Research Journal of Mathematical and Statistical Sciences ISSN 2320–6047
Vol. **3(7),** 1-8, July **(2015)** *Res. J. Mathematical and Statistical Sci.*

Reliability Evaluation of Engineering System Using Modified Weibull Distribution

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Available online at: www.isca.in, www.isca.me
Received 2nd April 2015, revised 23rd June 2015, accepted 6th July 2015

Abstract

Growing concern about the increasing number of disasters, safety is the main criterion to design every system. In this paper a complex bridge system is considered which includes five independent components whose longevity follows modified Weibull distribution function. The main aim of this paper is to enhance the system reliability by reduction method, warm duplication method and cold duplication method. In each method four sets of components are considered for improvement and their reliability functions reformulated. The three methods are compared with a numerical example.

Keywords: Modified Weibull distribution, reduction method, warm duplication method, cold duplication method, Reliability function.

Introduction

In modern society safety has become a key concept to design every system. In real world applications system reliability plays important role as system reliability is directly proportional to system safety, therefore, it has become necessary to work on system safety and hence on reliability theory. In literature various kinds of systems have been studied and the reliabilities of the systems have been improved by various methods. For instance, Misra[1] used sequential simplex search, Beraha and Misra[2] used random search algorithm to solve reliability optimization problem. Kusum Deep and Dipti[3] used self organizing migrating genetic algorithm to optimize the reliability as well as cost of the three complex systems. Mohan and Shanker[4] using their Random Search Technique to solved a complex bridge network problem of reliability optimization. Sarah[5] studied reliability equivalence from simple parallel and series systems to some complex systems and consider a radar system, which consists of three non-identical and independent components, in an aircraft. Ghasem Ezzati and Abbas Rasouli[6] investigated system reliability using three different methods and assumed that the components involved in the system are independent and their longevity follows linear-exponential distribution function. Ezzati et. al.[7] proposed a new method based on the conjugate gradient method called "Conjugate Gradient Analysis Method", to apply in the reliability analysis problems. To improve system reliability various distribution functions viz; Generalized Linear Exponential Distribution[8], Weibull distribution[9], Exponentiated Modified Weibull Extension Distribution[10] and Exponentiated Generalized Linear Exponential Distribution[11] are studied in the literature. Adil H. Khan and T.R. Jan[12,13] estimated the system reliabilities using exponential distributions and finite mixture of Lindley distributions. For three-parameter Weibull distribution Muraleedharan[14] independently derived the characteristic function and deduced the moment generating function from it and satisfies the tests to verify a function to be a characteristic function. Kusum Lata Singh and R.S. Srivastava[15] derived the pdf of Inverse Maxwell distribution, studied its properties and discussed its suitability as a survival model by obtaining its hazard and survival functions.

Reliability Function of Complex Bridge Systems

The function that is particularly used to define the concern, which is most important in today's electronic world for system safety, is reliability function. If $\omega(t)$ is the subset of all fault free items until the time t and let we have system with n items. Then the random variable $\frac{\omega(t)}{n}$ shows the system reliability as n tends to infinity, that is; $R(t) = \lim_{n\to\infty} \frac{\omega(t)}{n}$.

Reliability is often defined as the probability that a system or an item will consistently perform its function under the given condition for a given period of time without failure. Thus, reliability function is defined as

$$R(t) = P(T > t)\ , \quad t > 0$$

$$R(t) = \int_t^\infty f(x)dx$$

where, $f(x)$ is a probability density function.

Reliability engineers often required to work with systems connected in parallel, series and bridged combinations and to calculate their reliabilities. In such combinations, engineers for calculating the reliabilities often apply very convoluted block reliability formulas. If in a system n independent items are in series combinations, then the reliability of the system is equal to the product of reliabilities of the n items and is given as follows

$$R_s(t) = R_1(t).R_2(t) \dots R_n(t)$$
$$R_s(t) = \prod_{i=1}^{n} R_i(t)$$

And, if n independent items are in parallel combinations then the reliability of the system is below
$$R_p(t) = 1 - (1 - R_1(t)).(1 - R_2(t)) \dots (1 - R_n(t))$$
$$R_p(t) = 1 - \prod_{i=1}^{n} (1 - R_i(t))$$

In this paper, we have considered the system in which five independent components are connected in complex bridge configuration shown below in figure-1

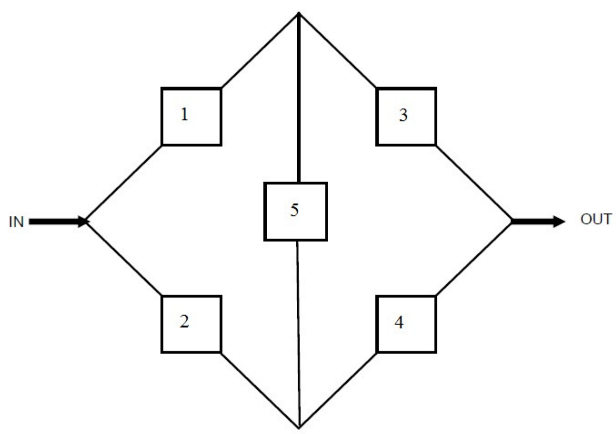

Figure-1
Complex bridge configuration

Each rectangular block in the diagram denotes a component with reliability $R_i, i = 1,2,3,4,5$. The reliability function[3] of the system can be computed by using minimal path or minimal cut sets and can expressed as

$$\begin{aligned} R_b(t) = {} & R_1R_3 + R_2R_4 + R_1R_4R_5 + R_2R_3R_5 - R_1R_3R_4R_5 \\ & - R_1R_2R_4R_5 - R_1R_2R_3R_4 - R_1R_2R_3R_5 \\ & - R_2R_3R_4R_5 + 2R_1R_2R_3R_4R_5 \end{aligned}$$
$$R_b(t) = R_5(R_1 + R_2 - R_1R_2)(R_3 + R_4 - R_3R_4) + (1 - R_5)(R_1R_3 + R_2R_4 - R_1R_2R_3R_4) \quad (1)$$

Minimum path set is defined as the minimum set of components which by functioning ensure the functioning of the structure and the minimal path sets of the above complex bridge structure are $\{1,3\}, \{2,4\}, \{1,4,5\}$ and $\{2,3,5\}$. Also, the minimal cut set is defined as the minimal set of components which by failing guarantee the failure of the structure, the minimal cut sets of the above complex bridge structure are $\{1,2\}, \{3,4\}, \{1,4,5\}$ and $\{2,3,5\}$. The minimal path sets represents the bridge configuration as parallel-series combinations while as minimal cut sets represents as series-parallel combinations. The minimal path sets and minimal cut sets representing complex bridge structure is shown below in figure-2.

Reliability Improvement based on Modified Weibull Distribution: The system reliability can be improved by using the three methods which are often considered for the improvement of the system reliability. The three methods are as follows: i. Reduction Method (RM): In this method the failure parameter of components are reduced by multiplying a factor ρ where $0 < \rho < 1$. ii. Warm Duplication Method (WDM): In this method components selected for the improvement are added directly to themselves in parallel without using any on/off switch for activation. iii. Cold Duplication Method (CDM): In this method components for the improvement are added to themselves in parallel with using on/off switch for activation.

