left(\frac{\hat{Y}_i - L_i}{T_i - L_i}\right)^t & \text{, if } L_i \le \hat{Y}_i \le T_i \\ \left(\frac{\hat{Y}_i - U_i}{T_i - U_i}\right)^q & \text{, if } T_i < \hat{Y}_i \le U_i \end{cases}$$

Here r, s, t and q are the user-specified exponential parameters that determine the shape of desirability function and in this work we take all values of these as 1.

Harrington[9] first suggested a geometric mean approach for composite desirability, which is expressed as

$$\lambda = (d_1 d_2 d_3 \ldots\ldots d_m)^{\frac{1}{m}}$$

Where there are m responses.

Later on, Derringer and Suich[10] suggested a weighted composite desirability function, expressed as

$$\lambda = \left(d_1^{w_1} d_2^{w_2} d_3^{w_3} \ldots .. d_m^{w_m}\right)^{1/\sum_{i=1}^{m} w_i}$$

Kim and Lin[11] extended the concept and employed a "minimum operator" to aggregate the individual desirability function and proposed a composite desirability:

$\lambda = \text{minimum}\,\{d_1, d_2, d_3, \ldots \ldots, d_m\},\ 0 \leq \lambda \leq 1$

In this study "minimum operator" desirability approach were selected.

Results and Discussion

In this section we discuss the regression models, parameter settings and all relevant results. Firstly, for multi-response optimization case, we consider three datasets and modeling these by OLS approach.

First dataset is taken from Mukherjee[12] and the fitted OLS model is

$$\hat{y}_1 = 1.575144 - 0.06466x_4 - 0.23420x_6 + 0.112997x_8 - 0.03210x_9$$

$$\hat{y}_2 = 0.000433 - 0.00009x_1 - 0.00008x_2 + 0.000159x_3 + 0.000021x_4 - 0.00004x_7 + 0.000042x_9 - 0.00003x_{10}$$

$$\hat{y}_3 = 7.01139 + 0.002453x_1 + 0.001082x_4 - 0.001817x_7 + 0.005376x_8 - 0.00168x_9$$

$$\hat{y}_4 = 0.003218 - 0.000997x_3 + 0.000912x_5 - 0.00055x_6$$

$$\hat{y}_5 = 0.003246 - 0.00037x_1 + 0.000248x_3 + 0.000584x_5 - 0.00025x_6$$

Here first three response variables are NTB type and last two response variables are STB type and other details are given as follows:

Table-1
Category of responses and other parameters of Mukherjee[12] dataset

Response variable	Type	Lower bound	Upper bound	Target
1st	NTB	1.41421	1.87083	1.64252
2nd	NTB	0.00028	0.00049	0.00039
3rd	NTB	97.00	97.02	97.01
4th	STB	0.0	0.01	–
5th	STB	0.0	0.01	–

Second dataset is taken from Shah et al.[13] and the fitted OLS model is

$$\hat{y}_1 = 1.884634 - 0.097383x_1 - 0.103901x_1^2$$

$$\hat{y}_2 = 22.648811 + 5.614804x_1 - 0.217981x_2 + 7.830449x_1^2 + 2.580000x_1x_2$$

$$\hat{y}_3 = 18.956432 + 0.744422x_1 - 0.012007x_2 - 1.070972x_3 + 3.215186x_1^2 + 1.341815x_3^2 + 1.8x_1x_2 + 2.1x_1x_3$$

$$\hat{y}_4 = 51.910033 + 2.436441x_1 - 3.428739x_1^2$$

In this case 1st and last response variables are LTB type and other two are STB type and also other details are given as follows:

Table-2
Category of responses and other parameters of Shah et al.[13] dataset

Response variable	Type	Lower bound	Upper bound
1st	LTB	1.7	1.92
2nd	STB	20.16	21
3rd	STB	18.50	20
4th	LTB	45	52.70

