# Persistent and Permanent Point of Views of Two Stages DNA Splicing Languages 

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#### Abstract

Yusof-Goode ( $Y-G$ ) splicing system was formulated by Yusof in 2012 to present the existence relation between formal language theory and molecular biology in a convenient approach. In terms of biology, the recombinant deoxyribonucleic acid (DNA) molecules can often be split with the existence of actual restriction enzymes. For this property of the recombinant DNA strands is called persistent. Therefore, determining the persistency of the hybrid templets of DNA strands, after acting restriction enzymes on the initial DNA strands, by providing mathematical proof is considered as a new contribution in the areas of DNA molecular. In this work, the persistent and permanent aspects of two stages splicing languages are investigated and discussed via Y-G approach. This investigation focuses on number of cutting sites in initial strings as well as sequences factors of splicing rules. Accordingly, the persistency and permanency of the above splicing languages with respect to two initial strings (with two cutting sites) and two rulesare presented as lemmas, theorems, and corollaries. Nevertheless, an example is provided, which shows the real meaning of theorem in terms of biology.


Keywords: Permanent, persistent, two stages Y-G splicing languages.

## Introduction

Deoxyribonucleic acid or DNA is a longchain of nucleotides made up of repeating components. The components of each nucleotide arenitrogen involving four nucleobases such as adenine $(A)$, guanine $(G)$, cytosine $(C)$ and thymine $(T)$, and a monosaccaharide sugar as well as a phosphate group. The four nucleobases of DNA is classified into two families of purine and pyrimidine. In a DNA, for adenine and guanine is called purine, while for cytosine and thymine is called pyrimidine. Since DNA exists in the form of double-stranded DNA (dsDNA), thusthe two single strands are joined together by hydrogen bonding between nucleotides bases. There are two hydrogen bonding between adenine and thymineand three hydrogen bonding between guanine and cytosine, and viceversa. In dsDNA, the two strandsare complementaryto each and run into two opposite directions. Due to this property of dsDNA, the lower single strand can be determined by its upper single strand and vice-versa ${ }^{1}$. However, a restriction enzyme (RE) is an enzyme that finds specific sequence of the nitrogenous bases usually 4-6 base pairs in length in the strand of DNA and carves it from phospodiester bond between adjacent bases, resulting in a molecule with staggered or blunt ends ${ }^{2}$. Then, the DNA fragments are re-joined with their complementary staggered ends or blunt ends by ligase to produce new DNA strands. Naturally, the vast majority of RE have been isolated from bacteria, where they accomplish a hostdefense function for the cell.

Y-G splicing system is the latest model of splicing system, which was introduced byYusof ${ }^{3}$. This splicing system studies the splitting and ligating characteristics of DNA under the effect of appropriate restriction enzyme and a ligase in a translucent approach. The notion of splicing system was first proposed by Head under a framework of formal language theory ${ }^{4}$. Furthermore, the splicing language, which is generated by splicing system, was discussed. Besides, persistent splicing system was defined and itsrelation with strictly locally testable languagewas proven. Later, Gatterdamdefined the notion of permanent and proved that permanent is a proper subset of persistent ${ }^{5}$. Although the permanent splicing system is proper subset of persistent, the splicing languages generate by these two splicing systems are equal ${ }^{6}$. Karimiproved that if there are two persistent (or permanent) splicing systems, the union and intersection of these two splicing systems are not persistent or permanent ${ }^{7}$. Some sufficient conditions for persistency of splicing system were provided ${ }^{8}$. The concepts of crossing preserved and self-closed were introduced and theirrelations with a persistent splicing system were presented ${ }^{9}$. Besides, the concept of extended crossing preserved was introduced and the relation between self-closed, extended crossing preserved and permanent splicing system was given. Common crossing splicing system was defined and its relationwitha self-closed and persistent splicing system was investigated ${ }^{10}$. Some sufficient conditions for splicing system to be permanent were provided ${ }^{11}$. An analysis on four different models of splicing systems namely, Head, Paun, Goode-Pixton and Y-G as well as splicing languages produce by them is investigated ${ }^{12}$. The persistent and permanent points of views of some non-semi
simple splicing system were discussed using Y-G approach ${ }^{3}$. In this research, the persistent and permanent points of views of two stages splicing languages based on crossing sites and contexts of splicing rules and cutting sites of initial strings are investigated and presentedaslemmas, theorems and corollaries.

## Methodology

Preliminaries: In this section, the definitions of Y-G splicing system, persistent, permanent and crossing disjoint are given, wherethese concepts are the key pointsin this research. Since this study is based on Y-G approach, this splicing system is defined as below.

Definition 1: Yusof-Goode Splicing system ${ }^{3}$ : If $r \in R$, where $r=(a, x, b: c, x, d)$ and $\quad s_{1}=\alpha \operatorname{axb} \beta$ and $s_{2}=\gamma c x d \delta$ are elements of $I$, then splicing $s_{1}$ and $s_{2}$ using $r$ produce the initial string $I$ together with $\alpha a x d \delta$ and $\gamma c x b \beta$, presented in either order where $\alpha, \beta, \gamma, \delta, a, b, c$ and $d \in A^{*}$ are free monoid generated by $A$ with the concatenation operation and 1 as the identity element. $\square$

Biologically, the new formed hybrid DNA strands are not usually persistent and permanent. Therefore, to investigate the persistency and permanency of two stages recombinant DNA molecules, these two important properties of splicing system are defined in the following.

Persistent ${ }^{4}$ : Let $S=(A, I, R)$ be a splicing system. Then $S$ is persistent if for each pair of strings $u c x d v$ and pexfq, in $A^{*}$ with $(c, x, d)$ and $(e, x, f)$ patterns of the same hands: if $y$ is a sub segment of $u c x$ (respectively $x f q$ ) that is crossing of a site in $u c x d v$ (respectively $p e x f q$ ) then this same sub segment $y$ of ucxfq contains an occurrence of a crossing of a site in ucxfq . $\square$

In the next, the definition of permanent is stated.
Permanent ${ }^{5}$ : Let $S=(A, I, R)$ be a splicing system. Then $S$ is permanent if for each pair of strings $u c x d v$ and $\operatorname{pexfq}$, in $A^{*}$ with $(c, x, d)$ and $(e, x, f)$ patterns of the same hands: if $y$ is a sub segment of $u c x$ (respectively $x f q$ ) that is crossing of a site in $u c x d v$ (respectively pexfq) then this same sub segment $y$ of $u c x f q$ is an occurrence of a crossing of a site in $u c x f q$.

When the restriction enzymes are chosen from any supplier, for example, New England BioLabs (NEB), their behaviors as well as the splicing sites are not always the same. In terms of DNA recombination, the splicing sites of restriction enzymes play an important role on the number of generating DNA molecules at
stage one and stage two as well as their persistent and permanent properties. Thus, the concept of crossing disjoint (disjoint splicing sites) splicing system using is defined below.

Crossing Disjoint ${ }^{13}$ : A splicing system $S=(A, I, B, C)$ is crossing disjoint if there do not exist patterns $(a, x, b)$ in $B$ and $(c, x, d)$ in $C$ with same $\operatorname{crossing} x$.