The difference between WDM and CDM is that in CDM spare component is activated manually after the main item is failed but in case of WDM no activation is required. In each improvement method the sets of system which are considered for improvements are $\{1,2\}, \{1,3\}, \{5\}$ and $\{2,3,5\}$.

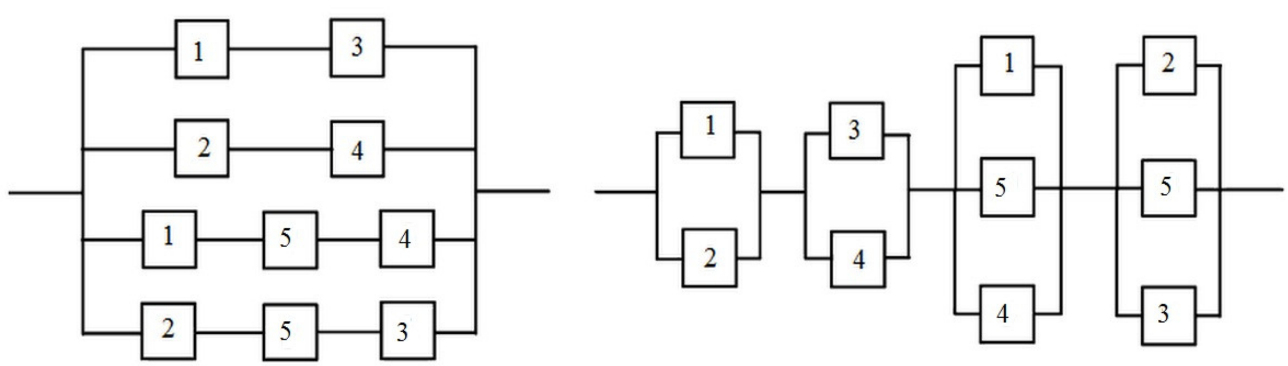

Figure-2
Minimal Path and Minimal Cut representation for complex bridge configuration

Also, it is supposed that the random variables $'t'$ of the complex bridge system follow the Modified Weibull Distribution (MWD) with probability density function

$$f(t) = (\alpha + \beta\mu t^{\mu-1})e^{-(\alpha t+\beta t^{\mu})}\ ,\ \ t > 0, \alpha, \beta, \mu \geq 0 \qquad (2)$$

and the reliability function of MWD is given as

$$R(t) = e^{-(\alpha t+\beta t^{\mu})}\ ,\ \ t > 0, \alpha, \beta, \mu \geq 0$$

where, α, β and μ are constants are known as failure parameters.

Thus based on the equation (1), the reliability function of the considered system assuming that the random variable follow MWD is obtained as follows

$$R_s(t) = e^{-(\alpha_5 t+\beta_5 t^{\mu_5})}\left[e^{-(\alpha_1 t+\beta_1 t^{\mu_1})} + e^{-(\alpha_2 t+\beta_2 t^{\mu_2})} - e^{-((\alpha_1+\alpha_2)x+\beta_1 t^{\mu_1}+\beta_2 t^{\mu_2})}\right] \times \left[e^{-(\alpha_3 t+\beta_3 t^{\mu_3})} + e^{-(\alpha_4 t+\beta_4 t^{\mu_4})} - e^{-((\alpha_3+\alpha_4)t+\beta_3 t^{\mu_3}+\beta_4 t^{\mu_4})}\right] + \left[1 - e^{-(\alpha_5 t+\beta_5 t^{\mu_5})}\right] \left[e^{-((\alpha_1+\alpha_3)t+\beta_1 t^{\mu_1}+\beta_3 t^{\mu_3})} + e^{-((\alpha_2+\alpha_4)t+\beta_2 t^{\mu_2}+\beta_4 t^{\mu_4})} - e^{-((\alpha_1+\alpha_2+\alpha_3+\alpha_4)t+\beta_1 t^{\mu_1}+\beta_2 t^{\mu_2}+\beta_3 t^{\mu_3}+\beta_4 t^{\mu_4})}\right] \qquad (4)$$

where, $\alpha_i, \beta_i,$ and $\mu_i, i = 1, 2, 3, 4, 5$ are the failure parameters corresponding to the component i. When the system is improved the above reliability function can be reformulated.

If a component of a system is improved by using RM, WDM or CDM its reliability function would be $R_i^{\rho}(t), R_i^{W}(t)$ or $R_i^{c}(t)$ respectively and is as below:

$R_i^{\rho}(t) = e^{-\rho(\alpha_i t+\beta_i t^{\mu_i})}$, $0 < \rho < 1$ and is called reduction factor.

$$R_i^{W}(t) = e^{-(\alpha_i t+\beta_i t^{\mu_i})}\left(2 - e^{-(\alpha_i t+\beta_i t^{\mu_i})}\right)$$

$$R_i^{c}(t) = (1 + \alpha_i t + \beta_i t^{\mu_i})e^{-(\alpha_i t+\beta_i t^{\mu_i})}$$

where, $\alpha_i, \beta_i,$ and μ_i are the failure parameters of the selected component, selected for the improvement. Also, it should be noted that numerical methods should be used to find the mean time to failure as means cannot be derived analytically and explicitly when random variable follow MWD.

In the following subsection the three improvement methods of reliability of the system are discussed when different sets of components are considered for improvement. The different sets considered for improvements are $S_1 = \{1,2\}, S_2 = \{1,3\}, S_3 = \{5\}$ and $S_4 = \{2,3,5\}$.

Reduction Method: The reliability function of the improved system using reduction method, when $S_1 = \{1,2\}$ is selected for improvement is given by

$$R_{S_1}^{\rho}(t) = R_5\left(R_1^{\rho} + R_2^{\rho} - R_1^{\rho}R_2^{\rho}\right)(R_3 + R_4 - R_3R_4) + (1 - R_5)\left(R_1^{\rho}R_3 + R_2^{\rho}R_4 - R_1^{\rho}R_2^{\rho}R_3R_4\right)$$

where, R_1^{ρ}, R_2^{ρ} are the reliability functions of the components 1 and 2, obtained after reduction and $R_3,\ R_4,\ R_5$ are the original reliabilities of the components 3, 4 and 5 respectively. Thus,

$$R_{S_1}^{\rho}(t) = e^{-(\alpha_5 t+\beta_5 t^{\mu_5})}\left[e^{-\rho(\alpha_1 t+\beta_1 t^{\mu_1})} + e^{-\rho(\alpha_2 t+\beta_2 t^{\mu_2})} - e^{-\rho((\alpha_1+\alpha_2)t+\beta_1 t^{\mu_1}+\beta_2 t^{\mu_2})}\right] \times \left[e^{-(\alpha_3 t+\beta_3 t^{\mu_3})} + e^{-(\alpha_4 t+\beta_4 t^{\mu_4})} - e^{-((\alpha_3+\alpha_4)t+\beta_3 t^{\mu_3}+\beta_4 t^{\mu_4})}\right] + \left[1 - e^{-(\alpha_5 t+\beta_5 t^{\mu_5})}\right] \times \left[e^{-((\rho\alpha_1+\alpha_3)t+\rho\beta_1 t^{\mu_1}+\beta_3 t^{\mu_3})} + e^{-((\rho\alpha_2+\alpha_4)t+\rho\beta_2 t^{\mu_2}+\beta_4 t^{\mu_4})} - e^{-((\rho\alpha_1+\rho\alpha_2+\alpha_3+\alpha_4)t+\rho\beta_1 t^{\mu_1}+\rho\beta_2 t^{\mu_2}+\beta_3 t^{\mu_3}+\beta_4 t^{\mu_4})}\right] \qquad (5)$$

