Research Journal of Mathematical and Statistical Sciences ______________________________E-ISSN 2320-6047
Vol. **4(7),** 1-6, August **(2016)** *Res. J. Mathematical and Statistical Sci.*

ABS Solution of Linear Programming Problems with Fuzzy Variables

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Available online at: www.isca.in, www.isca.me
Received 5th January 2016, revised 9th August 2016, accepted 11th August 2016

Abstract

ABS method has been broadly used to solve linear and nonlinear system of equations containing a good number of variables and constraints. In this paper, ABS method is being applied to solve linear programming problems with fuzzy variables provided that degeneracy has been treated correctly. This can be verified by graphical method and simplex method of linear programming problems.

Keywords: ABS algorithm, Linear programming Problems and fuzzy variables.
AMS Subject Classification: 65F30

Introduction

Abaffy, Broyden and Spedicato introduced ABS algorithms to find the solution of determined or undetermined linear systems. Later on, this algorithm has been extended to solve linear and non linear equations of different nature.

One of the most important techniques among all the applied operations research techniques is Linear Programming.

The study of linear programming had been done by so many researchers from different point of views for more than a half century. But still it has enough potential with which one can develop new approach in order to best fit the real life problems within the limitations of linear programming.

Fuzzy linear programming problems play an important role in fuzzy modeling which can formulate in actual environment. Tanaka *et al*[1] first proposed the fuzzy linear programming. Zimmerman[2] developed a method to solve fuzzy linear programming problems by using multiobjective linear programming technique.

A method has been proposed by Campos and Verdegay[3] to solve linear programming problems containing fuzzy coefficients in requirement vector components and in matrix both.

Feng *et al*[4] presented how to apply the ABS algorithm to simplex method and the dual simplex method. Feng *et al*[5] also developed a method to solve linear programming problems with fuzzy coefficients in constraints.

Lin[6] discussed a method for solving fuzzy linear programming problems based on the satisfaction degree of the constraints.

Fuzzy linear programming problems containing objective function components as fuzzy numbers were discussed by Zhang *et al*[7]. To solve a system containing linear equations and linear inequalities, an algorithm known as IABS-MPVT algorithm has been developed by Emilio Spedicato *et al*[8].

Hamid Esmaeili *et al* [9] presented that ABS algorithms can be used to solve full rank linear inequalities and linear programming problems where the number of variables is either greater than or equal to number of inequalities.

Fuzzy linear programming problems based on fuzzy relations was proposed by Ramik[10].

Later Fuzzy linear programming problems were solved by Nasseri[11] by the technique of classical linear programming. After that, Fuzzy linear programming problems with fuzzy parameters were solved by Ebrahimnejad and Nessari[12] by using the complementary slackness theorem.

A new primal- dual algorithm for solving linear programming problems with fuzzy variables was proposed by Ebrahimnejad et al[13] by using duality theorems.

Preliminaries

Following concepts will be used throughout this paper which is based on the function principle.

A fuzzy number $\tilde{a}$ is a triangular fuzzy number denoted by (a_1, a_2, a_3), where a_1, a_2 and a_3 are real numbers and its membership function is given below.

$$\mu_{\tilde{a}}(x)=\begin{cases} (x-a_1)/(a_2-a_1) & for \quad a_1\le x\le a_2 \\ (a_3-x)/(a_3-a_2) & for \quad a_2\le x\le a_3 \\ 0 & otherwise \end{cases}$$

Let (a_1, a_2, a_3) and (b_1,b_2,b_3) be two triangular fuzzy numbers. Then

$(a_1, a_2, a_3) \oplus (b_1, b_2, b_3) = (a_1 + b_1, a_2 + b_2, a_3 + b_3)$

$(a_1, a_2, a_3) \ominus (b_1, b_2, b_3) = (a_1 - b_1, a_2 - b_2, a_3 - b_3)$

$\lambda (a_1, a_2, a_3) = (\lambda a_1, \lambda a_2, \lambda a_3)$ for $\lambda \ge 0$

$\lambda (a_1, a_2, a_3) = (\lambda a_3, \lambda a_2, \lambda a_1)$ for $\lambda < 0$

Let $\tilde{A} = (a_1, a_2, a_3)$ be in F (R)
Then

$\tilde{A}$ is called positive if $a_i \ge 0$, for all i = 1 to 3;

$\tilde{A}$ is called integer if $a_i \ge 0$, for all i = 1 to 3 are integers and

$\tilde{A}$ is called symmetric if $a_2 - a_1 = a_3 - a_2$.