Since Y-G splicing system is being used in this research, the above definition can be defined as: A Y-G splicing system $S=(A, I, R)$ is crossing disjoint if there does not exist pattern $(a, x, b: a, x, b)$ in $R$ with same crossing $x . \square$

## Results and Discussions

This section discusses on persistent as well as permanent points of views of two stages DNA splicing languages. Somelemmas, theorems, and corollaries regarding to characteristics of two stages splicing languages with respect to number of cutting sites of initial strings as well ascrossing sites and sequences of splicing rules are provided in terms of persistent and permanent points of views. In biological perspective, the provided mathematical theorems predict whether the new forming DNA fragments after accomplishing the recombination process at two stages (stage one and stage two ) will be split by acting of existence RE or not. It means that if the recombinant DNA strands are persistent, then they can be cut by the existence of RE. However, if the recombinant DNA molecules are nonpersistent, then they cannot be cut by the existence of same RE. In other words, the recombination process is stopped. To understand the concept of two stages splicing languages,its definition is first presented below.

Two Stages Splicing Languages: Let $S=(A, I, R)$ is a splicing system. Furthermore, let $L=L(S)$ is the set of stage one splicing languages produced by splicing system $S$ and $L^{\prime}=L^{\prime}(S)$ is the set of stage two splicing languages produced by $S$ that consists of $L=L(S)$ and all splicing languages that can be resulted by splicing $L$. Then, the union of stage one and stage two splicing languages are called two stages splicing languages.ㅁ

Lemma 1 discusses on the persistency of two stages DNA splicing languages with respect to two initial strings (with two cutting sites) and two rules, where first rule is used on first initial string, and second rule is used on second initial string.

Lemma 1: If the crossing sites of the rules in a Y-G splicing system be disjoint and non-palindromic so that the first rule cuts the first initial string (with two cutting sites) and the second rule cuts the second initial string (with two cutting sites)
from two specific places, respectively, then the set of two stages splicing languages, which is produced by Y-G splicing system, is persistent.

Proof: Suppose that $S=(A, I, R)$ be a Y-G splicing system. Thus the rules $r_{1}, r_{2} \in R$ are presented in the forms of $r_{1}=\left(a_{1}, a_{1} a_{2}, a_{2}: a_{1}, a_{1} a_{2}, a_{2}\right)$ and $r_{2}=\left(b_{1}, b_{1} b_{2}, b_{2}: b_{1}, b_{1} b_{2}, b_{2}\right)$, respectively where $a_{1}$ is not complementary with $a_{2}, b_{1}$ is not complementary with $b_{2}$ and vice- versa, $a_{1}, a_{2}, b_{1}, b_{2} \in A^{*}$ .Since the patterns have disjoint crossings, therefore the fragments of initial strings after cutting by the existing splicing rules cannot join together. To show the two stages splicing languages are persistent, the splicing languages at two stages need to be obtained. Let $s_{1}$ and $s_{2}$ be two initial strings in $A^{*}$ which are presented the forms $s_{1}=\alpha a_{1} a_{1} a_{2} a_{2} a_{1} a_{1} a_{2} a_{2} \beta$ and $s_{2}=\gamma b_{1} b_{1} b_{2} b_{2} b_{1} b_{1} b_{2} b_{2} \delta$, respectively such that string $s_{1}$ can be cut by $r_{1}$ and string $s_{2}$ can be cut by $r_{2}$ from two specific sequences, and by splicing the following splicing languages can be generated at stage one.
$\alpha a_{1}\left(a_{1} a_{2} a_{2} a_{1}\right)^{k} a_{1} a_{2} a_{2} \beta, \gamma b_{1}\left(b_{1} b_{2} b_{2} b_{1}\right)^{k} b_{1} b_{2} b_{2} \delta$,
where $k \in \mathbb{N}$.
Now the splicing languages of stage two is obtained. By applying the rules $r_{1}$ and $r_{2}$ on the resulted splicing languages
$\alpha a_{1}\left(a_{1} a_{2} a_{2} a_{1}\right)^{k} a_{1} a_{2} a_{2} \beta$ and $\quad \gamma b_{1}\left(b_{1} b_{2} b_{2} b_{1}\right)^{k} b_{1} b_{2} b_{2} \delta$, respectively the following two long DNA splicing languages will be produced at stage two.
$\alpha a_{1}\left(a_{1} a_{2} a_{2} a_{1}\right)^{n} a_{1} a_{2} a_{2} \beta, \quad \gamma b_{1}\left(b_{1} b_{2} b_{2} b_{1}\right)^{n} b_{1} b_{2} b_{2} \delta, \quad$ where $n=k+i, i=0,1,2, \ldots, k$

To show that the above splicing languages are persistent the patterns with same crossing should be considered. Therefore, According to definition of persistent, if $a_{1} a_{2}$ bea sub segment of $\alpha a_{1} a_{1} a_{2}$ and $b_{1} b_{2}$ be a sub segment of $\gamma b_{1} b_{1} b_{2}$. These $a_{1} a_{2}$ and $b_{1} b_{2}$ is a crossing of $\alpha a_{1} a_{1} a_{2} a_{2} a_{1} a_{1} a_{2} a_{2} \beta$ and $\gamma b_{1} b_{1} b_{2} b_{2} b_{1} b_{1} b_{2} b_{2} \delta$, respectively. These same sub segments $a_{1} a_{2}$ and $b_{1} b_{2}$, respectively contain an occurrence of the crossing of a site in the resulted DNA splicing languages $\alpha a_{1}\left(a_{1} a_{2} a_{2} a_{1}\right)^{k} a_{1} a_{2} a_{2} \beta$ and $\gamma b_{1}\left(b_{1} b_{2} b_{2} b_{1}\right)^{k} b_{1} b_{2} b_{2} \delta$, respectively as well the rest of the above yielding splicing languages. Thus, the two stages splicing languages are persistent.

In the following lemma, the persistency of two stages DNA splicing languages is proven according to two initial string (with two cutting sites) and two rules, where first rule is applied on first initial string and both rules are used on second string.

Lemma 2: If the crossing sites of the rules in a Y-G splicing system be disjoint and non-palindromic so that the first rule cuts the first initial string (with two cutting sites) and both splicing rules cut the second initial string (with two cutting sites) from two specific places, respectively, then the set of two stages splicing languages, which is produced by Y-G splicing system, is persistent.

Proof: Suppose that $S=(A, I, R)$ be a Y-G splicing system. Thus the rules $r_{1}, r_{2} \in R$ are presented the forms $r_{1}=\left(a_{1}, a_{1} a_{2}, a_{2}: a_{1}, a_{1} a_{2}, a_{2}\right)$ and $r_{2}=\left(b_{1}, b_{1} b_{2}, b_{2}: b_{1}, b_{1} b_{2}, b_{2}\right)$, where $a_{1}$ is not complementary with $a_{2}, b_{1}$ is not complementary with $b_{2}$ and vice- versa, and $a_{1}, a_{2}, b_{1}, b_{2} \in A^{*}$ .Since the patterns have disjoint crossings, thus the fragments of initial strings after cutting by the existing splicing rules cannot re-join together. To show the two stages splicing languages are persistent; the splicing languages at two stages need to be obtained. Let $s_{1}$ and $s_{2}$ be two initial strings in $A^{*}$ which are presented in the forms $s_{1}=\alpha a_{1} a_{1} a_{2} a_{2} a_{1} a_{1} a_{2} a_{2} \beta$ and $s_{2}=\gamma a_{1} a_{1} a_{2} a_{2} b_{1} b_{1} b_{2} b_{2} \delta$ such that string $s_{1}$ can be cut by $r_{1}$ and string $s_{2}$ can be cut by both rules $r_{1}$ and $r_{2}$ from two specific places. Splicing them using above rules the following splicing languages will be resulted at stage one.
$\alpha a_{1}\left(a_{1} a_{2} a_{2} a_{1}\right)^{k} a_{1} a_{2} a_{2} \beta, \alpha a_{1}\left(a_{1} a_{2} a_{2} a_{1}\right)^{k} a_{1} a_{2} a_{2} b_{1} b_{1} b_{2} b_{2} \delta$,
$\gamma a_{1}\left(a_{1} a_{2} a_{2} a_{1}\right)^{k} a_{1} a_{2} a_{2} \beta, \gamma a_{1}\left(a_{1} a_{2} a_{2} a_{1}\right)^{k} a_{1} a_{2} a_{2} b_{1} b_{1} b_{2} b_{2} \delta$, where $k \in \mathbb{N}$.