When $S_2 = \{1,3\}$ is selected for improvement the improved reliability function of the system is given by

$$R_{S_2}^{\rho}(t) = R_5\left(R_1^{\rho} + R_2 - R_1^{\rho}R_2\right)\left(R_3^{\rho} + R_4 - R_3^{\rho}R_4\right) + (1 - R_5)\left(R_1^{\rho}R_3^{\rho} + R_2R_4 - R_1^{\rho}R_2R_3^{\rho}R_4\right)$$

and we have

$$R_{S_2}^{\rho}(t) = e^{-(\alpha_5 t+\beta_5 t^{\mu_5})}\left[e^{-\rho(\alpha_1 t+\beta_1 t^{\mu_1})} + e^{-(\alpha_2 t+\beta_2 t^{\mu_2})} - e^{-((\rho\alpha_1+\alpha_2)t+\rho\beta_1 t^{\mu_1}+\beta_2 t^{\mu_2})}\right] \times \left[e^{-\rho(\alpha_3 t+\beta_3 t^{\mu_3})} + e^{-(\alpha_4 t+\beta_4 t^{\mu_4})} - e^{-((\rho\alpha_3+\alpha_4)t+\rho\beta_3 t^{\mu_3}+\beta_4 t^{\mu_4})}\right] + \left[1 - e^{-(\alpha_5 t+\beta_5 t^{\mu_5})}\right] \left[e^{-\rho((\alpha_1+\alpha_3)t+\beta_1 t^{\mu_1}+\beta_3 t^{\mu_3})} + e^{-((\alpha_2+\alpha_4)t+\beta_2 t^{\mu_2}+\beta_4 t^{\mu_4})} - e^{-((\rho\alpha_1+\alpha_2+\rho\alpha_3+\alpha_4)t+\rho\beta_1 t^{\mu_1}+\beta_2 t^{\mu_2}+\rho\beta_3 t^{\mu_3}+\beta_4 t^{\mu_4})}\right] \qquad (6)$$

When $S_3 = \{5\}$ is selected for improvement the improved reliability function of the system is given by

$$R_{S_3}^{\rho}(t) = R_5^{\rho}(R_1 + R_2 - R_1R_2)(R_3 + R_4 - R_3R_4) + \left(1 - R_5^{\rho}\right)(R_1R_3 + R_2R_4 - R_1R_2R_3R_4)$$

and then, we have

$$R_{S_3}^{\rho}(t) = e^{-\rho(\alpha_5 t+\beta_5 t^{\mu_5})}\left[e^{-(\alpha_1 t+\beta_1 t^{\mu_1})} + e^{-(\alpha_2 t+\beta_2 t^{\mu_2})} - e^{-((\alpha_1+\alpha_2)t+\beta_1 t^{\mu_1}+\beta_2 t^{\mu_2})}\right] \times \left[e^{-(\alpha_3 t+\beta_3 t^{\mu_3})} + e^{-(\alpha_4 t+\beta_4 t^{\mu_4})} - e^{-((\alpha_3+\alpha_4)t+\beta_3 t^{\mu_3}+\beta_4 t^{\mu_4})}\right] + \left[1 - e^{-\rho(\alpha_5 t+\beta_5 t^{\mu_5})}\right] \times \left[e^{-((\alpha_1+\alpha_3)t+\beta_1 t^{\mu_1}+\beta_3 t^{\mu_3})} + e^{-((\alpha_2+\alpha_4)t+\beta_2 t^{\mu_2}+\beta_4 t^{\mu_4})} - e^{-((\alpha_1+\alpha_2+\alpha_3+\alpha_4)t+\beta_1 t^{\mu_1}+\beta_2 t^{\mu_2}+\beta_3 t^{\mu_3}+\beta_4 t^{\mu_4})}\right] \qquad (7)$$

Further when $S_4 = \{2,3,5\}$ is selected for improvement, the improved reliability function of the system is given by

$$R_{S_4}^{\rho}(t) = R_5^{\rho}\left(R_1 + R_2^{\rho} - R_1R_2^{\rho}\right)\left(R_3^{\rho} + R_4 - R_3^{\rho}R_4\right) + \left(1 - R_5^{\rho}\right)\left(R_1R_3^{\rho} + R_2^{\rho}R_4 - R_1R_2^{\rho}R_3^{\rho}R_4\right)$$

and then, we have

$$R_{S_4}^{\rho}(t) = e^{-\rho(\alpha_5 t+\beta_5 t^{\mu_5})}\left[e^{-(\alpha_1 t+\beta_1 t^{\mu_1})} + e^{-\rho(\alpha_2 t+\beta_2 t^{\mu_2})} - e^{-((\alpha_1+\rho\alpha_2)t+\beta_1 t^{\mu_1}+\rho\beta_2 t^{\mu_2})}\right] \times \left[e^{-\rho(\alpha_3 t+\beta_3 t^{\mu_3})} + e^{-(\alpha_4 t+\beta_4 t^{\mu_4})} - e^{-((\rho\alpha_3+\alpha_4)t+\rho\beta_3 t^{\mu_3}+\beta_4 t^{\mu_4})}\right] + \left[1 - e^{-\rho(\alpha_5 t+\beta_5 t^{\mu_5})}\right] \times$$

$$\left[e^{-((\alpha_1+\rho\alpha_3)t+\beta_1 t^{\mu_1}+\rho\beta_3 t^{\mu_3})}+e^{-((\rho\alpha_2+\alpha_4)t+\rho\beta_2 t^{\mu_2}+\beta_4 t^{\mu_4})}-e^{-((\alpha_1+\rho\alpha_2+\rho\alpha_3+\alpha_4)t+\beta_1 t^{\mu_1}+\rho\beta_2 t^{\mu_2}+\rho\beta_3 t^{\mu_3}+\beta_4 t^{\mu_4})}\right] \quad (8)$$

Warm Duplication Method: Under this method when the subsets $S_1=\{1,2\}, S_2=\{1,3\}, S_3=\{5\}$ and $S_4=\{2,3,5\}$ of the complex bridge system are considered for improvements the improved reliability functions are given below:

$$R_{S_1}^W(t)=R_5[R_1(2-R_1)+R_2(2-R_2)-R_1(2-R_1)R_2(2-R_2)][R_3+R_4-R_3R_4]+[1-R_5][R_1(2-R_1)R_3+R_2(2-R_2)R_4-R_1(2-R_1)R_2(2-R_2)R_3R_4]$$

$$R_{S_2}^W(t)=R_5[R_1(2-R_1)+R_2-R_1(2-R_1)R_2][R_3(2-R_3)+R_4-R_3(2-R_3)R_4]+[1-R_5][R_1(2-R_1)R_3(2-R_3)+R_2R_4-R_1(2-R_1)R_2R_3(2-R_3)R_4]$$

$$R_{S_3}^W(t)=R_5(2-R_5)[R_1+R_2-R_1R_2][R_3+R_4-R_3R_4]+[1-R_5(2-R_5)][R_1R_3+R_2R_4-R_1R_2R_3R_4]$$

$$R_{S_4}^W(t)=R_5(2-R_5)[R_1+R_2(2-R_2)-R_1R_2(2-R_2)][R_3(2-R_3)+R_4-R_3(2-R_3)R_4]+[1-R_5(2-R_5)][R_1R_3(2-R_3)+R_2(2-R_2)R_4-R_1R_2(2-R_2)R_3(2-R_3)R_4]$$

