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Estimation of Ratio of Two Population Means using Regression Estimators in Presence of Non – Response

Narendra Kumar and K.M. Patel
Department of Statistics, Udai Pratap Autonomous College, Varanasi U.P. INDIA

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Abstract

This paper summarises conventional and alternative estimators of the ratio of two population means of study characters using regression estimator with known population mean of auxiliary character in the presence of non-response. The expression of bias and mean square error of the proposed estimators are derived. After comparison with the corresponding estimators in case of fixed sample size, it is observed that the proposed estimators is always efficient than the existing estimators. An empirical study is carried out to support the theoretical results.

Keywords: Population mean, study character, auxiliary character, ratio estimator, regression estimator, relative bias, mean square error, non-response.

Introduction

Some times, we need estimation of ratio of two population means in the field of agricultural, socio-economic and medical science. For example, in calculation of growth index rate in the children; we need to divide the weight of children by the height of children

The information on auxiliary variables play a significant role to increase the efficiency of the estimators for estimation of ratio of two population means.

Using the information on auxiliary variable, various research works for estimation of ratio of two population means have been done by Singh[1], Tripathi[2,3], Upadhyaya and Singh[4], Srivastava[5,6] *et al* , Singh[7,8] *et al* and Singh and Singh[9].

During conducting the sample survey some times we find that information on all units selected in the sample is not available due to the problem of non-response. To deal with the problem of non-response in mail-survey Hansen and Hurwitz[10] suggested a technique a sub sampling from non-respondents.

Further El-Badry[11] proposed a method to sending several ways of questionnaire by mail. Using Hansen and Hurwitz technique, various research works for estimation of population mean in case of known and unknown population means of auxiliary characters in the presence of non-response have been done by Cochran[12], Rao[13], Khare and Pandey[14].

The estimation of ratio of two population means using auxiliary variables in the presence of non- response have been proposed by Khare and Pandey[15], Khare and Sinha[16,17,18,19],Khare[20,21] *et al.*

The proposed estimators

Let Y_{ip}(i = 1,2), and X_p(p =1,2,.........,N) be the non- negative value of p-th unit of the population on the study characters y_i(i = 1,2) and the auxiliary character x with their population means $\bar{Y}_i$ (i = 1,2), and $\bar{X}$ respectively.

Using Hansen and Hurwitz technique, a sample of size n is drawn from the finite population of size N, which can be considered as divided into two groups such as Response group of size N_1 and Non-response group of size N_2, using simple random sampling without replacement (SRSWOR) technique and $N_1 + N_2$ = N. It is found that in sample of size n(<N), $n_1$units supply information on y_i(i = 1,2) and n_2 refuse to respond. So, we may regard the sample of n_1 respondents as a simple random sample from the response group and the sample of n_2 units as a simple random sample from the non- response group. Let m denote the size of the sub-sample from n_2 non-respondents drawn randomly and is enumerated by direct interview such that k = n_2/m ,(k > 1). Hence, the Hansen – Hurwitz[10] estimator for population mean $\bar{Y}_i$ of study character y_i(i = 1,2) based on (n_1 + m) units is given as

$$\bar{y}_{iw} = \frac{n_1}{n}\bar{y}_{i(1)} + \frac{n_2}{n}\bar{y}_{i(2)m} \qquad (1)$$

Where, $\bar{y}_{i(1)}$ and $\bar{y}_{i(2)m}$ are the sample means of y_i(i = 1,2) characters based on n_1 and m units respectively.

And also the variance of Hansen- Hurwitz[10] unbiased estimator of study characters y_i(i = 1,2) is given by

$V(\bar{y}_{iw}) = (\frac{1}{n} - \frac{1}{N}) S_{yi}^2 + \frac{W2(k-1)}{n} S_{yi(2)}^2$ (2)

Where, $W_2 = \frac{N_2}{N}$, stratum weight of non- responding groups of size N_2.

$S_{yi}^2 = \frac{1}{N-1}\sum_{p=1}^{N}(Y_{ip} - \bar{Y}_i)^2$; the population mean square of charactersy_i(i = 1,2) for entire population.

And, $S_{yi(2)}^2 = \frac{1}{N_2-1}\sum_{p=1}^{N_2}(Y_{ip(2)} - \bar{Y}_{i(2)})^2$; the population mean square of Characters y_i(i = 1,2) for non- responding groups of size N_2.

Similarly, the Hansen – Hurwitz[10] estimator for population mean $\bar{X}$ of auxiliary character x , based on (n_1 + m) units is given as

$\bar{x}_w = \frac{n_1}{n}\bar{x}_1 + \frac{n_2}{n}\bar{x}_{2(m)}$ (3)

Where, $\bar{x}_1$ and $\bar{x}_{2(m)}$ are the sample means of x character based on n_1 and m units respectively.