When splicing operation takes place among the resulted DNA splicing languages, the following DNA splicing languages will be generated at stage two.

$$
\begin{aligned}
& \alpha a_{1}\left(a_{1} a_{2} a_{2} a_{1}\right)^{n} a_{1} a_{2} a_{2} \beta, \alpha a_{1}\left(a_{1} a_{2} a_{2} a_{1}\right)^{n} a_{1} a_{2} a_{2} b_{1} b_{1} b_{2} b_{2} \delta, \\
& \gamma a_{1}\left(a_{1} a_{2} a_{2} a_{1}\right)^{n} a_{1} a_{2} a_{2} \beta, \gamma a_{1}\left(a_{1} a_{2} a_{2} a_{1}\right)^{n} a_{1} a_{2} a_{2} b_{1} b_{1} b_{2} b_{2} \delta, \\
& \text { where } n=k+i, i=0,1,2, \ldots, k .
\end{aligned}
$$

To prove the above sets of splicing languages are persistent, the patterns with same crossing site need to be considered. If $a_{1} a_{2}$ be a sub segment of $\alpha a_{1} a_{1} a_{2}$, that is crossing of a site in $\alpha a_{1} a_{1} a_{2} a_{2} a_{1} a_{1} a_{2} a_{2} \beta$. This same sub segment $a_{1} a_{2}$ contains an occurrence of crossing of a site in the yielding string $\alpha a_{1}\left(a_{1} a_{2} a_{2} a_{1}\right)^{k} a_{1} a_{2} a_{2} b_{1} b_{1} b_{2} b_{2} \delta$. Hence, the two stages splicing languages are persistent.

In the next lemma, the persistency of two stages DNA splicing languages is presented with respect to a Y-G splicing system consisting of two initial strings (with two cutting sites) and two
rules where both rules are used on first initial string and second rule is used on second initial string.

Lemma 3: If the crossing sites of the rules in a Y-G splicing system be disjoint and non-palindromic so that both splicing rules cut the first initial string (with two cutting sites) and the second rule cuts the second initial string (with two cutting sites) from two specific places, respectively, then the set of two stages splicing languages, which is produced by Y-G splicing system, is persistent. $\quad$

Proof: Suppose that $S=(A, I, R)$ be a Y-G splicing system. Thus the rules $r_{1}, r_{2} \in R$ are presented the forms $r_{1}=\left(a_{1}, a_{1} a_{2}, a_{2}: a_{1}, a_{1} a_{2}, a_{2}\right)$ and $r_{2}=\left(b_{1}, b_{1} b_{2}, b_{2}: b_{1}, b_{1} b_{2}, b_{2}\right)$, where $a_{1}$ is not complementary with $a_{2}, b_{1}$ is not complementary with $b_{2}$ and vice- versa, $a_{1}, a_{2}, b_{1}, b_{2} \in A^{*}$. Since the patterns have disjoint crossings, thus the fragments of initial strings after cutting by the existing splicing rules cannot recombine together. To show the two stages splicing languages are persistent, the splicing languages at two stages need to be obtained. Assume $s_{1}$ and $s_{2}$ be two initial strings in $I \in A^{*}$ have a form as $s_{1}=\alpha a_{1} a_{1} a_{2} a_{2} b_{1} b_{1} b_{2} b_{2} \beta$ and $s_{2}=\gamma b_{1} b_{1} b_{2} b_{2} b_{1} b_{1} b_{2} b_{2} \delta$ such that string $s_{2}$ can be cut by $r_{2}$ and string $s_{1}$ can be cut by both rules $r_{1}$ and $r_{2}$ from two specific places and by splicing the following splicing languages will be the generating splicing languages at stage two.

$$
\begin{aligned}
& \gamma b_{1}\left(b_{1} b_{2} b_{2} b_{1}\right)^{k} b_{1} b_{2} b_{2} \delta, \gamma b_{1}\left(b_{1} b_{2} b_{2} b_{1}\right)^{k} b_{1} b_{2} b_{2} \beta, \\
& \alpha a_{1} a_{1} a_{2} a_{2} b_{1}\left(b_{1} b_{2} b_{2} b_{1}\right)^{k} b_{1} b_{2} b_{2} \delta, \\
& \alpha a_{1} a_{1} a_{2} a_{2} b_{1}\left(b_{1} b_{2} b_{2} b_{1}\right)^{k} b_{1} b_{2} b_{2} \beta,
\end{aligned}
$$

where $k \in \mathbb{N}$.
Since the resulted DNA splicing languages can be cut by the rules $r_{1}$ and $r_{2}$, thus by adding the rules on the generated splicing languages of stage one the following DNA splicing languages will be obtained at stage two.
$\gamma b_{1}\left(b_{1} b_{2} b_{2} b_{1}\right)^{n} b_{1} b_{2} b_{2} \delta, \gamma b_{1}\left(b_{1} b_{2} b_{2} b_{1}\right)^{n} b_{1} b_{2} b_{2} \beta$,
$\alpha a_{1} a_{1} a_{2} a_{2} b_{1}\left(b_{1} b_{2} b_{2} b_{1}\right)^{n} b_{1} b_{2} b_{2} \delta$,
$\alpha a_{1} a_{1} a_{2} a_{2} b_{1}\left(b_{1} b_{2} b_{2} b_{1}\right)^{n} b_{1} b_{2} b_{2} \beta$,
where $n=k+i, i=0,1,2, \ldots, k$.

The proof for persistency of above sets of splicing languages follows Lemma 2.

In Lemma 4, the persistency of two stages DNA splicing languages is discussed with respect to a Y-G splicing system
consisting of two initial strings (with two cutting sites) and two rules where both rules are used on each of the initial string.

Lemma 4: If the crossing sites of the rules in a Y-G splicing system be disjoint and non-palindromic so that both splicing rules cut both of the initial strings (with two cutting sites) from two specific places, respectively, thenthe set of two stages splicing languages, that is produced by Y-G splicing system, is persistent. $\square$

Proof: Suppose that $S=(A, I, R)$ be a Y-G splicing system. Thus the rules $r_{1}, r_{2} \in R$ are presented the forms $r_{1}=\left(a_{1}, a_{1} a_{2}, a_{2}: a_{1}, a_{1} a_{2}, a_{2}\right)$ and $r_{2}=\left(b_{1}, b_{1} b_{2}, b_{2}: b_{1}, b_{1} b_{2}, b_{2}\right)$, respectivelywhere $a_{1}$ is not complementary with $a_{2}, b_{1}$ is not complementary with $b_{2}$ and vice- versa, $a_{1}, a_{2}, b_{1}, b_{2} \in A^{*}$ .Since the patterns have disjoint crossings, therefore, the fragments of initial strings after cutting by the existing splicing rules will not re-join together. To show the two stages splicing languages are persistent, the splicing languages at two stages need to be obtained. Suppose that $s_{1}$ and $s_{2}$ be two initial strings in $I \in A^{*}$ have a form as $s_{1}=\alpha a_{1} a_{1} a_{2} a_{2} b_{1} b_{1} b_{2} b_{2} \beta$ and $s_{2}=\gamma a_{1} a_{1} a_{2} a_{2} b_{1} b_{1} b_{2} b_{2} \delta$ such that strings $s_{1}$ and $s_{2}$ can be cut by both rules $r_{1}$ and $r_{2}$ from two specific sequences and by splicing them the following splicing languages will be generated at stage one namely, $\alpha a_{1} a_{1} a_{2} a_{2} b_{1} b_{1} b_{2} b_{2} \delta$,
$\gamma a_{1} a_{1} a_{2} a_{2} b_{1} b_{1} b_{2} b_{2} \beta$
However, by applying both rules $r_{1}$ and $r_{2}$ on the resulted DNA splicing languages of first stage, no distinct DNA splicing languages will be generated at stage two.