Finally, the reliability functions of the improved system can be expressed as

$$R_{S_1}^W(t)=e^{-(\alpha_5 t+\beta_5 t^{\mu_5})}\left[Ae^{-(\alpha_1 t+\beta_1 t^{\mu_1})}+Be^{-(\alpha_2 t+\beta_2 t^{\mu_2})}-ABe^{-((\alpha_1+\alpha_2)t+\beta_1 t^{\mu_1}+\beta_2 t^{\mu_2})}\right]\times\left[e^{-(\alpha_3 t+\beta_3 t^{\mu_3})}+e^{-(\alpha_4 t+\beta_4 t^{\mu_4})}-e^{-((\alpha_3+\alpha_4)t+\beta_3 t^{\mu_3}+\beta_4 t^{\mu_4})}\right]+\left[-e^{-(\alpha_5 t+\beta_5 t^{\mu_5})}\right]\times\left[Ae^{-((\alpha_1+\alpha_3)t+\beta_1 t^{\mu_1}+\beta_3 t^{\mu_3})}+Be^{-((\alpha_2+\alpha_4)t+\beta_2 t^{\mu_2}+\beta_4 t^{\mu_4})}-ABe^{-((\alpha_1+\alpha_2+\alpha_3+\alpha_4)t+\beta_1 t^{\mu_1}+\beta_2 t^{\mu_2}+\beta_3 t^{\mu_3}+\beta_4 t^{\mu_4})}\right] \quad (9)$$

$$R_{S_2}^W(t)=e^{-(\alpha_5 t+\beta_5 t^{\mu_5})}\left[Ae^{-(\alpha_1 t+\beta_1 t^{\mu_1})}+e^{-(\alpha_2 t+\beta_2 t^{\mu_2})}-Ae^{-((\alpha_1+\alpha_2)t+\beta_1 t^{\mu_1}+\beta_2 t^{\mu_2})}\right]\times\left[Ce^{-(\alpha_3 t+\beta_3 t^{\mu_3})}+e^{-(\alpha_4 t+\beta_4 t^{\mu_4})}-Ce^{-((\alpha_3+\alpha_4)t+\beta_3 t^{\mu_3}+\beta_4 t^{\mu_4})}\right]+\left[1-e^{-(\alpha_5 t+\beta_5 t^{\mu_5})}\right]\times\left[ACe^{-((\alpha_1+\alpha_3)t+\beta_1 t^{\mu_1}+\beta_3 t^{\mu_3})}+e^{-((\alpha_2+\alpha_4)t+\beta_2 t^{\mu_2}+\beta_4 t^{\mu_4})}-ACe^{-((\alpha_1+\alpha_2+\alpha_3+\alpha_4)t+\beta_1 t^{\mu_1}+\beta_2 t^{\mu_2}+\beta_3 t^{\mu_3}+\beta_4 t^{\mu_4})}\right] \quad (10)$$

$$R_{S_3}^W(t)=Ee^{-(\alpha_5 t+\beta_5 t^{\mu_5})}\left[e^{-(\alpha_1 t+\beta_1 t^{\mu_1})}+e^{-(\alpha_2 t+\beta_2 t^{\mu_2})}-e^{-((\alpha_1+\alpha_2)t+\beta_1 t^{\mu_1}+\beta_2 t^{\mu_2})}\right]\times\left[e^{-(\alpha_3 t+\beta_3 t^{\mu_3})}+e^{-(\alpha_4 t+\beta_4 t^{\mu_4})}-e^{-((\alpha_3+\alpha_4)t+\beta_3 t^{\mu_3}+\beta_4 t^{\mu_4})}\right]+\left[1-Ee^{-(\alpha_5 t+\beta_5 t^{\mu_5})}\right]\times\left[e^{-((\alpha_1+\alpha_3)t+\beta_1 t^{\mu_1}+\beta_3 t^{\mu_3})}+e^{-((\alpha_2+\alpha_4)t+\beta_2 t^{\mu_2}+\beta_4 t^{\mu_4})}-e^{-((\alpha_1+\alpha_2+\alpha_3+\alpha_4)t+\beta_1 t^{\mu_1}+\beta_2 t^{\mu_2}+\beta_3 t^{\mu_3}+\beta_4 t^{\mu_4})}\right] \quad (11)$$

$$R_{S_4}^W(t)=Ee^{-(\alpha_5 t+\beta_5 t^{\mu_5})}\left[e^{-(\alpha_1 t+\beta_1 t^{\mu_1})}+Be^{-(\alpha_2 t+\beta_2 t^{\mu_2})}-Be^{-((\alpha_1+\alpha_2)t+\beta_1 t^{\mu_1}+\beta_2 t^{\mu_2})}\right]\times\left[Ce^{-(\alpha_3 t+\beta_3 t^{\mu_3})}+e^{-(\alpha_4 t+\beta_4 t^{\mu_4})}-Ce^{-((\alpha_3+\alpha_4)t+\beta_3 t^{\mu_3}+\beta_4 t^{\mu_4})}\right]+\left[1-Ee^{-(\alpha_5 t+\beta_5 t^{\mu_5})}\right]\times\left[Ce^{-((\alpha_1+\alpha_3)t+\beta_1 t^{\mu_1}+\beta_3 t^{\mu_3})}+Be^{-((\alpha_2+\alpha_4)t+\beta_2 t^{\mu_2}+\beta_4 t^{\mu_4})}-BCe^{-((\alpha_1+\alpha_2+\alpha_3+\alpha_4)t+\beta_1 t^{\mu_1}+\beta_2 t^{\mu_2}+\beta_3 t^{\mu_3}+\beta_4 t^{\mu_4})}\right] \quad (12)$$

where, $A=\left(2-e^{-(\alpha_1 t+\beta_1 t^{\mu_1})}\right)$, $B=\left(2-e^{-(\alpha_2 t+\beta_2 t^{\mu_2})}\right)$, $C=\left(2-e^{-(\alpha_3 t+\beta_3 t^{\mu_3})}\right)$, $D=\left(2-e^{-(\alpha_4 t+\beta_4 t^{\mu_4})}\right)$, $E=\left(2-e^{-(\alpha_5 t+\beta_5 t^{\mu_5})}\right)$

