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A Note on Q-Fuzzy Ideal of Quotient Near-Ring Group

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Abstract

In this paper we shall study on Q-fuzzy ideal of quotient N-group. We give the definition of Q-fuzzy subnear-ring and Q-fuzzy ideal of N-group. We shall investigate some of their properties and prove some characterizations on quotient N-group with the help of Q-fuzzy subnear-ring and Q-fuzzy ideal.

Keywords: Q-fuzzy subnear-ring, Q-fuzzy ideal, quotient N-group.

Introduction

Zadeh[1] introduce fuzzy set in 1965. The idea of the fuzzy ideal in near-ring is discussed by Zaid[2]. Solarairaju *et al.*[3] introduce the new structures of Q-fuzzy groups. On the other hand Muhammad Akram[4] introduces the T-fuzzy Ideals in Near-Ring. Muhammad Akram[4] also introduce about quotient near-ring. Bartakur *et al*[5]. has discussed on Q-fuzzy N-subgroup and Q-fuzy ideal of an N-group. Basumatary *et al*[6]. has discussed on Q-fuzzy ideal and Q-fuzzy quotient near-ring. Generally in this work, we shall study quotient N-group with the help of Q-fuzzy[2] ideals and some of their properties.

Preliminaries

Definition: Consider near-ring N and E as an additive group. Then E is said to be near-ring group or left N-group if there exist a mapping $N\times E \longrightarrow E$, $(n, e) \longrightarrow ne$ such that
(n+m)e=ne+me
(nm)e=n(me)
1.e=e, for all n, m∊N and x∊E.
Unless otherwise stated we denote the zero element of E by 0.
Note: Our discussion by an N-group we mean left N-group.

Definition: Consider a set E as a non empty set. Then a function
$\gamma : A \rightarrow [0, 1]$ is a fuzzy subset of E.

Definition: A function $\gamma : E\times Q \rightarrow [0, 1]$ is called Q-fuzzy[2] set in E, where Q be a set and E be group respectively.

Definition: Consider a function "f" from a set A to B and a Q-fuzzy[2] set μ in A. Then μ is a Q- fuzzy[2] set in B defined by

$$f(\mu)(y, q)=\begin{cases} \sup_{x\in f^{-1}(y)}\mu(x, q) : f^{-1}(y) \neq \phi \\ 0 \quad : \text{ otherwise} \end{cases}$$

Definition: Let Im(λ) denote the image set of λ. Let λ be a Q-fuzzy[2] set in a set E. For " t" in [0, 1] the set λ_t={ x∊E, q∊Q; λ(x, Q)≥t} is called Q-level subset of λ .

Definition: Consider γ a Q- fuzzy[2] subset in a near-ring E, then γ is Q- fuzzy[2] subnear-ring of E if it holds the conditions
1. $\gamma(p-s, q) \geq \gamma(p, q) \wedge \gamma(s, q)$
2. $\gamma(ps, q) \geq \gamma(p, q) \wedge \gamma(s, q)$; for p, s∊E and q∊Q.

Definition: A Q-fuzzy[2] subnear-ring γ in E is called Q-fuzzy[2] ideal
1. $\gamma(p+s-p, q) \geq \gamma(s, q)$
2. $\gamma(ps, q) \geq \gamma(s, q)$
3. $\gamma((p+z)s-ps, q) \geq \gamma(z, q)$; p, s, z∊E, q∊Q

Definition: Consider γ a Q-fuzzy[2] subset of an N-group E. Then γ is said to be Q-fuzzy[2] subnear-ring of E if for all n∊N, p, s∊E the following holds:
i. $\gamma(p-s, q) \geq \gamma(p, q) \wedge \gamma(s, q)$
ii. $\gamma(np, q) \geq \gamma(p, q)$

Definition: A Q-fuzzy[2] subnear-ring γ in N-group E is called Q-fuzzy[2] ideal if for n∊N and
p, s∊E the following condition holds:
1. $\gamma(p+s, q) \geq \gamma(p, q) \wedge \gamma(s, q)$
2. $\gamma(p+s-p, q) \geq \gamma(s, q)$
3. $\gamma(n(p+s)-np, q) \geq \gamma(s, q)$

Theorem: Let E and F be two N-groups and $h:E \longrightarrow F$ be an N-epimorphism. Suppose γ be a Q-fuzzy[2] ideal of E then h(γ) is Q-fuzzy[2] ideal of F.
Proof: We have γ is Q- fuzzy ideal on N-group E.