Now it is shown that the above set of splicing languages is persistent. According to definition of persistent if $a_{1} a_{2}$ and $b_{1} b_{2}$ be sub segments of $\alpha a_{1} a_{1} a_{2}$ (respectively $b_{1} b_{2} b_{2} \delta$ ), these are crossings of the sites in $\alpha a_{1} a_{1} a_{2} a_{2} b_{1} b_{1} b_{2} b_{2} \beta$ (respectively $\gamma a_{1} a_{1} a_{2} a_{2} b_{1} b_{1} b_{2} b_{2} \delta$ ). These same sub segments $a_{1} a_{2}$ and $b_{1} b_{2}$ contain an occurrence of crossing of a site in the yielding string $\alpha a_{1} a_{1} a_{2} a_{2} b_{1} b_{1} b_{2} b_{2} \delta$ and $\gamma a_{1} a_{1} a_{2} a_{2} b_{1} b_{1} b_{2} b_{2} \beta$. Hence, the two stages splicing languages are persistent.

In the following theorem, the persistency of two stages DNA splicing languages according to Y-G splicing system consisting two initial strings (with two cutting sites) and two rules with disjoint non-palindromic crossing sites is investigated.

Theorem 1: The set of two stages splicing languages, which is produced by Y-G splicing system consisting two initial strings (with two cutting sites) and two rules with disjoint crossing sites and non-palindromic sequences, is persistent.

Proof: Suppose that $S=(A, I, R)$ be a Y-G splicing system. Thus, the rules $r_{1}, r_{2} \in R$ are presented in the forms $r_{1}=\left(a_{1}, a_{1} a_{2}, a_{2}: a_{1}, a_{1} a_{2}, a_{2}\right)$ and $r_{2}=\left(b_{1}, b_{1} b_{2}, b_{2}: b_{1}, b_{1} b_{2}, b_{2}\right)$, respectivelywhere $a_{1}$ is not complementary with $a_{2}, b_{1}$ is not complementary with $b_{2}$ and vice- versa, $a_{1}, a_{2}, b_{1}, b_{2} \in A^{*}$. Since the patterns have disjoint crossings, therefore the fragments of initial strings after cutting by the existing splicing rules cannot recombine together. According to the number of cutting sites of initial strings four cases happen.

Case 1: $r_{1}$ is used on $s_{1}$ and $r_{2}$ is used on $s_{2}$.
Case 2: $r_{1}$ is used on $s_{1}$ and both $r_{1}$ and $r_{2}$ are used on $s_{2}$
Case 3: both $r_{1}$ and $r_{2}$ are used on $s_{1}$ and $r_{2}$ is used on $s_{2}$.
Case 4: both $r_{1}$ and $r_{2}$ are used on $s_{1}$ and $s_{2}$.

Since the cases of Theorem 1have compatibility with the lemmas that have been proven above, therefore this theorem can be proven by combining the proofs of Lemma 1, Lemma 2, Lemma 3 and Lemma 4, respectively.

The following corollary, which is resulted from Theorem 1, indicates that the set of two stages splicing languages with the above conditions is also permanent.

Corollary 1: The set of two stages splicing languages, that is produced by Y-G splicing system consisting two initial strings (with two cutting sites) and two rules with disjoint crossing sites and non-palindromic sequences, is permanent.

Lemma 5 Presentsthe persistency of two stages DNA splicing languages at the existence of two initial strings (with two cutting sites) and two palindromic rules, where the first rule is used on first initial string and second rule on second initial string.

Lemma 5: If the crossing sites of the rules in a Y-G splicing system be disjoint and palindromic so that the first rule cuts the first initial string (with two cutting sites) and the second rule cuts the second initial string (with two cutting sites) from two specific places, respectively, thenthe set of two stages splicing languages, that is produced by Y-G splicing system, is persistent. $\square$

Proof: Assume $S=(A, I, R)$ be a Y-G splicing system that consists of two initial strings $s_{1}, s_{2} \in I$ and two rules $r_{1}, r_{2} \in R$. Thus, the rules $r_{1}, r_{2} \in R$ are presented in the forms of $r_{1}=\left(a_{1}, a_{1} a_{2}, a_{2}: a_{1}, a_{1} a_{2}, a_{2}\right)$ and $r_{2}=\left(b_{1}, b_{1} b_{2}, b_{2}: b_{1}, b_{1} b_{2}, b_{2}\right)$, respectively where $a_{1}$ is complementary with $a_{2}, b_{1}$ is complementary with $b_{2}$ and vice- versa, $a_{1}, a_{2}, b_{1}, b_{2} \in A^{*}$. To prove the persistency of the generating splicing languages at
two stages with respect to recognition sites of initial strings four cases need be considered. Suppose $s_{1}$ and $s_{2}$ are two initial strings in $I$ such that the string $s_{1}$ can be cut by rule $r_{1}$ and string $s_{2}$ can be cut by rule $r_{2}$ from two specific sequences. Let $s_{1}=\alpha a_{1} a_{1} a_{2} a_{2} a_{1} a_{1} a_{2} a_{2} \beta$ and $s_{2}=\gamma b_{1} b_{1} b_{2} b_{2} b_{1} b_{1} b_{2} b_{2} \delta$ be two initial strings in $I \in A^{*}, \quad$ and $\quad \alpha^{\prime}, \beta^{\prime}, \gamma^{\prime}$ and $\delta^{\prime}$ are complemented of $\alpha, \beta, \gamma$ and $\delta$, respectively. Since the crossings of rules are disjoint, splicing $s_{1}$ itself and $s_{2}$ itself the following six DNA splicing languages will be produced at stage one.
$\alpha a_{1}\left(a_{1} a_{2} a_{2} a_{1}\right)^{k} a_{1} a_{2} a_{2} \beta, \alpha a_{1}\left(a_{1} a_{2} a_{2} a_{1}\right)^{k} a_{1} a_{2} a_{2} \alpha^{\prime}$,
$\beta^{\prime} a_{1}\left(a_{1} a_{2} a_{2} a_{1}\right)^{k} a_{1} a_{2} a_{2} \beta$
$\gamma b_{1}\left(b_{1} b_{2} b_{2} b_{1}\right)^{k} b_{1} b_{2} b_{2} \delta, \gamma b_{1}\left(b_{1} b_{2} b_{2} b_{1}\right)^{k} b_{1} b_{2} b_{2} \gamma^{\prime}$,
$\delta^{\prime} b_{1}\left(b_{1} b_{2} b_{2} b_{1}\right)^{k} b_{1} b_{2} b_{2} \delta$,
where $k \in \mathbb{N}$.