If each element of a fuzzy vector $b=(b_i)_{mxn}$ is called nonnegative real fuzzy number ,then it is said to be nonnegative and denoted by $b\ge 0$ i.e ; $b_i\ge 0$, i = 1, 2, ….., m.

Consider the following m × n fuzzy linear system with nonnegative real fuzzy numbers:
$Ax\le b$
Where $A = (a_{ij})$ is a nonnegative crisp matrix and $x = (x_j)$, $b = (b_i)$ nonnegative fuzzy vectors and x_i ,b_i ϵ F(R), for all $1\le j \le n$ and $1 \le i \le m$ where F(R) is the set of all real triangular fuzzy numbers.

ABS Algorithm

Let $x_1 \in R^n$ be an arbitrary vector. Let $H_1 \in R^{n,n}$ .. be an arbitrary nonsingular matrix. (1)

Cycle for $i = 1, \ldots, n$ (2)

Let $z_i \in R^n$ .. be a vector arbitrary save for the condition :

$$z_i^T H_i a_i \ne 0 \tag{3}$$

Compute search vector p_i :

$$p_i = H_i^T z_i \tag{4}$$

Compute step size α_i :

$$\alpha_i = \frac{a_i^T x_i - b_i}{p_i^T a_i} \tag{5}$$

Which is well defined with regard to (3) to (4)

Compute the new approximation of the solution using

$$x_{i+1} = x_i - \alpha_i p_i \tag{6}$$

If $i = n$ stop; x_{n+1} solve the system (1).

Let $w_i \in R^n$ be a vector arbitrary save for the condition:

$$w_i^T H_i a_i = 1, \tag{7}$$

and to update the matrix H_i :

$$H_{i+1} = H_i - H_i a_i w_i^T H_i \tag{8}$$

There are three eligible parameters Matrix H_1 and two systems f vectors z_i and w_i in the general version of the ABS algorithm. By a suitable choice of these three parameters the new algorithms or a new formulation of the classical algorithms can be created.