Cold Duplication Method: Using this method when the subsets $S_1=\{1,2\}, S_2=\{1,3\}, S_3=\{5\}$ and $S_4=\{2,3,5\}$ of the complex bridge system are considered for improvements the improved reliability functions are given below:

$$R_{S_1}^C(t)=R_5[A'R_1+B'R_2-A'B'R_1R_2][R_3+R_4-R_3R_4]+[1-R_5][A'R_1R_3+B'R_2R_4-A'B'R_1R_2R_3R_4]$$

$$R_{S_2}^C(t)=R_5[A'R_1+R_2-A'R_1R_2][C'R_3+R_4-C'R_3R_4]+[1-R_5][A'C'R_1R_3+R_2R_4-A'C'R_1R_2R_3R_4]$$

$$R_{S_3}^C(t)=E'R_5[R_1+R_2-R_1R_2][R_3+R_4-R_3R_4]+[1-E'R_5][R_1R_3+R_2R_4-R_1R_2R_3R_4]$$

$$R_{S_4}^C(t)=E'R_5[R_1+B'R_2-B'R_1R_2][C'R_3+R_4-C'R_3R_4]+[1-E'R_5][C'R_1R_3+B'R_2R_4-B'C'R_1R_2R_3R_4]$$

Finally, the reliability functions of the improved system can be expressed as

$$R_{S_1}^C(t)=e^{-(\alpha_5 t+\beta_5 t^{\mu_5})}\left[A'e^{-(\alpha_1 t+\beta_1 t^{\mu_1})}+B'e^{-(\alpha_2 t+\beta_2 t^{\mu_2})}-A'B'e^{-((\alpha_1+\alpha_2)t+\beta_1 t^{\mu_1}+\beta_2 t^{\mu_2})}\right]\times\left[e^{-(\alpha_3 t+\beta_3 t^{\mu_3})}+e^{-(\alpha_4 t+\beta_4 t^{\mu_4})}-e^{-((\alpha_3+\alpha_4)t+\beta_3 t^{\mu_3}+\beta_4 t^{\mu_4})}\right]+\left[1-e^{-(\alpha_5 t+\beta_5 t^{\mu_5})}\right]\times\left[A'e^{-((\alpha_1+\alpha_3)t+\beta_1 t^{\mu_1}+\beta_3 t^{\mu_3})}+B'e^{-((\alpha_2+\alpha_4)t+\beta_2 t^{\mu_2}+\beta_4 t^{\mu_4})}-A'B'e^{-((\alpha_1+\alpha_2+\alpha_3+\alpha_4)t+\beta_1 t^{\mu_1}+\beta_2 t^{\mu_2}+\beta_3 t^{\mu_3}+\beta_4 t^{\mu_4})}\right] \quad (13)$$

$$R_{S_2}^C(t)=e^{-(\alpha_5 t+\beta_5 t^{\mu_5})}\left[A'e^{-(\alpha_1 t+\beta_1 t^{\mu_1})}+e^{-(\alpha_2 t+\beta_2 t^{\mu_2})}-A'e^{-((\alpha_1+\alpha_2)t+\beta_1 t^{\mu_1}+\beta_2 t^{\mu_2})}\right]\times\left[C'e^{-(\alpha_3 t+\beta_3 t^{\mu_3})}+e^{-(\alpha_4 t+\beta_4 t^{\mu_4})}-C'e^{-((\alpha_3+\alpha_4)t+\beta_3 t^{\mu_3}+\beta_4 t^{\mu_4})}\right]+\left[1-e^{-(\alpha_5 t+\beta_5 t^{\mu_5})}\right]\times\left[A'C'e^{-((\alpha_1+\alpha_3)t+\beta_1 t^{\mu_1}+\beta_3 t^{\mu_3})}+e^{-((\alpha_2+\alpha_4)t+\beta_2 t^{\mu_2}+\beta_4 t^{\mu_4})}-A'C'e^{-((\alpha_1+\alpha_2+\alpha_3+\alpha_4)t+\beta_1 t^{\mu_1}+\beta_2 t^{\mu_2}+\beta_3 t^{\mu_3}+\beta_4 t^{\mu_4})}\right] \quad (14)$$

$$R^C_{S_3}(t) = E'e^{-(\alpha_5 t+\beta_5 t^{\mu_5})}\left[e^{-(\alpha_1 t+\beta_1 t^{\mu_1})} + e^{-(\alpha_2 t+\beta_2 t^{\mu_2})} - e^{-((\alpha_1+\alpha_2)t+\beta_1 t^{\mu_1}+\beta_2 t^{\mu_2})}\right] \times \left[e^{-(\alpha_3 t+\beta_3 t^{\mu_3})} + e^{-(\alpha_4 t+\beta_4 t^{\mu_4})} - e^{-((\alpha_3+\alpha_4)t+\beta_3 t^{\mu_3}+\beta_4 t^{\mu_4})}\right] + \left[1 - E'e^{-(\alpha_5 t+\beta_5 t^{\mu_5})}\right] \times \left[e^{-((\alpha_1+\alpha_3)t+\beta_1 t^{\mu_1}+\beta_3 t^{\mu_3})} + e^{-((\alpha_2+\alpha_4)t+\beta_2 t^{\mu_2}+\beta_4 t^{\mu_4})} - e^{-((\alpha_1+\alpha_2+\alpha_3+\alpha_4)t+\beta_1 t^{\mu_1}+\beta_2 t^{\mu_2}+\beta_3 t^{\mu_3}+\beta_4 t^{\mu_4})}\right] \quad (15)$$

$$R^C_{S_4}(t) = E'e^{-(\alpha_5 t+\beta_5 t^{\mu_5})}\left[e^{-(\alpha_1 t+\beta_1 t^{\mu_1})} + B'e^{-(\alpha_2 t+\beta_2 t^{\mu_2})} - B'e^{-((\alpha_1+\alpha_2)t+\beta_1 t^{\mu_1}+\beta_2 t^{\mu_2})}\right] \times \left[C'e^{-(\alpha_3 t+\beta_3 t^{\mu_3})} + e^{-(\alpha_4 t+\beta_4 t^{\mu_4})} - C'e^{-((\alpha_3+\alpha_4)t+\beta_3 t^{\mu_3}+\beta_4 t^{\mu_4})}\right] + \left[1 - E'e^{-(\alpha_5 t+\beta_5 t^{\mu_5})}\right] \times \left[C'e^{-((\alpha_1+\alpha_3)t+\beta_1 t^{\mu_1}+\beta_3 t^{\mu_3})} + B'e^{-((\alpha_2+\alpha_4)t+\beta_2 t^{\mu_2}+\beta_4 t^{\mu_4})} - B'C'e^{-((\alpha_1+\alpha_2+\alpha_3+\alpha_4)t+\beta_1 t^{\mu_1}+\beta_2 t^{\mu_2}+\beta_3 t^{\mu_3}+\beta_4 t^{\mu_4})}\right] \quad (16)$$

where,

$$A' = (1 + \alpha_1 t + \beta_1 t^{\mu_1}), \quad B' = (1 + \alpha_2 t + \beta_2 t^{\mu_2}),$$
$$C' = (1 + \alpha_3 t + \beta_3 t^{\mu_3}),$$
$$D' = (1 + \alpha_4 t + \beta_4 t^{\mu_4}),$$
$$E' = (1 + \alpha_5 t + \beta_5 t^{\mu_5})$$

Data Analysis

In this section for some specific values of the failure parameters we have plotted the graphs of the reliability functions of original system and improved system. Firstly, we have compared reliability functions of the original and improved systems where the improvement method is reduction method. After that we have compared reliabilities of improved system for different reduction factors. And at last we have compared reduction method, warm duplication method and cold duplication method for specific values of failure parameters and reduction factor.