Now $h(\gamma)(u-v, q)= \bigvee_{h(w)=u-v} \gamma(w, q)$

$= \bigvee_{h(p-s)=u-v} \gamma(p-s, q)$

$= \bigvee_{h(p)=u, h(s)=v} \gamma(p-s, q)$

$\geq [\bigvee_{h(p)=u} \gamma(p, q)] \wedge [\bigvee_{h(s)=v} \gamma(s, q)]$

$= h(\gamma)(u, q) \wedge h(\gamma)(v, q)$.

Now let p϶F, n϶N and q϶Q, so we have z϶E such that h(z)=p and hence h(nz)=np.

$h(\gamma)(np, q)= \{ \bigvee_{h(m)=np} \gamma(m, q) : m϶E, q϶Q \}$

$\geq \{ \bigvee_{h(nz)=np} \gamma(nz, q) : nz϶E, q϶Q \}$

$= \{ \bigvee_{nh(z)=np} \gamma(nz, q) : nz϶E, q϶Q \}$

$\geq \{ \bigvee_{h(z)=p} \gamma(z, q) : z϶E, q϶Q \}$

$= h(\gamma)(p, q).$

Thus h(γ) is Q- fuzzy² subnearing of F.
Let u, v϶F and q϶Q, so p, m in E such that h(p)=u and h(m)=v.

Now $h(\gamma)(u+v, q)= \bigvee_{h(w)=u+v} \gamma(w, q)$

$= \bigvee_{h(p+m)=u+v} \gamma(p+m, q)$

$= \bigvee_{h(p)=u, h(m)=v} \gamma(p+m, q)$

$\geq [\bigvee_{h(p)=u} \gamma(p, q)] \wedge [\bigvee_{h(m)=v} \gamma(m, q)]$

$\geq h(\gamma)(u, q) \wedge h(\gamma)(v, q).$

Let p, m϶F so a, b϶E such that h(a)=p, h(b)=m and q϶Q.

$h(\gamma)(p+m-p, q)= \bigvee_{h(u)=p+m-p} \gamma(u, q)$

Now since we have h(b+a-b)=m+p-m, therefore
$h(\gamma)(m+p-m, q) \geq \gamma(b+a-b, q)$
$\geq \gamma(a, q)$, whenever h(a)=p

$\geq \bigvee_{h(a)=p} \gamma(a, q)$

$= h(\gamma)(p, q)$

Also let n϶N and p, m϶F. Since "h" is N-epimorphism so, we have a, b϶E such that h(a)=p, h(b)=m
Now

$h(\gamma)(n(m+p)-np, q)= \bigvee_{h(u)=n(p+m)-np} \gamma(u, q)$

Now since we have h[n(a+b)-na]=n(p+m)-np, therefore
$h(\gamma)(n(p+m)-np, q) \geq \gamma(n(a+b)-na, q)$
$\geq \gamma(b, q)$, whenever h(b)=m

$\geq \bigvee_{h(b)=m} \gamma(b, q)$

$= h(\gamma)(m, q)$
Thus h(γ) is Q-fuzzy² ideal of F.

Theorem: Consider an ideal K of N-group E. Consider a Q-fuzzy ideal γ of E, let us consider Q-fuzzy set ϕ of E/K such that $\phi(x+K, q) = \sup_{a϶K} \gamma(x+a, q)$ then ϕ is Q-fuzzy ideal of the quotient N-group E/K with respect to K.

Proof: Here ϕ is clearly well define as if we consider two elements a, b in N-group E so that (a+K) is equal to (b+K).

Then we have b=a+m for some m in K.