To get the resulted DNA splicing languages of stage two, the rules $r_{1}$ and $r_{2}$ are added on the splicing languages of stage one, the generated DNA splicing languages at stage two are as below.
$\alpha a_{1}\left(a_{1} a_{2} a_{2} a_{1}\right)^{n} a_{1} a_{2} a_{2} \beta, \alpha a_{1}\left(a_{1} a_{2} a_{2} a_{1}\right)^{n} a_{1} a_{2} a_{2} \alpha^{\prime}$,
$\beta^{\prime} a_{1}\left(a_{1} a_{2} a_{2} a_{1}\right)^{n} a_{1} a_{2} a_{2} \beta$,
$\gamma b_{1}\left(b_{1} b_{2} b_{2} b_{1}\right)^{n} b_{1} b_{2} b_{2} \delta, \gamma b_{1}\left(b_{1} b_{2} b_{2} b_{1}\right)^{n} b_{1} b_{2} b_{2} \gamma^{\prime}$,
$\delta^{\prime} b_{1}\left(b_{1} b_{2} b_{2} b_{1}\right)^{n} b_{1} b_{2} b_{2} \delta$,
where $n=k+i, i=0,1,2, \ldots, k$.
To prove that the families of two stages splicing languages are persistent, the pattern with same crossing should be considered. According to definition of persistent by taking $a_{1} a_{2}$ as sub segment of $\alpha a_{1} a_{1} a_{2}$, that is crossing of $\alpha a_{1} a_{1} a_{2} a_{2} a_{1} a_{1} a_{2} a_{2} \beta$. Thus, this sub segment $a_{1} a_{2}$ also contains an occurrence of the crossing of a site in the resulted DNA splicing language $\alpha a_{1}\left(a_{1} a_{2} a_{2} a_{1}\right)^{k} a_{1} a_{2} a_{2} \alpha^{\prime}$ and rest of other above splicing languages that have a pattern of $\left(a_{1}, a_{1} a_{2}, a_{2}: a_{1}, a_{1} a_{2}, a_{2}\right)$. Also, the same method the persistency of the above splicing languages, which have a pattern of $\left(b_{1}, b_{1} b_{2}, b_{2}: b_{1}, b_{1} b_{2}, b_{2}\right)$, is proven.Consequently, the set of two stages splicing languages is persistent.

In Lemma 6, the persistent aspect of two stages DNA splicing languages is discussed at the existence of two initial strings
(with two cutting sites) and two rules, so that first rule is applied on first initial string and both of splicing rules are used on second initial string.

Lemma 6: If the crossing sites of the rules in a Y-G splicing system be disjoint and palindromic so that the first rule cuts the first initial string (with two cutting sites) and both splicing rules cut the second initial string (with two cutting sites) from two specific places, respectively, then the set of two stages splicing languages, that is produced by Y-G splicing system, is persistent. $\square$

Proof: Assume $S=(A, I, R)$ be a Y-G splicing system that consists of two initial strings $s_{1}, s_{2} \in I$ and two rules $r_{1}, r_{2} \in R$. Thus, the rules $r_{1}, r_{2} \in R$ are presented in the forms of $r_{1}=\left(a_{1}, a_{1} a_{2}, a_{2}: a_{1}, a_{1} a_{2}, a_{2}\right)$ and $r_{2}=\left(b_{1}, b_{1} b_{2}, b_{2}: b_{1}, b_{1} b_{2}, b_{2}\right)$, respectively where $a_{1}$ is complementary with $a_{2}, b_{1}$ is complementary with $b_{2}$ and vice- versa, $a_{1}, a_{2}, b_{1}, b_{2} \in A^{*}$. Therefore, for the persistency of the generating splicing languages at two stages with respect to recognition sites of initial strings four cases need be considered. Suppose $s_{1}$ and $s_{2}$ are two initial strings in $I$ that have two cutting sites and the string $s_{1}$ can be cut by $r_{1}$ and string $s_{2}$ can cut by both rules $r_{1}, r_{2} \in R \quad$ from two specific places. Assume $s_{1}=\alpha a_{1} a_{1} a_{2} a_{2} a_{1} a_{1} a_{2} a_{2} \beta$ and $s_{2}=\gamma a_{1} a_{1} a_{2} a_{2} b_{1} b_{1} b_{2} b_{2} \delta$ are two initial strings in $I \in A^{*}, \quad$ and $\alpha^{\prime}, \beta^{\prime}, \gamma^{\prime}$ and $\delta^{\prime}$ are complemented of $\alpha, \beta, \gamma$ and $\delta$, respectively. Applying the rules on initial strings and splicing the fragments of them with their complementary ends, the following thirteen DNA splicing languages will be generated at stage one namely,
$\alpha a_{1}\left(a_{1} a_{2} a_{2} a_{1}\right)^{k} a_{1} a_{2} a_{2} \beta$,
$\alpha a_{1}\left(a_{1} a_{2} a_{2} a_{1}\right)^{k} a_{1} a_{2} a_{2} \alpha^{\prime}$,
$\beta^{\prime} a_{1}\left(a_{1} a_{2} a_{2} a_{1}\right)^{k} a_{1} a_{2} a_{2} \beta, \alpha a_{1}\left(a_{1} a_{2} a_{2} a_{1}\right)^{k} a_{1} a_{2} a_{2} b_{1} b_{1} b_{2} b_{2} \delta$,
$\alpha a_{1}\left(a_{1} a_{2} a_{2} a_{1}\right)^{k} a_{1} a_{2} a_{2} \gamma^{\prime}, \beta^{\prime} a_{1}\left(a_{1} a_{2} a_{2} a_{1}\right)^{k} a_{1} a_{2} a_{2} b_{1} b_{1} b_{2} b_{2} \delta$,
$\gamma a_{1}\left(a_{1} a_{2} a_{2} a_{1}\right)^{k} a_{1} a_{2} a_{2} \beta, \gamma a_{1}\left(a_{1} a_{2} a_{2} b_{1} \cup b_{1} b_{2} b_{2} a_{1}\right)^{k} a_{1} a_{2} a_{2} \gamma^{\prime}$,
$\delta^{\prime} b_{1}\left(b_{1} b_{2} b_{2} a_{1} \cup a_{1} a_{2} a_{2} b_{1}\right)^{k} b_{1} b_{2} b_{2} \delta$,
$\gamma a_{1} a_{1} a_{2} a_{2} b_{1}\left(b_{1} b_{2} b_{2} a_{1} \cup a_{1} a_{2} a_{2} b_{1}\right)^{k} b_{1} b_{2} b_{2} \delta$,
$\alpha a_{1}\left(a_{1} a_{2} a_{2} b_{1} \cup b_{1} b_{2} b_{2} a_{1}\right)^{k} a_{1} a_{2} a_{2} \gamma^{\prime}$,
$\gamma a_{1}\left(a_{1} a_{2} a_{2} a_{1}\right)^{k} a_{1} a_{2} a_{2} b_{1} b_{1} b_{2} b_{2} \delta$,
$\gamma a_{1}\left(a_{1} a_{2} a_{2} b_{1} \cup b_{1} b_{2} b_{2} a_{1}\right)^{k} a_{1} a_{2} a_{2} \beta$,
where $k \in \mathbb{N}$.