Reduction Method for Different Sets: The specific values of failure parameters are given below

Table-1

i	1	2	3	4	5
α_i	0.5	1.8	0.85	1.6	0.75
β_i	1.2	1.4	0.9	1.1	1.3
μ_i	1.1	2.2	1.9	0.95	2.3

Also, the reduction factor (ρ) is supposed to be 0.6

Thus, from the equation (4) the reliability function of the original system would be

$$R_s(t) = e^{-(0.75x+1.3x^{2.3})}\left[e^{-(0.5x+1.2x^{1.1})} + e^{-(1.8x+1.4x^{2.2})} - e^{-(2.3x+1.2x^{1.1}+1.4x^{2.2})}\right] \times \left[e^{-(0.85x+0.9x^{1.9})} + e^{-(1.6x+1.1x^{0.95})} - e^{-(2.45x+0.9x^{1.9}+1.1x^{0.95})}\right] + \left[1 - e^{-(0.75x+1.3x^{2.3})}\right] \times \left[e^{-(1.35x+1.2x^{1.1}+0.9x^{1.9})} + e^{-(3.4x+1.4x^{2.2}+1.1x^{0.95})} - e^{-(4.75x+1.2x^{1.1}+1.4x^{2.2}+0.9x^{1.9}+1.1x^{0.95})}\right]$$

Also, from equations (5), (6), (7) and (8) the reliability functions of the improved systems when different sets ($S_1 = \{1,2\}, S_2 = \{1,3\}, S_3 = \{5\}$ and $S_4 = \{2,3,5\}$) are considered for improvement would be, respectively:

$$R^\rho_{S_1}(t) = e^{-(0.75x+1.3x^{2.3})}\left[e^{-(0.3x+0.72x^{1.1})} + e^{-(1.08x+0.84x^{2.2})} - e^{-(1.38x+0.72x^{1.1}+0.84x^{2.2})}\right] \times \left[e^{-(0.85x+0.9x^{1.9})} + e^{-(1.6x+1.1x^{0.95})} - e^{-(2.45x+0.9x^{1.9}+1.1x^{0.95})}\right] + \left[1 - e^{-(0.75x+1.3x^{2.3})}\right] \times \left[e^{-(1.15x+0.72x^{1.1}+0.9x^{1.9})} + e^{-(2.68x+0.84x^{2.2}+1.1x^{0.95})} - e^{-(3.83x+0.72x^{1.1}+0.84x^{2.2}+0.9x^{1.9}+1.1x^{0.95})}\right]$$

$$R^\rho_{S_2}(t) = e^{-(0.75x+1.3x^{2.3})}\left[e^{-(0.3x+0.72x^{1.1})} + e^{-(1.8x+1.4x^{2.2})} - e^{-(2.1x+0.72x^{1.1}+1.4x^{2.2})}\right] \times \left[e^{-(0.51x+0.54x^{1.9})} + e^{-(1.6x+1.1x^{0.95})} - e^{-(2.11x+0.54x^{1.9}+1.1x^{0.95})}\right] + \left[1 - e^{-(0.75x+1.3x^{2.3})}\right] \times \left[e^{-(0.81x+0.72x^{1.1}+0.54x^{1.9})} + e^{-(3.4x+1.4x^{2.2}+1.1x^{0.95})} - e^{-(4.21x+0.72x^{1.1}+1.4x^{2.2}+0.54x^{1.9}+1.1x^{0.95})}\right]$$

$$R^\rho_{S_3}(t) = e^{-(0.45x+0.78x^{2.3})}\left[e^{-(0.5x+1.2x^{1.1})} + e^{-(1.8x+1.4x^{2.2})} - e^{-(2.3x+1.2x^{1.1}+1.4x^{2.2})}\right] \times \left[e^{-(0.85x+0.9x^{1.9})} + e^{-(1.6x+1.1x^{0.95})} - e^{-(2.45x+0.9x^{1.9}+1.1x^{0.95})}\right] + \left[1 - e^{-(0.45x+0.78x^{2.3})}\right] \times \left[e^{-(1.35x+1.2x^{1.1}+0.9x^{1.9})} + e^{-(3.4x+1.4x^{2.2}+1.1x^{0.95})} - e^{-(4.75x+1.2x^{1.1}+1.4x^{2.2}+0.9x^{1.9}+1.1x^{0.95})}\right]$$

$$R^\rho_{S_4}(t) = e^{-(0.45x+0.78x^{2.3})}\left[e^{-(0.5x+1.2x^{1.1})} + e^{-(1.08x+0.84x^{2.2})} - e^{-(1.58x+1.2x^{1.1}+0.84x^{2.2})}\right] \times \left[e^{-(0.51x+0.54x^{1.9})} + e^{-(1.6x+1.1x^{0.95})} - e^{-(2.11x+0.54x^{1.9}+1.1x^{0.95})}\right] + \left[1 - e^{-(0.45x+0.78x^{2.3})}\right] \times \left[e^{-(1.01x+1.2x^{1.1}+0.54x^{1.9})} + e^{-(2.68x+0.84x^{2.2}+1.1x^{0.95})} - e^{-(3.69x+1.2x^{1.1}+0.84x^{2.2}+0.54x^{1.9}+1.1x^{0.95})}\right]$$