Now $\phi(b+K, q)= \bigvee_{z\in K} \gamma(b+z, q)$

$= \bigvee_{z\in K} \gamma(a+m+z, q)$

$= \bigvee_{m+z=p\in K} \gamma(a+p, q)$

$= \phi(a+K, q).$

Now we try to show ϕ is Q-fuzzy² subnearing in E/K.
Consider (p+K), (m+K) be two elements of E/K.
$\Phi((p+K)-(m+K), q)= \phi((p-m)+K, q)$

$= \bigvee_{z\in K} \gamma((p-m)+z, q)$

$= \bigvee_{u-v=z\in K} \gamma((p-m+(u-v), q)$

$= \bigvee_{u-v=z\in K} \gamma((p+u)-(m+v), q)$

$\geq [\bigvee_{u\in K} \gamma((p+u), q)] \wedge [\bigvee_{v\in K} \gamma((m+v), q)]$

$= \phi(p+K, q) \wedge \phi(m+K, q)$

Let n϶N and p be an element of E.

$\Phi(n(p+K), q) = \Phi(np+K, q)= \bigvee_{z\in K} \gamma(np+z, q)$

$= \bigvee_{uv=z\in K} \gamma(np+uv, q)$

$\geq \bigvee_{u\in K} \gamma(p+u, q)$

$= \phi(p+K, q)$
Hence ϕ is Q-fuzzy² subnearing in E/K.

Now we try to show ϕ is Q- fuzzy² ideal in E/K.
Consider (p+K), (m+K) be two elements of E/K.
$\Phi((p+K)+(m+K), q)= \phi((p+m)+K, q)$

$= \bigvee_{z\in K} \gamma((p+m)+z, q)$

$= \bigvee_{u+v=z\in K} \gamma((p+m)+(u+v), q)$

$= \bigvee_{u+v=z\in K} \gamma((p+u)+(m+v), q)$

$\geq [\bigvee_{u\in K} \gamma((p+u), q)] \wedge [\bigvee_{v\in K} \gamma((m+v), q)]$

$= \phi(p+K, q) \wedge \phi(m+K, q)$
$\Phi((p+K)+(m+K)-(p+K), q)= \phi((p+m-p)+K, q)$

$= \bigvee_{z\in K} \gamma((p+m-p)+z, q)$

$\geq \bigvee_{z\in K} \gamma(m+z, q)$

$= \phi(m+K, q)$

Let n϶N and (p+K), (m+K) be two elements of E/K.
Now
$\Phi(n((p+K)+(m+K))-n(p+K), q)= \phi((n(p+m)-np)+K, q)$

$= \bigvee_{z\in K} \gamma(n(p+m)-np)+z, q)$

$\geq \bigvee_{z\in K} \gamma(nm+z, q)$

$\geq \bigvee_{z\in K} \gamma(m+z, q)$
$=\phi(m+K, q)$
Thus ϕ is Q-fuzzy^2 ideal of E/K.

Theorem: Consider an ideal K of N-group E. Then we can have one to one mapping between the set of Q-fuzzy^2 ideals γ of E so that $\gamma(0, q)$ is equal to $\gamma(s, q)$ for all "s" in K and the set ϕ, the set of all Q-fuzzy^2 ideal of E/K.

Proof: Let γ be Q-fuzzy^2 ideal of E then following theorem[3.2] we can show
$\phi(x+K, q) = \sup_{a\epsilon K}\gamma(x+a, q)$ is Q-fuzzy^2 ideal of E/K.
Also we have $\gamma(0, q)=\gamma(s, q)$.

Now we have from definition[2.9]
$\gamma(a+s, q)\geq\gamma(a, q)$.
Also $\gamma(a, q)=\gamma(a+s-s, q)\geq\gamma(a+s, q)$.
Thus we have $\gamma(a+s, q)=\gamma(a, q)$, for all $s\epsilon K$.
Thus $\phi(a+K, q)$ is equal to $\gamma(a, q)$.
So the corresponding $\gamma \longmapsto \phi$ is one to one.

Now consider ϕ be Q-fuzzy^2 ideal in E/K. Define Q-fuzzy^2 set γ in E so that $\gamma(a, q)$ is equal to $\phi(a+K, q)$, for all $a\epsilon K$.

Let p and m be two element of E and $n\epsilon N$
$\gamma(p-m, q)=\phi((p-m)+K, q)$
$=\phi((p+K)-(m+K), q)$
$\geq\phi(p+K, q) \wedge \phi(m+K, q)$
$= \gamma(p, q) \wedge \gamma(m, q)$
$\gamma(np, q)=\phi(np+K, q)$
$\geq \phi(p+K, q)$
$= \gamma(p, q)$.