And also the variance of Hansen- Hurwitz[10] unbiased estimator of auxiliary character x is given by

$V(\bar{x}_w) = (\frac{1}{n} - \frac{1}{N}) S_x^2 + \frac{W2(k-1)}{n} S_{x2}^2$ (4)

Where,

$S_x^2 = \frac{1}{N-1}\sum_{p=1}^{N}(X_p - \bar{X})^2$; the population mean square of characters x for entire population of size N.

And, $S_{x2}^2 = \frac{1}{N_2-1}\sum_{p=1}^{N_2}(X_{p(2)} - \bar{X}_2)^2$; the population mean square of character x for non- responding groups of size N_2.

Let Rw ($= \frac{\bar{y}_{1w}}{\bar{y}_{2w}}$) denotes a conventional estimator for the estimation of ratio of two population means R ($=\frac{\bar{Y}_1}{\bar{Y}_2}$) of study characters y_i(i = 1,2).

In this case, when population mean of auxiliary character x is known, the conventional (S_1) and alternative (S2) estimators for estimation of ratio of two population means in presence of non-response is defined as

$S_1 = Rw\frac{\bar{x}_w}{\bar{X}}$ (5)

and $S_2 = Rw\frac{\bar{x}}{\bar{X}}$ (6)

Where, $\bar{x} = \frac{1}{n}\sum_{p=1}^{n} x_p$;the sample mean of auxiliary character of size n.

Now, the relative bias and mean square error of estimators S1 and S2 is given as

$R.B.(S_1) = R.B.(Rw) + (\frac{1}{n}\text{-}\frac{1}{N})[C_{xy_1} - C_{xy_2}] + \frac{W2(k-1)}{n}[C_{xy_1(2)} - C_{xy_2(2)}]$ (7)

$M.S.E.(S_1) = M.S.E.(Rw) + R^2(\frac{1}{n} - \frac{1}{N})[C_x^2 + 2(C_{xy_1} - C_{xy_2})] + \frac{W2(k-1)}{n}[C_{x(2)}^2 + 2(C_{xy_1(2)} - C_{xy_2(2)})]$ (8)

And, $R.B.(S_2) = R.B.(Rw) + (\frac{1}{n} - \frac{1}{N})[C_{xy_1} - C_{xy_2}]$ (9)

$M.S.E.(S_2) = M.S.E.(Rw) + R^2(\frac{1}{n} - \frac{1}{N})[C_x^2 + 2(C_{xy_1} - C_{xy_2})]$ (10)

Where, $R.B.(Rw) = (\frac{1}{n} - \frac{1}{N})[C_{y_2}^2 - C_{y_1y_2}] + \frac{W2(k-1)}{n}[C_{y_2(2)}^2 - C_{y_1y_2(2)}]$ (11)

And, $M.S.E.(Rw) = R^2\{(\frac{1}{n} - \frac{1}{N})[C_{y_1}^2 + C_{y_2}^2 - 2C_{y_1y_2}] + \frac{W2(k-1)}{n}[C_{y_1(2)}^2 + C_{y_2(2)}^2 - 2C_{y_1y_2(2)}]\}$ (12)

Now, we suggest two different type estimators, first, the conventional estimator (S_{11}), when incomplete information on both study characters y_i(i = 1,2) and auxiliary character x and second ,the alternative estimator(S_{21}), when incomplete information on only study characters y_i(i = 1,2) which are given as

$S_{11} = \frac{\bar{y}_{1w} + b_{xy_1}(\bar{X} - \bar{x}_w)}{\bar{y}_{2w} + b_{xy_2}(\bar{X} - \bar{x}_w)}$ (13)

And, $S_{21} = \frac{\bar{y}_{1w} + b_{xy_1}(\bar{X} - \bar{x})}{\bar{y}_{2w} + b_{xy_2}(\bar{X} - \bar{x})}$ (14)

Where, b_{xy_i} (i =1,2) $= \frac{s_{xy_i}}{s_x^2} = \frac{r_{xy_i}s_x s_{y_i}}{s_x^2}$; the sample regression coefficient of y_i(i =1,2) and x.

$s_x^2 = \frac{1}{n-1}\sum_p^n(x_p - \bar{x})^2$; the sample mean square of characters x based on n units.

$s_{yi}^2 = \frac{1}{n-1}\sum_p^n(y_{ip} - \bar{y}_i)^2$; the sample mean square of characters y_i(i = 1,2) of size n.