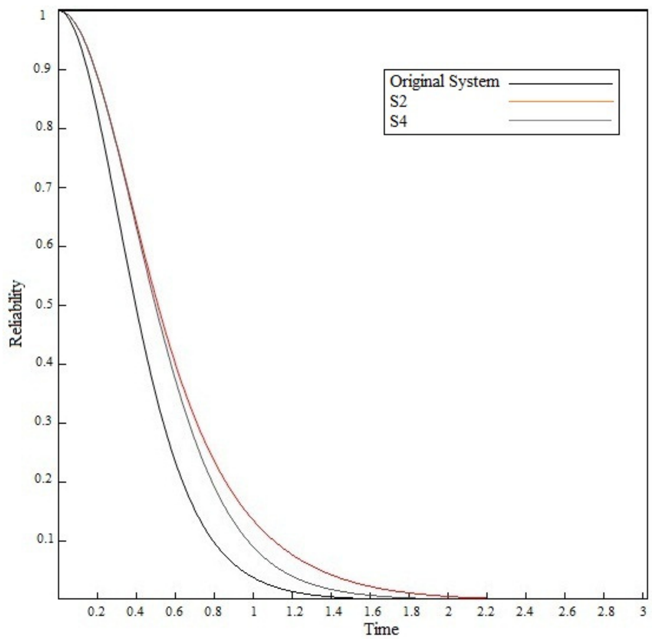

Figure-3
Reliability Functions

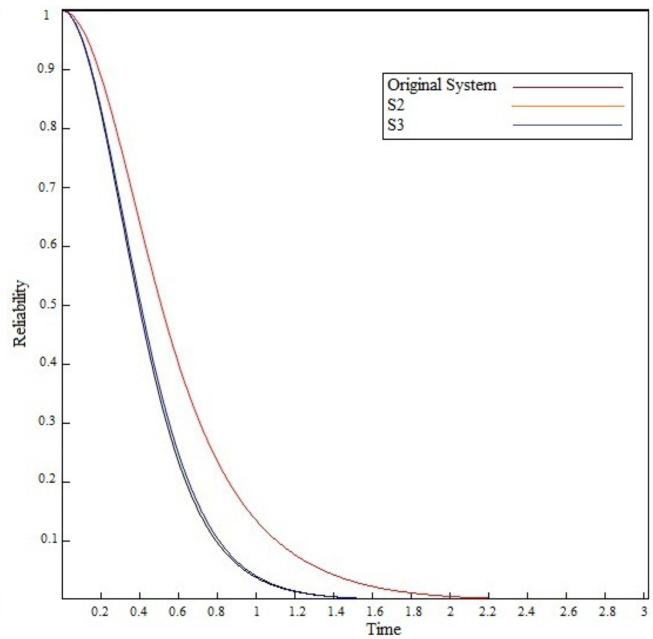

Figure-4
Reliability Functions

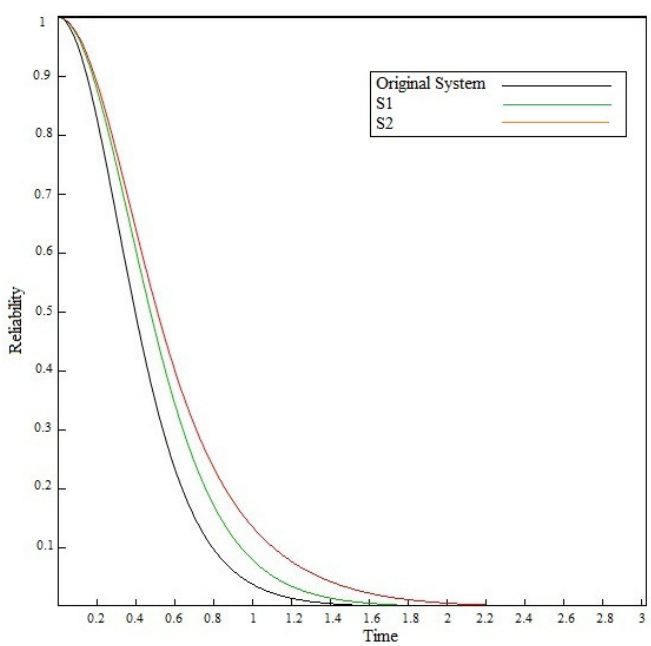

Figure-5
Reliability Functions

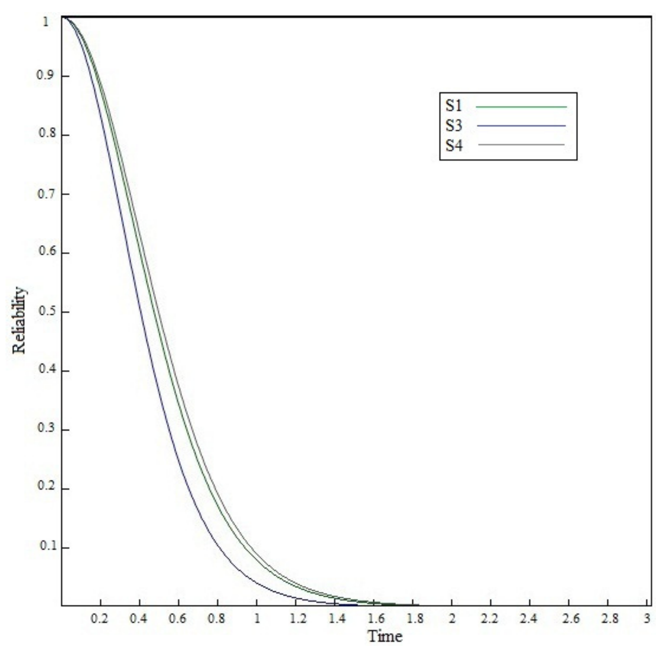

Figure-6
Reliability Functions

It is clear from the above figures of the reliability functions; reduction method of improving system reliability in our problem has a consistent trend to improve the system reliability. All improved systems have higher reliability than the original system and the when the system is improved by reducing $S_2 = \{1,3\}$ set of components, reliability of system reaches maximum and then S_4, S_1, S_3 are in the next positions respectively.

Reduction Method with Different Reduction Factors and Comparison between Different Improvement Methods: In this subsection, the components of the system which are considered for the improvement is $S_2 = \{1,3\}$ set, and if the components are improved by the reduction method with different reducing factors $\rho_1 = 0.3$, $\rho_2 = 0.6$, $\rho_3 = 0.9$,t hen from equation (6) reliability functions of the improved systems for different reducing factors are given as follows:

$$\begin{aligned} R_{S_2}^{\rho_1}(t) = {} & e^{-(0.75x+1.3x^{2.3})}\big[e^{-(0.15x+0.36x^{1.1})} + e^{-(1.8x+1.4x^{2.2})} - e^{-(1.95x+0.36x^{1.1}+1.4x^{2.2})}\big] \\ & \times \big[e^{-(0.255x+0.27x^{1.9})} + e^{-(1.6x+1.1x^{0.95})} - e^{-(1.855x+0.27x^{1.9}+1.1x^{0.95})}\big] + \big[1 - e^{-(0.75x+1.3x^{2.3})}\big] \\ & \times \big[e^{-(0.405x+0.36x^{1.1}+0.27x^{1.9})} + e^{-(3.4x+1.4x^{2.2}+1.1x^{0.95})} - e^{-(3.805x+0.36x^{1.1}+1.4x^{2.2}+0.27x^{1.9}+1.1x^{0.95})}\big] \end{aligned}$$

$$\begin{aligned} R_{S_2}^{\rho_2}(t) = {} & e^{-(0.75x+1.3x^{2.3})}\big[e^{-(0.3x+0.72x^{1.1})} + e^{-(1.8x+1.4x^{2.2})} - e^{-(2.1x+0.72x^{1.1}+1.4x^{2.2})}\big] \\ & \times \big[e^{-(0.51x+0.54x^{1.9})} + e^{-(1.6x+1.1x^{0.95})} - e^{-(2.11x+0.54x^{1.9}+1.1x^{0.95})}\big] + \big[1 - e^{-(0.75x+1.3x^{2.3})}\big] \\ & \times \big[e^{-(0.81x+0.72x^{1.1}+0.54x^{1.9})} + e^{-(3.4x+1.4x^{2.2}+1.1x^{0.95})} - e^{-(4.21x+0.72x^{1.1}+1.4x^{2.2}+0.54x^{1.9}+1.1x^{0.95})}\big] \end{aligned}$$

$$\begin{aligned} R_{S_2}^{\rho_3}(t) = {} & e^{-(0.75x+1.3x^{2.3})}\big[e^{-(0.45x+1.08x^{1.1})} + e^{-(1.8x+1.4x^{2.2})} - e^{-(2.25x+1.08x^{1.1}+1.4x^{2.2})}\big] \\ & \times \big[e^{-(0.765x+0.81x^{1.9})} + e^{-(1.6x+1.1x^{0.95})} - e^{-(2.365x+0.81x^{1.9}+1.1x^{0.95})}\big] + \big[1 - e^{-(0.75x+1.3x^{2.3})}\big] \\ & \times \big[e^{-(1.215x+1.08x^{1.1}+0.81x^{1.9})} + e^{-(3.4x+1.4x^{2.2}+1.1x^{0.95})} - e^{-(4.615x+1.08x^{1.1}+1.4x^{2.2}+0.81x^{1.9}+1.1x^{0.95})}\big] \end{aligned}$$