Hence γ is Q-fuzzy^2 subnear-ring in E.
Now let p, m be two element N-group E and n be an element of N.
$\gamma(p+m, q)=\phi((p+m)+K, q)$
$=\phi((p+K)+(m+K), q)$
$\geq\phi(p+K, q) \wedge \phi(m+K, q)$
$= \gamma(p, q) \wedge \gamma(m, q)$
$\gamma(p+m-p, q)=\phi((p+m-p)+K, q)$
$=\phi((p+K)+(m+K)-(p+K), q)$
$\geq\phi(m+K, q)$
$=\gamma(m, q)$
$\gamma(n(p+m)-np), q)= \phi((n(p+m)-np)+K, q)$
$= \Phi(n((p+K)+(m+K))-n(p+K), q)$
$\geq\phi(m+K, q)$
$=\gamma(m, q)$

Thus γ is Q-fuzzy^2 ideal in N-group E. Clearly $\gamma(a, q)$ is equal to $\phi(a+K, q)$ which is again equal to $\phi(K, q)$, for all "a" in K. This indicates that $\gamma(0, q)$ is equal to $\gamma(s, q)$ for all "s" in K.

Theorem: Let us consider K be an ideal of an N-group E. We can have then Q-fuzzy^2 ideal γ of N-group E so that $\gamma(0, q)$ is "t" and λ_t is K, for $t\epsilon[0, 1]$, where λ_t is called Q- level subset of λ.
Proof: It is straightforward.

Theorem: Consider a Q-fuzzy^2 ideal γ of a N-group E also $\gamma(o, q)$ is "t". Then ϕ is Q-fuzzy^2 ideal of E/λ_t, where ϕ is constructed as $\phi(p+\lambda_t, q)=\gamma(p, q)$, for $p\epsilon E$ and λ_t is called Q-level subset of λ.
Proof: The prove is straightforward.

Conclusion

In this work, we have defined the definition of Q-fuzzy^2 subnearing, Q-fuzzy^2 ideal of N-group. We introduced the definition of Q-fuzzy^2 subnear-ring, Q-fuzzy^2 ideal of N-group. With the help of Q-fuzzy^2 subnear-ring and Q-fuzzy^2 ideal we have discussed on Q-fuzzy^2 quotient N-group and proved some theorems on Q-fuzzy quotient N-group.

Reference

1. Zadeh L.A., Fuzzy sets, *Information and Control*, **8,** 338-353 **(1965)**
2. Zaid A., On fuzzy sub near-rings, *Fuzzy Sets and System*, **44**, 139-146 (**1991**)
3. Solairaju A., Nagarajan R., A new structure and construction of Q-fuzzy group, *Advance in Fuzzy Mathematics*, **4**, 512-517 (**2009**)
4. Akram M., On T-Fuzzy Ideals in Nearings, *International Journal of Mathematics and Mathematical Sciences*, doi:10.1155/2007/73514 (**2007**)
5. Bartakur G.K., Khargharia J., On Q-fuzzy N-subgroup and Q-fuzy Ideal of an N-group, *International Journal of Fuzzy Mathematics and Systems*, **3**(2), 153-164 **(2013)**
6. Basumatary B., Bartakur G.K., On Q-fuzzy Ideal and Q-fuzzy Quotient Near-Ring, *Research Journal of Mathematical and Statistical Sciences*, **2(7),** 4-6, **(2014)**
7. Mourad O.M., On Q-Fuzzy R-Subgroups of Near-Rings, *International Mathematical Forum*, **8**, 387-393 **(2013)**
8. Bhakad S.K. and Das P., Fuzzy subrings and ideals redefined, *Fuzzy Sets and System*, **81**, 383-393 (**1996**)
9. Bhakad S.K., Das P., On the definition of a fuzzy subgroup, *Fuzzy Sets and System*, **51**, 235-241(**1992**)
10. Devi R., On Intuitionistic Q-Fuzzy Ideal of Near-Rings, NIFS, **15(3)** 25-32 (**2009**)