And, r_{xy_i} be the sample correlation coefficient between y_i(i =1,2) and x.

Relative Bias and Mean Square Error

For expression of Bias and mean square error, Let us assume $\bar{y}_{1w} = \bar{Y}_1 + \varepsilon_1$; $\bar{y}_{2w} = \bar{Y}_2 + \varepsilon_2$; $\bar{x}_w = \bar{X} + \varepsilon_3$; $\bar{x} = \bar{X} + \varepsilon_4$, $s_{xy_1} = S_{xy_1} + \varepsilon_5$; $s_{xy_2} = S_{xy_2} + \varepsilon_6$; $s_x^2 = S_x^2 + \varepsilon_7$

Such that - $E(\varepsilon_1) = E(\varepsilon_2) = E(\varepsilon_3) = E(\varepsilon_4) = E(\varepsilon_5) = E(\varepsilon_6) = E(\varepsilon_7) = 0$

Now, expressing S_{11} in terms of ε_i 's , we get

$S_{11} = \{(\bar{Y}_1 + \varepsilon_1) + (\frac{S_{xy_1} + \varepsilon_5}{S_x^2 + \varepsilon_7})(-\varepsilon_3)\}/\{(\bar{Y}_2 + \varepsilon_2) + (\frac{S_{xy_2} + \varepsilon_6}{S_x^2 + \varepsilon_7})(-\varepsilon_3)\}$

$=\{\ \bar{Y}_1(1+\frac{\varepsilon_1}{\bar{Y}_1})+\beta_1\ (1+\frac{\varepsilon_5}{S_{xy_1}})\ (1+\frac{\varepsilon_7}{S_x^2})^{-1}(-\varepsilon_3)\}\{\bar{Y}_2(1+\frac{\varepsilon_2}{\bar{Y}_2})+\beta_2\ (1+\frac{\varepsilon_6}{S_{xy_2}})\ (1+\frac{\varepsilon_7}{S_x^2})^{-1}(-\varepsilon_3)\}^{-1}$

Where, $\beta_i=\frac{S_{xy_i}}{S_x^2}$ (i = 1,2)

Expressing using binomial theorem and neglecting 3rd and higher terms , we get

$S_{11}=R\{\ 1+\frac{\varepsilon_1}{\bar{Y}_1}-\frac{\varepsilon_2}{\bar{Y}_2}+(\frac{\beta_2}{\bar{Y}_2}-\frac{\beta_1}{\bar{Y}_1})\varepsilon_3+(\frac{\varepsilon_2^2}{\bar{Y}_2^2}-\frac{\varepsilon_1}{\bar{Y}_1}\frac{\varepsilon_2}{\bar{Y}_2})+(\frac{\beta_2^2}{\bar{Y}_2^2}-\frac{\beta_1}{\bar{Y}_1}\frac{\beta_2}{\bar{Y}_2})\varepsilon_3^2-(\frac{\beta_2}{\bar{Y}_2}-\frac{\beta_1}{\bar{Y}_1})\frac{\varepsilon_3\varepsilon_7}{S_x^2}-(\frac{2\beta_2}{\bar{Y}_2}-\frac{\beta_1}{\bar{Y}_1})\frac{\varepsilon_2\varepsilon_3}{\bar{Y}_2}+\frac{\beta_2}{\bar{Y}_2}\frac{\varepsilon_3\varepsilon_6}{S_{xy_2}}-\frac{\beta_1}{\bar{Y}_1}\frac{\varepsilon_3\varepsilon_5}{S_{xy_1}}+\frac{\beta_2}{\bar{Y}_2}\frac{\varepsilon_1\varepsilon_3}{\bar{Y}_1}\}$

OR, $S_{11}-R=R\{\ \frac{\varepsilon_1}{\bar{Y}_1}-\frac{\varepsilon_2}{\bar{Y}_2}+(\frac{\beta_2}{\bar{Y}_2}-\frac{\beta_1}{\bar{Y}_1})\varepsilon_3+(\frac{\varepsilon_2^2}{\bar{Y}_2^2}-\frac{\varepsilon_1}{\bar{Y}_1}\frac{\varepsilon_2}{\bar{Y}_2})+(\frac{\beta_2^2}{\bar{Y}_2^2}-\frac{\beta_1}{\bar{Y}_1}\frac{\beta_2}{\bar{Y}_2})\varepsilon_3^2-(\frac{\beta_2}{\bar{Y}_2}-\frac{\beta_1}{\bar{Y}_1})\frac{\varepsilon_3\varepsilon_7}{S_x^2}-(\frac{2\beta_2}{\bar{Y}_2}-\frac{\beta_1}{\bar{Y}_1})\frac{\varepsilon_2\varepsilon_3}{\bar{Y}_2}+\frac{\beta_2}{\bar{Y}_2}\frac{\varepsilon_3\varepsilon_6}{S_{xy_2}}-\frac{\beta_1}{\bar{Y}_1}\frac{\varepsilon_3\varepsilon_5}{S_{xy_1}}+\frac{\beta_2}{\bar{Y}_2}\frac{\varepsilon_1\varepsilon_3}{\bar{Y}_1}\}$ (15)