To generate the splicing languages of stage two, the resulted DNA splicing languages are spliced using rules $r_{1}$ and $r_{2}$. When splicing occurs the following DNA splicing languages will be formed as listed below.

$$
\begin{aligned}
& \alpha a_{1}\left(a_{1} a_{2} a_{2} a_{1}\right)^{n} a_{1} a_{2} a_{2} \beta, \alpha a_{1}\left(a_{1} a_{2} a_{2} a_{1}\right)^{n} a_{1} a_{2} a_{2} \alpha^{\prime}, \\
& \beta^{\prime} a_{1}\left(a_{1} a_{2} a_{2} a_{1}\right)^{n} a_{1} a_{2} a_{2} \beta, \alpha a_{1}\left(a_{1} a_{2} a_{2} a_{1}\right)^{n} a_{1} a_{2} a_{2} b_{1} b_{1} b_{2} b_{2} \delta, \\
& \alpha a_{1}\left(a_{1} a_{2} a_{2} a_{1}\right)^{n} a_{1} a_{2} a_{2} \gamma^{\prime}, \beta^{\prime} a_{1}\left(a_{1} a_{2} a_{2} a_{1}\right)^{n} a_{1} a_{2} a_{2} b_{1} b_{1} b_{2} b_{2} \delta, \\
& \gamma a_{1}\left(a_{1} a_{2} a_{2} a_{1}\right)^{n} a_{1} a_{2} a_{2} \beta, \gamma a_{1}\left(a_{1} a_{2} a_{2} b_{1} \cup b_{1} b_{2} b_{2} a_{1}\right)^{n} a_{1} a_{2} a_{2} \gamma^{\prime}, \\
& \delta^{\prime} b_{1}\left(b_{1} b_{2} b_{2} a_{1} \cup a_{1} a_{2} a_{2} b_{1}\right)^{n} b_{1} b_{2} b_{2} \delta, \\
& \gamma a_{1} a_{1} a_{2} a_{2} b_{1}\left(b_{1} b_{2} b_{2} a_{1} \cup a_{1} a_{2} a_{2} b_{1}\right)^{n} b_{1} b_{2} b_{2} \delta, \\
& \alpha a_{1}\left(a_{1} a_{2} a_{2} b_{1} \cup b_{1} b_{2} b_{2} a_{1}\right)^{n} a_{1} a_{2} a_{2} \gamma^{\prime}, \\
& \gamma a_{1}\left(a_{1} a_{2} a_{2} a_{1}\right)^{n} a_{1} a_{2} a_{2} b_{1} b_{1} b_{2} b_{2} \delta, \\
& \gamma a_{1}\left(a_{1} a_{2} a_{2} b_{1} \cup b_{1} b_{2} b_{2} a_{1}\right)^{n} a_{1} a_{2} a_{2} \beta, \\
& \text { where } n=k+i, i=0,1,2, \ldots, k .
\end{aligned}
$$

To prove that the families of the above two stages splicing languages are persistent, the pattern with same crossing should be considered. According to definition of persistent by taking $y=a_{1} a_{2} \quad$ as $\quad$ a sub segment of $\alpha a_{1} a_{1} a_{2}$ (respectively $a_{1} a_{2} a_{2} b_{1} b_{1} b_{2} b_{2} \delta$ ), that is crossing of $\alpha a_{1} a_{1} a_{2} a_{2} a_{1} a_{1} a_{2} a_{2} \beta$. Thus, this $y=a_{1} a_{2}$ also contains an occurrence of the crossing of a site in $\alpha a_{1}\left(a_{1} a_{2} a_{2} a_{1}\right)^{k} a_{1} a_{2} a_{2} b_{1} b_{1} b_{2} b_{2} \delta$, as well crossing of the rest of the splicing languages, which have a pattern of $\left(a_{1}, a_{1} a_{2}, a_{2}: a_{1}, a_{1} a_{2}, a_{2}\right)$. By using the same approach the persistency of two stages splicing languages, which have a pattern of $\left(b_{1}, b_{1} b_{2}, b_{2}: b_{1}, b_{1} b_{2}, b_{2}\right)$, is proven. Consequently, the above two stages splicing languages are persistent.

In the next lemma, the persistent point of view of two stages DNA splicing languages is presented at the existence of two initial strings and two palindromic rules, where both of splicing rules are applied on first initial strings and second on second initial string.

Lemma 7: If the crossing sites of the rules in a Y-G splicing system be disjoint and palindromic so that both splicing rules cut the first initial string (with two cutting sites) and the second rule cuts the second initial string (with two cutting sites) from two specific places, respectively, then the set of two stages splicing languages, which is produced by Y-G splicing system, is persistent.

Proof: Assume $S=(A, I, R)$ be a Y-G splicing system that consists the two initial strings $s_{1}, s_{2} \in I$ and two rules $r_{1}, r_{2} \in R$.

Thus, the rules $r_{1}, r_{2} \in R$ are presented in the forms of $r_{1}=\left(a_{1}, a_{1} a_{2}, a_{2}: a_{1}, a_{1} a_{2}, a_{2}\right)$ and $r_{2}=\left(b_{1}, b_{1} b_{2}, b_{2}: b_{1}, b_{1} b_{2}, b_{2}\right)$, respectively where $a_{1}$ is complementary with $a_{2}, b_{1}$ is complementary with $b_{2}$ and vice- versa, $a_{1}, a_{2}, b_{1}, b_{2} \in A^{*}$. Therefore, for the persistency of the generating splicing languages at two stages with respect to recognition sites of initial strings four cases need be considered. Suppose $s_{1}$ and $s_{2}$ are two initial strings in $I$ that have two cutting sites and the string $s_{1}$ can be cut by both rules $r_{1}$ and $r_{2}$ and string $s_{2}$ can cut by rule $r_{2}$ from two specific places. Assume $s_{1}=\alpha a_{1} a_{1} a_{2} a_{2} b_{1} b_{1} b_{2} b_{2} \beta$ and $s_{2}=\gamma b_{1} b_{1} b_{2} b_{2} b_{1} b_{1} b_{2} b_{2} \delta$ be two initial strings. in $I \in A^{*}, \quad$ and $\quad \alpha^{\prime}, \beta^{\prime}, \gamma^{\prime}$ and $\delta^{\prime}$ are complemented of $\alpha, \beta, \gamma$ and $\delta$, respectively. Applying the rules on initial strings and splicing the fragments of them with their complementary ends, the following thirteen DNA splicing languages will be generated at stage one namely,

$$
\begin{aligned}
& \alpha a_{1}\left(a_{1} a_{2} a_{2} b_{1} \cup b_{1} b_{2} b_{2} a_{1}\right)^{k} a_{1} a_{2} a_{2} \alpha^{\prime}, \\
& \beta^{\prime} b_{1}\left(b_{1} b_{2} b_{2} a_{1} \cup a_{1} a_{2} a_{2} b_{1}\right)^{k} b_{1} b_{2} b_{2} \beta, \gamma b_{1}\left(b_{1} b_{2} b_{2} b_{1}\right)^{k} b_{1} b_{2} b_{2} \delta, \\
& \gamma b_{1}\left(b_{1} b_{2} b_{2} b_{1}\right)^{k} b_{1} b_{2} b_{2} \gamma^{\prime}, \delta^{\prime} b_{1}\left(b_{1} b_{2} b_{2} b_{1}\right)^{k} b_{1} b_{2} b_{2} \delta, \\
& \gamma b_{1}\left(b_{1} b_{2} b_{2} b_{1}\right)^{k} b_{1} b_{2} b_{2} \beta, \gamma b_{1}\left(b_{1} b_{2} b_{2} b_{1}\right)^{k} b_{1} b_{2} b_{2} a_{1} a_{1} a_{2} a_{2} \alpha^{\prime}, \\
& \delta^{\prime} b_{1}\left(b_{1} b_{2} b_{2} b_{1}\right)^{k} b_{1} b_{2} b_{2} \beta, \delta^{\prime} b_{1}\left(b_{1} b_{2} b_{2} b_{1}\right)^{k} b_{1} b_{2} b_{2} a_{1} a_{1} a_{2} a_{2} \alpha^{\prime}, \\
& \alpha a_{1} a_{1} a_{2} a_{2} b_{1}\left(b_{1} b_{2} b_{2} b_{1}\right)^{k} b_{1} b_{2} b_{2} \beta, \\
& \beta^{\prime} b_{1}\left(b_{1} b_{2} b_{2} a_{1} \cup a_{1} a_{2} a_{2} b_{1}\right)^{k} b_{1} b_{2} b_{2} \delta, \\
& \beta^{\prime} b_{1}\left(b_{1} b_{2} b_{2} a_{1} \cup a_{1} a_{2} a_{2} b_{1}\right)^{k} b_{1} b_{2} b_{2} \gamma^{\prime}, \\
& \beta^{\prime} b_{1}\left(b_{1} b_{2} b_{2} a_{1} \cup a_{1} a_{2} a_{2} b_{1}\right)^{k} b_{1} b_{2} b_{2} a_{1} a_{1} a_{2} a_{2} \alpha^{\prime}, \\
& \text { where } k \in \mathbb{N} .
\end{aligned}
$$

Since the resulted DNA splicing languages contains the cleavage patterns of splicing rules, if splicing takes place among them the only change that will arise in the DNA splicing languages of stage two is the value of $k \in \mathbb{N}$, and the repeating sequences will have the power $n$, where $n=k+i, i=0,1,2, \ldots, k$.