Also, using values of failure parameters given in table-1 and from equations (10) and (14), the reliability functions of improved system when the system is improved by the warm duplication method and cold duplication method are as follows:

$$\begin{aligned} R_{S_2}^{W}(t) = {} & e^{-(0.75x+1.3x^{2.3})}\big[\big(2 - e^{-(0.5x+1.2x^{1.1})}\big) * e^{-(0.5x+1.2x^{1.1})} + e^{-(1.8x+1.4x^{2.2})} \\ & - \big(2 - e^{-(0.5x+1.2x^{1.1})}\big)e^{-(2.3x+1.2x^{1.1}+1.4x^{2.2})}\big]\big[\big(2 - e^{-(0.85x+0.9x^{1.9})}\big)e^{-(0.85x+0.9x^{1.9})} \\ & + e^{-(1.6x+1.1x^{0.95})} - \big(2 - e^{-(0.85x+0.9x^{1.9})}\big)e^{-(2.45x+0.9x^{1.9}+1.1x^{0.95})}\big] + \big[1 - e^{-(0.75x+1.3x^{2.3})}\big] \\ & \times \big[\big(2 - e^{-(0.5x+1.2x^{1.1})}\big)\big(2 - e^{-(0.85x+0.9x^{1.9})}\big)e^{-(1.35x+1.2x^{1.1}+0.9x^{1.9})} + e^{-(3.4x+1.4x^{2.2}+1.1x^{0.95})} \\ & - \big(2 - e^{-(0.5x+1.2x^{1.1})}\big)\big(2 - e^{-(0.85x+0.9x^{1.9})}\big)e^{-(4.75x+1.2x^{1.1}+1.4x^{2.2}+0.9x^{1.9}+1.1x^{0.95})}\big] \end{aligned}$$

$$\begin{aligned} R_{S_2}^{W}(t) = {} & e^{-(0.75x+1.3x^{2.3})}\big[(1+0.5x+1.2x^{1.1})e^{-(0.5x+1.2x^{1.1})} + e^{-(1.8x+1.4x^{2.2})} \\ & - (1+0.5x+1.2x^{1.1})e^{-(2.3x+1.2x^{1.1}+1.4x^{2.2})}\big] \\ & \times \big[(1+0.85x+0.9x^{1.9})e^{-(0.85x+0.9x^{1.9})} + e^{-(1.6x+1.1x^{0.95})} - (1+0.85x+0.9x^{1.9})e^{-(2.45x+0.9x^{1.9}+1.1x^{0.95})}\big] \\ & + \big[1 - e^{-(0.75x+1.3x^{2.3})}\big]\big[(1+0.5x+1.2x^{1.1})(1+0.85x+0.9x^{1.9})e^{-(1.35x+1.2x^{1.1}+0.9x^{1.9})} + e^{-(3.4x+1.4x^{2.2}+1.1x^{0.95})} \\ & - (1+0.5x+1.2x^{1.1})(1+0.85x+0.9x^{1.9})e^{-(4.75x+1.2x^{1.1}+1.4x^{2.2}+0.9x^{1.9}+1.1x^{0.95})}\big] \end{aligned}$$

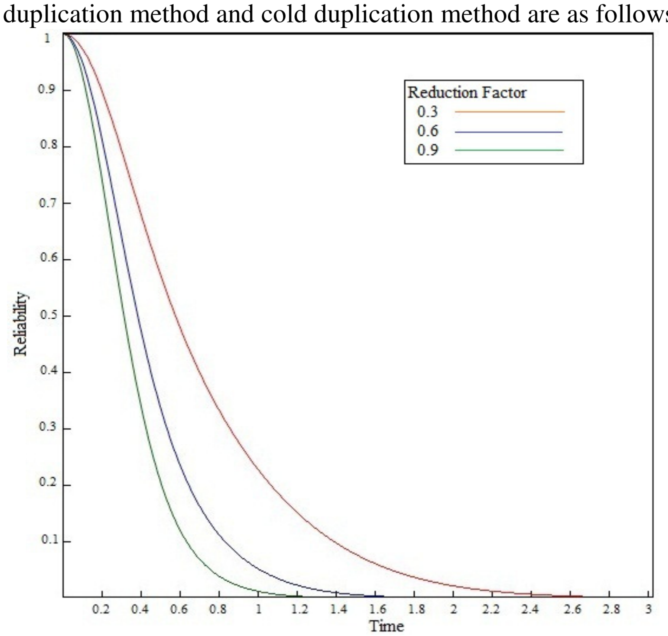

Figure-7
Graph between reliability of the system with reductions factor

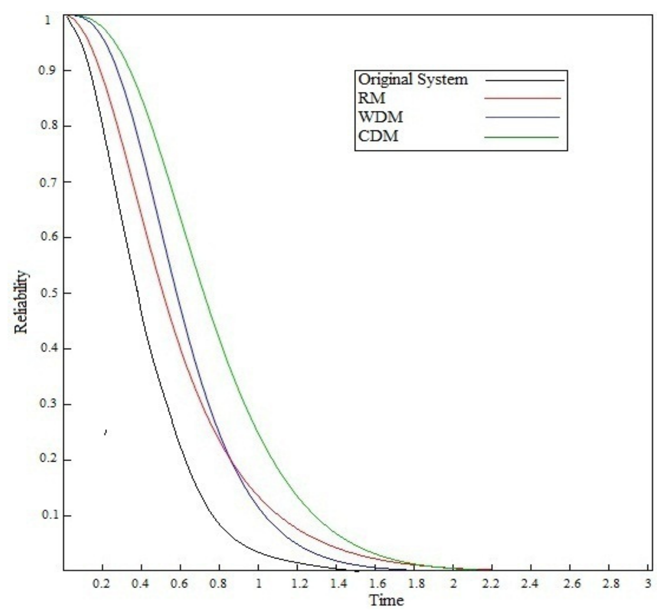

Figure-8
Graph between reliability of the system with original system

It is clear from the figure 7 that reliability of the system increases as the reductions factor decreases. Also, it can be observed that the difference between reliabilities in this experiment is quite large; this is due to the fact that the difference between given reduction factors are also large. Thus in order to keep system safe for longer time we should use the reduction factor as small as possible.

Also, from figure 8 it is obvious that the cold duplication method works better than the warm duplication method where as the original system has always reliability level minimum. In this experiment, the reduction factor is 0.6 based on that we can say; initially reduction method is better than warm duplicate method but the system improved with warm duplication method works for longer time than reduction method. But it should be noted that if we use smaller reduction factor reliability of system increases at faster rate, thus we can say in our problem reduction method with smaller reduction factor is better one and cold duplication method is better than warm duplication method.

Conclusion

In this paper, we have used modified Weibull distribution and reformulated the reliability function of the complex bridge system by three improvement methods, reduction method (RM), warm duplication method (WDM) and cold duplication method (CDM). In all the cases it has been shown the reliability of improved system is higher than original system. Further by a numerical example, it is observed that in reduction method reliability of the system increases with decrease in the reduction factor and cold duplication method improves system reliability much better than the warm duplication method.

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