Third dataset is taken from Khuri and Conlon[14] and the fitted OLS model is

$$\hat{y}_1 = 051.156 - 176.083x_1 + 57.58333x_3 + 21.8333x_4$$

$$\hat{y}_2 = 11.01563 + 1.083333x_1 + 3.691667x_2 + 1.6x_3 - 1.5x_5$$

$$\hat{y}_3 = 8.82500 - 10.1208x_1 - 8.67917x_2 + 1.370833x_5$$

$$\hat{y}_4 = 92.14375 - 8.24583x_1 + 7.512500x_2 + 2.379167x_3 + 1.662500x_5$$

In this case all response variables are LTB type and other details about responses are as follows:

Table-3
Category of responses and other parameters of Khuri and Conlon[14] dataset

Response variable	Type	Lower bound	Upper bound
1st	LTB	651	1363
2nd	LTB	4	36
3rd	LTB	36.8	86.5
4th	LTB	50.5	113

Also for all three models we keep the bounds of explanatory variables between -1 to $+1$. After applying the max-min desirability approach by ant colony strategy, we get the following results:

Table-4
Results of MRO problems by CACO

Dataset	Optimum desirability value
Mukherjee[12]	0.67159
Shah et al.[13]	0.471862
Khuri and Conlon[14]	0.412984

The best setting of the parameters of ant colony strategy are $m = 100, r = 200, mut = 0.5, cp = 1, b = 10$. All statistical working done in SAS and the program or simulations were performed in Matlab 7.0 environment.

Conclusion

Multi-response optimization problems may or not correlated can be reasonably optimized by ant colony strategy with max-min desirable function approach. Although seemingly unrelated regression (SUR) model is more efficient than ordinary least square (OLS) model when the dataset are correlated otherwise these two model are equivalent. However scope exists to improve and compare the CACO with other meta-heuristics to solve or optimize the multi-response problems reasonably by SUR based model approach.

References

1. M. Dorigo (1992). Optimization, Learning and Natural Algorithms. Ph.D. Thesis, ipartimento di Elettronica, Politecnico di Milano, Italy.

2. Bilchev G. and Parmee I.C. (1995). The ant colony metaphor for searching continuous design spaces. Lecture Notes in Computer Science, New York, Springer- Verlag, 993, 25-39.

3. Wodrich M. and Bilchev G. (1997). Cooperative distributed search: the ants way. Control Cybernetics, 26, 413.

4. Mathur M., Karale S.B., Priye S., Jayaraman V.K. and Kulkarni B.D. (2000). Ant colony approach to continuous function optimization. *Ind. Eng. Chem. Res.*, 39, 3814-3822.

5. Jayaraman V.K., Kulkarni B.D., Karale S. and Shelokar P. (2000). Ant colony framework for optimal design and scheduling of batch plants. *Computer and Chemical Engineering*, 24, 1901-1912.

6. Jayaraman V.K., Kulkarni B.D., Gupta K., Rajesh J. and Kusumaker H.S. (2001). Dynamic optimization of fed-batch bioreactors using the ant algorithm. *Biotechnol. Prog.*, 17, 81-88.

7. Prabhaharan G., Asokan P. and Rajendran S. (2005). Sensitivity-based conceptual design and tolerance allocation using the continuous ants colony algorithm (CACO). *Int J Adv Manuf Technol*, 25, 516-526.

8. Socha K. and Dorigo M. (2008). Ant colony optimization for continuous domains. *European Journal of Operation Research*, 185, 1155-1173.

9. Harrington E.C. (1965). The desirability function. *Industrial Quality Control*, 21(10), 51-57.

10. Derringer G. and Suich R. (1980). Simultaneous optimization of several response variables. *Journal of Quality Technology*, 12(4), 214-219.

11. Kim K.J. and Lin D.K.J. (2000). Simultaneous optimization of mechanical properties of steel by maximizing exponential desirability functions. *Journal of the Royal Statistical Society Applied Statistics Series C*, 49(3), 311-325.

12. Mukherjee I. (2006). Modeling and optimization of abrasive metal cutting processes. Ph.D. Thesis, IIT Kharagpur, India.

13. Shah H.K., Montgomery D.C. and Carlyle W.M. (2004). Response surface modeling and optimization in multiresponse experiments using seemingly unrelated regressions. *Quality Engineering*, 16(3), 387-397.

14. Khuri A.l. and Conlon M. (1981). Simultaneous optimization of multiple responses represented by polynomial regression functions. *Technometrices*, 23(4), 363-375.