To prove that the families of the above two stages splicing languages are persistent, the pattern with same crossing should be considered. By taking $b_{1} b_{2}$ as a sub segment of $\gamma b_{1} b_{1} b_{2}$, that is crossing of $\gamma b_{1} b_{1} b_{2} b_{2} b_{1} b_{1} b_{2} b_{2} \delta$. Thus, this $b_{1} b_{2}$ also contains an occurrence of the crossing of a site in $\gamma b_{1}\left(b_{1} b_{2} b_{2} b_{1}\right)^{k} b_{1} b_{2} b_{2} \beta$ , as well crossing of the rest of the splicing languages that have a pattern of $\left(b_{1}, b_{1} b_{2}, b_{2}: b_{1}, b_{1} b_{2}, b_{2}\right)$. By using the same approach the persistency of two stages splicing languages, which have a pattern of $\left(a_{1}, a_{1} a_{2}, a_{2}: a_{1}, a_{1} a_{2}, a_{2}\right)$ is proven.

Consequently, the above two stages splicing languages are persistent.

In the last lemma, the persistency of two stages DNA splicing languages is proven using Y-G approach with respect to two initial string and two palindromic rules, where both of splicing rules are applied on both of initial strings.

Lemma 8: If the crossing sites of the rules in a Y-G splicing system be disjoint and palindromic so that both splicing rules cut both of the initial strings (with two cutting sites) from two specific places, respectively, then the set of two stages splicing languages, that is produced by Y-G splicing system, is persistent. $\square$

Proof: Assume $S=(A, I, R)$ be a Y-G splicing system that consists of two initial strings $s_{1}, s_{2} \in I$ and two rules $r_{1}, r_{2} \in R$. Thus, the rules $r_{1}, r_{2} \in R$ are presented in the forms of $r_{1}=\left(a_{1}, a_{1} a_{2}, a_{2}: a_{1}, a_{1} a_{2}, a_{2}\right)$ and $r_{2}=\left(b_{1}, b_{1} b_{2}, b_{2}: b_{1}, b_{1} b_{2}, b_{2}\right)$, respectively where $a_{1}$ is complementary with $a_{2}, b_{1}$ is complementary with $b_{2}$ and vice- versa, $a_{1}, a_{2}, b_{1}, b_{2} \in A^{*}$. Therefore, for the persistency of the generating splicing languages at two stages with respect to recognition sites of initial strings four cases need be considered. Suppose $s_{1}$ and $s_{2}$ are two initial strings in $I$ such that the strings $s_{1}$ and $s_{2}$ can be cut by both rules $r_{1}$ and $r_{2}$ from two specific sequences. Let $s_{1}=\alpha a_{1} a_{1} a_{2} a_{2} b_{1} b_{1} b_{2} b_{2} \beta$ and $s_{2}=\gamma a_{1} a_{1} a_{2} a_{2} b_{1} b_{1} b_{2} b_{2} \delta$ be two initial strings in $I \in A^{*}, \quad$ and $\quad \alpha^{\prime}, \beta^{\prime}, \gamma^{\prime}$ and $\delta^{\prime}$ are complemented of $\alpha, \beta, \gamma$ and $\delta$, respectively.Apply the rules $r_{1}$ and $r_{2}$ on the initial strings $s_{1}$ and $s_{2}$, the following ten DNA splicing languages will be produced at stage one.

```
\(\alpha a_{1}\left(a_{1} a_{2} a_{2} b_{1} \cup b_{1} b_{2} b_{2} a_{1}\right)^{k} a_{1} a_{2} a_{2} \alpha^{\prime}\),
\(\beta^{\prime} b_{1}\left(b_{1} b_{2} b_{2} a_{1} \cup a_{1} a_{2} a_{2} b_{1}\right)^{k} b_{1} b_{2} b_{2} \beta\),
\(\gamma a_{1}\left(a_{1} a_{2} a_{2} b_{1} \cup b_{1} b_{2} b_{2} a_{1}\right)^{k} a_{1} a_{2} a_{2} \gamma^{\prime}\),
\(\delta^{\prime} b_{1}\left(b_{1} b_{2} b_{2} a_{1} \cup a_{1} a_{2} a_{2} b_{1}\right)^{k} b_{1} b_{2} b_{2} \delta\),
\(\alpha a_{1}\left(a_{1} a_{2} a_{2} b_{1} \cup b_{1} b_{2} b_{2} a_{1}\right)^{k} a_{1} a_{2} a_{2} \gamma^{\prime}\),
\(\beta^{\prime} b_{1}\left(b_{1} b_{2} b_{2} a_{1} \cup a_{1} a_{2} a_{2} b_{1}\right)^{k} b_{1} b_{2} b_{2} \delta\),
\(\gamma a_{1} a_{1} a_{2} a_{2} b_{1}\left(b_{1} b_{2} b_{2} a_{1} \cup a_{1} a_{2} a_{2} b_{1}\right)^{k} b_{1} b_{2} b_{2} \delta\),
\(\alpha a_{1} a_{1} a_{2} a_{2} b_{1}\left(b_{1} b_{2} b_{2} a_{1} \cup a_{1} a_{2} a_{2} b_{1}\right)^{k} b_{1} b_{2} b_{2} \delta\),
\(\alpha a_{1} a_{1} a_{2} a_{2} b_{1}\left(b_{1} b_{2} b_{2} a_{1} \cup a_{1} a_{2} a_{2} b_{1}\right)^{k} b_{1} b_{2} b_{2} \beta\),
\(\gamma a_{1} a_{1} a_{2} a_{2} b_{1}\left(b_{1} b_{2} b_{2} a_{1} \cup a_{1} a_{2} a_{2} b_{1}\right)^{k} b_{1} b_{2} b_{2} \beta\),
```

where $k \in \mathbb{N}$.