Numerical Example

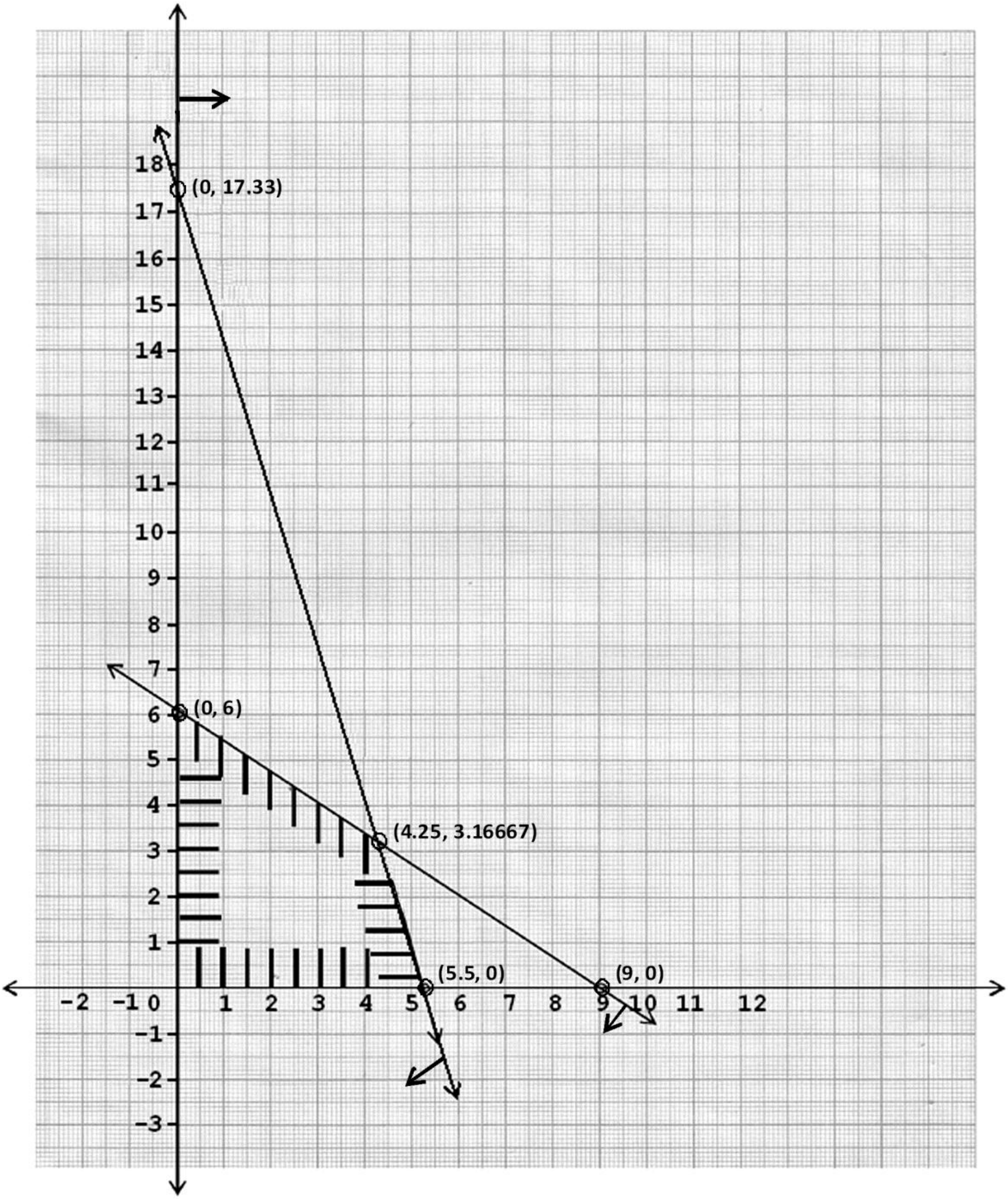
Consider the following linear programming problem with fuzzy variables.

$$\begin{aligned} &Max \quad \tilde{z} = 5\tilde{x}_1 \oplus 6\tilde{x}_2 \\ &Subject \quad to \quad 10\tilde{x}_1 \oplus 3\tilde{x}_2 = (48,52,48) \\ &\qquad 2\tilde{x}_1 \oplus 3\tilde{x}_2 = (12,18,12) \\ &\qquad x_1, x_2 \ge 0 \end{aligned}$$

The above problem then reduces to
Max $z = 5x_1 + 6x_2$

Subject to $10x_1 + 3x_2 \le 52$
$2x_1 + 3x_2 \le 18$
$x_1 \ge 0$
$x_2 \ge 0$

Solution by Graphical Method



Feasible solutions are
x1=0, x2=6, z =36
x1=5.2, x2=0, z =26
x1=4.25, x2=3.166667, z =40.25
As z is maximum at x1 =4.25, x2=3.166667, so it's optimal solution.

Solution by Simple Method

Max $z_2 = 5x_1 + 6x_2$
subject to $10x_1 + 3x_2 \leq 52$
$2x_1 + 3x_2 \leq 18$
$x_1 \geq 0$
$x_2 \geq 0$

Now, using dual simplex method
The solution of the problem (P_2) is $x_1 = 17/4$, $x_2 = 19/6$,
Max $z_2 = 161/4$.

Solution by ABS Method

$$A = \begin{bmatrix} 10 & 3 \\ 2 & 3 \end{bmatrix} \quad B = \begin{bmatrix} 52 \\ 18 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 10 & 2 \\ 3 & 3 \end{bmatrix} \quad H = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$z_1^T H_1 a_1 = [10 \quad 3] \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 10 \\ 3 \end{bmatrix}$$

$$= [10 \quad 3] \begin{bmatrix} 10 \\ 3 \end{bmatrix} = 100 + 9 = 109$$

$$P_1 = H_1^T Z_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 10 \\ 3 \end{bmatrix} = \begin{bmatrix} 10 \\ 3 \end{bmatrix}$$