Since the yielded DNA splicing languages have the sites two split by the rules $r_{1}$ and $r_{2}$, therefore splicing them via existence rules, produce the following splicing languages at stage two as listed below.

$$
\begin{aligned}
& \alpha a_{1}\left(a_{1} a_{2} a_{2} b_{1} \cup b_{1} b_{2} b_{2} a_{1}\right)^{n} a_{1} a_{2} a_{2} \alpha^{\prime}, \\
& \beta^{\prime} b_{1}\left(b_{1} b_{2} b_{2} a_{1} \cup a_{1} a_{2} a_{2} b_{1}\right)^{n} b_{1} b_{2} b_{2} \beta, \\
& \gamma a_{1}\left(a_{1} a_{2} a_{2} b_{1} \cup b_{1} b_{2} b_{2} a_{1}\right)^{n} a_{1} a_{2} a_{2} \gamma^{\prime}, \\
& \delta^{\prime} b_{1}\left(b_{1} b_{2} b_{2} a_{1} \cup a_{1} a_{2} a_{2} b_{1}\right)^{n} b_{1} b_{2} b_{2} \delta, \\
& \alpha a_{1}\left(a_{1} a_{2} a_{2} b_{1} \cup b_{1} b_{2} b_{2} a_{1}\right)^{n} a_{1} a_{2} a_{2} \gamma^{\prime}, \\
& \beta^{\prime} b_{1}\left(b_{1} b_{2} b_{2} a_{1} \cup a_{1} a_{2} a_{2} b_{1}\right)^{n} b_{1} b_{2} b_{2} \delta, \\
& \gamma a_{1} a_{1} a_{2} a_{2} b_{1}\left(b_{1} b_{2} b_{2} a_{1} \cup a_{1} a_{2} a_{2} b_{1}\right)^{n} b_{1} b_{2} b_{2} \delta, \\
& \alpha a_{1} a_{1} a_{2} a_{2} b_{1}\left(b_{1} b_{2} b_{2} a_{1} \cup a_{1} a_{2} a_{2} b_{1}\right)^{n} b_{1} b_{2} b_{2} \delta, \\
& \alpha a_{1} a_{1} a_{2} a_{2} b_{1}\left(b_{1} b_{2} b_{2} a_{1} \cup a_{1} a_{2} a_{2} b_{1}\right)^{n} b_{1} b_{2} b_{2} \beta, \\
& \gamma a_{1} a_{1} a_{2} a_{2} b_{1}\left(b_{1} b_{2} b_{2} a_{1} \cup a_{1} a_{2} a_{2} b_{1}\right)^{n} b_{1} b_{2} b_{2} \beta, \\
& \text { where } n=k+i, i=0,1,2, \ldots, k .
\end{aligned}
$$

To show the two stages splicing languages are persistent, the pattern with same crossing should be considered. According to definition of persistent if $y=a_{1} a_{2}$ be the sub segment of $\alpha a_{1} a_{1} a_{2}$, that is crossings of $\alpha a_{1} a_{1} a_{2} a_{2} b_{1} b_{1} b_{2} b_{2} \beta$. This sub segments $y=a_{1} a_{2}$ contains an occurrence of the crossing of a site in $\alpha a_{1}\left(a_{1} a_{2} a_{2} b_{1} \cup b_{1} b_{2} b_{2} a_{1}\right)^{k} a_{1} a_{2} a_{2} \alpha^{\prime}$, as well as crossing of the rest of the splicing languages that have a pattern of $\left(a_{1}, a_{1} a_{2}, a_{2}: a_{1}, a_{1} a_{2}, a_{2}\right)$. By using the same approach, the persistency of two stages splicing languages, which have a pattern of $\left(b_{1}, b_{1} b_{2}, b_{2}: b_{1}, b_{1} b_{2}, b_{2}\right)$, is proven. Hence, the set of two stages splicing languages is persistent.

In the next theorem, the persistency of two stages splicing languages is proven with respect to Y-G splicing system consisting two initial strings (with two cutting sites) and two rules with palindromic disjoint crossing sites.

Theorem2: The set of two stages splicing languages, that is produced by Y-G splicing system consisting two initial strings (with two cutting sites) and two rules with disjoint crossing sites and palindromic sequences, is persistent. $\square$
Proof: Assume $S=(A, I, R)$ be a Y-G splicing system that consists the two initial strings $s_{1}, s_{2} \in I$ and two rules $r_{1}, r_{2} \in R$. Thus, the rules $r_{1}, r_{2} \in R$ are presented as $r_{1}=\left(a_{1}, a_{1} a_{2}, a_{2}: a_{1}, a_{1} a_{2}, a_{2}\right)$ and $r_{2}=\left(b_{1}, b_{1} b_{2}, b_{2}: b_{1}, b_{1} b_{2}, b_{2}\right)$,
respectively where $a_{1}$ is complementary with $a_{2}, \quad b_{1}$ is complementary with $b_{2}$ and vice- versa, $a_{1}, a_{2}, b_{1}, b_{2} \in A^{*}$. Therefore, for the persistency of the generating splicing languages at two stages with respect to recognition sites of initial strings four cases need be considered.Based on the number of cutting sites of initial strings four cases need to be considered.

Case 1: $r_{1}$ is used on $s_{1}$ and $r_{2}$ is used on $s_{2}$.
Case 2: $r_{1}$ is used on $s_{1}$ and both $r_{1}$ and $r_{2}$ are used on $s_{2}$
Case 3:both $r_{1}$ and $r_{2}$ are used on $s_{1}$ and $r_{2}$ is used on $s_{2}$.
Case 4: both $r_{1}$ and $r_{2}$ are used on $s_{1}$ and $s_{2}$.

Since the cases of Theorem 2have compatibility with Lemma 5, Lemma 6, Lemma 7 and Lemma 8, respectively, thus the proof of this theorem can exactly be resulted from the above proven lemmas.

The following corollary can be directly resulted from Theorem 2 and definition of permanent, which indicates the set of the above two stages splicing languages is also permanent.

Corollary 2: The set of two stages splicing languages, that is produced by Y-G splicing system consisting two initial strings (with two cutting sites) and two rules with disjoint crossing sites and palindromic sequences, is permanent.

In the following, a biological example is provided concerning Lemma 6, which indicates the validating of the theorem in a real sense.

Example 1:Suppose that $S=(A, I, R)$ is a Y-G splicing system where $_{A}=\{[A / T],[G, C],[C / G],[T / A]\}$ and the set of splicing rule $R$ consists restriction enzymes namely $F s p B \mathrm{I}$ and $P s p L I$, which their cleavage patterns are represented as $5^{\prime} \ldots C$... $T A G .3^{\prime}$ and $5^{\prime} \ldots C \nabla$ GTACG...3',$~ r e s p e c t i v e l y . A s s u m e ~{ }_{s_{1}}=5^{\prime} \ldots M M M C T A G C T A G N N N . . .3^{\prime}$ $3^{\prime} \ldots G C A T G \mathbf{A} \ldots .5^{\prime}, \quad$ respectively.Assume $s_{s_{1}}={ }_{3^{\prime} . . \text { MMMGATCGATCNNN...5 }}$ and $s_{2}=\frac{5^{\prime} \ldots \text { XXXCGTACGCGTACGYYY .... } 3^{\prime}}{3^{\prime} \ldots X X X G C A T G C G C A T G C Y Y Y ~ . . .5 ~} 5^{\prime}$ are two initial DNA fragments in $I$, where $M, N, X, Y \in A$. Therefore, according to Lemma 6, the resulting two stages recombinant DNA strandsare persistent.

## Conclusion

In this paper, the concept of two stages splicing languages is introduced and its persistent and permanent aspects are investigated by providing somelemmas, theorems and corollaries. In the first four lemmas (Theorem 1), the persistency of two stages DNA splicing languages with respect to two initial strings (with two cutting sites) and two rules with disjoint non-palindromic crossing sites are presented. The difference of these lemmas is on applying the rules on initial
strings. In the last four lemmas (Theorem 2), the persistency of two stages DNA splicing languages are investigated based on two initial strings (with two cutting sites) and two rules with palindromic disjoint crossing sites. All in all, if the crossing sites of the rules are disjoint and the whole sequences of the rules are palindromic or only the crossing sites of the rules are non-palindromic, the two stages DNA splicing languages, which are produced by Y-G splicing system, are persistent as well as permanent.

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