$$= x_2 = x_1 - \left[\frac{a_1^T - b_1}{a_1^T P_1} \right] P_1$$

$$= \begin{bmatrix} 0 \\ 0 \end{bmatrix} - \left[\frac{[10 \quad 3]\begin{bmatrix} 0 \\ 0 \end{bmatrix} - 52}{[10 \quad 3]\begin{bmatrix} 10 \\ 3 \end{bmatrix}} \right] \begin{bmatrix} 10 \\ 3 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 0 \end{bmatrix} - \left[\frac{0 - 52}{100 + 9} \right] \begin{bmatrix} 10 \\ 3 \end{bmatrix} = \begin{bmatrix} 520/109 \\ 156/109 \end{bmatrix}$$

$$x_2 = \begin{bmatrix} 520/109 \\ 156/109 \end{bmatrix}$$

$$H_2 = H_1 - \frac{P_1 P_1^T}{P_1^T P_1}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \frac{\begin{bmatrix} 10 \\ 3 \end{bmatrix} [10 \quad 3]}{[10 \quad 3] \begin{bmatrix} 10 \\ 3 \end{bmatrix}}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \frac{\begin{bmatrix} 100 & 30 \\ 30 & 9 \end{bmatrix}}{100 + 9}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 100/109 & 30/109 \\ 30/109 & 9/109 \end{bmatrix}$$

$$= \begin{bmatrix} 9/109 & -30/109 \\ -30/109 & 100/109 \end{bmatrix}$$

$$P_2 = H_2^T Z_2 = \begin{bmatrix} 9/109 & -30/109 \\ -30/109 & 100/109 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{18 - 90}{109} \\ \frac{-60 + 300}{109} \end{bmatrix} = \begin{bmatrix} -72/109 \\ 240/109 \end{bmatrix}$$

$$x_3 = x_2 - \left[\frac{a_2^T x_2 - b_2}{a_2^T P_2}\right] P_2$$

$$= \begin{bmatrix} 520/109 \\ 156/109 \end{bmatrix} - \left[\frac{\begin{bmatrix} 2 & 3 \end{bmatrix}\begin{bmatrix} 520/109 \\ 156/109 \end{bmatrix} - 18}{\begin{bmatrix} 2 & 3 \end{bmatrix}\begin{bmatrix} -72/109 \\ 240/109 \end{bmatrix}}\right] \begin{bmatrix} -72/109 \\ 240/109 \end{bmatrix}$$

$$= \begin{bmatrix} 520/109 \\ 156/109 \end{bmatrix} - \left[\frac{\frac{1040+468}{109}}{\frac{-144+720}{109}}\right] \begin{bmatrix} -72/109 \\ 240/109 \end{bmatrix}$$

$$= \begin{bmatrix} 520/109 \\ 156/109 \end{bmatrix} + \left(\frac{1508-1961}{576}\right) \begin{bmatrix} -72/109 \\ 240/109 \end{bmatrix}$$

$$= \begin{bmatrix} 520/109 \\ 156/109 \end{bmatrix} + \left(\frac{454}{576}\right) \begin{bmatrix} -72/109 \\ 240/109 \end{bmatrix}$$

$$= \begin{bmatrix} 520/109 \\ 156/109 \end{bmatrix} + 0 \cdot 788194 \begin{bmatrix} -72/109 \\ 240/109 \end{bmatrix}$$

$$= \begin{bmatrix} 520/109 \\ 156/109 \end{bmatrix} + \begin{bmatrix} -56 \cdot 749968/109 \\ 189 \cdot 16656/109 \end{bmatrix}$$

$$= \begin{bmatrix} 463.250032/109 \\ 345.16656/109 \end{bmatrix} = \begin{bmatrix} 4.250000 \\ 3.166667 \end{bmatrix}$$

$$= \begin{bmatrix} x_1 = 4 \cdot 250000 \\ x_2 = 3 \cdot 166667 \end{bmatrix}$$

Conclusion

We have experimented the ABS method for Fuzzy Linear Programming using an extensive numerical example and the result is verified using traditional graphical and simplex methods.

If the degeneracy has been treated properly, then the feasible solution becomes optimal after n iteration given by the ABS method where n is the rank of the matrix.

Acknowledgement

The authors express heartiest thanks to Dr. Sanjay Jain for valuable suggestions while working.

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