Now, on taking expectation both sides of section (15), we get

Bias ($\boldsymbol{S_{11}}$) = $R[R.B.(Rw)+(\frac{1}{n}-\frac{1}{N})\{(\frac{\beta_2^2}{\bar{Y}_2^2}-\frac{\beta_1}{\bar{Y}_1}\frac{\beta_2}{\bar{Y}_2})S_x^2+(\frac{\beta_1}{\bar{Y}_1}-\frac{2\beta_2}{\bar{Y}_2})\frac{S_{xy_2}}{\bar{Y}_2}+\frac{\beta_2}{\bar{Y}_2}\frac{S_{xy_1}}{\bar{Y}_1}\}+\frac{W2(k-1)}{n}\{(\frac{\beta_2^2}{\bar{Y}_2^2}-\frac{\beta_1}{\bar{Y}_1}\frac{\beta_2}{\bar{Y}_2})S_{x(2)}^2+(\frac{\beta_1}{\bar{Y}_1}-\frac{2\beta_2}{\bar{Y}_2})\frac{S_{xy_2(2)}}{\bar{Y}_2}+\frac{\beta_2}{\bar{Y}_2}\frac{S_{xy_1(2)}}{\bar{Y}_1}\}+(\frac{1}{n}-\frac{1}{N})\ \{(\frac{\beta_1}{\bar{Y}_1}-\frac{\beta_2}{\bar{Y}_2})\frac{\mu_{30}}{S_x^2}+\frac{\beta_2}{\bar{Y}_2}\frac{\mu_{21(2)}}{S_{xy_2}}-\frac{\beta_1}{\bar{Y}_1}\frac{\mu_{21(1)}}{S_{xy_1}}\}$

Relative Bias ($\boldsymbol{S_{11}}$) = R.B.(Rw) $+(\frac{1}{n}-\frac{1}{N})[\rho_{xy_2}C_{y_2}A+\frac{\mu_{30}}{S_x^3}A-(\frac{\mu_{21(1)}}{\bar{Y}_1}-\frac{\mu_{21(2)}}{\bar{Y}_2})/S_x^2]+\frac{W2(k-1)}{n}[-\rho_{xy_2}C_{y_2}AB^2+\rho_{xy_2(2)}C_{y_2(2)}AB+\rho_{xy_2}C_{y_2}A_1B]$ (16)

Now, on squaring and taking expectation both sides after neglecting 3rd and higher terms of section (15), we get

$MSE(S_{11})=E[S11-R]^2$

$=MSE(Rw)-R^2(\frac{1}{n}-\frac{1}{N})A^2+R^2\frac{W2(k-1)}{n}AB[AB-2A_1]$ (17)

Now, expressing S21 in terms of ε_i ‘s , we get

$\boldsymbol{S_{21}}=\{\ \bar{Y}_1(1+\frac{\varepsilon_1}{\bar{Y}_1})+\beta_1\ (1+\frac{\varepsilon_5}{S_{xy_1}})\ (1+\frac{\varepsilon_7}{S_x^2})^{-1}(-\varepsilon_4)\}\{\bar{Y}_2(1+\frac{\varepsilon_2}{\bar{Y}_2})+\beta_2\ (1+\frac{\varepsilon_6}{S_{xy_2}})\ (1+\frac{\varepsilon_7}{S_x^2})^{-1}(-\varepsilon_4)\}^{-1}$

After following the procedure of expression o section (15) using binomial theorem and neglecting 3rd and higher terms , we get

$\boldsymbol{S_{21}}-R=R\{\ \frac{\varepsilon_1}{\bar{Y}_1}-\frac{\varepsilon_2}{\bar{Y}_2}+(\frac{\beta_2}{\bar{Y}_2}-\frac{\beta_1}{\bar{Y}_1})\varepsilon_4+(\frac{\varepsilon_2^2}{\bar{Y}_2^2}-\frac{\varepsilon_1}{\bar{Y}_1}\frac{\varepsilon_2}{\bar{Y}_2})+(\frac{\beta_2^2}{\bar{Y}_2^2}-\frac{\beta_1}{\bar{Y}_1}\frac{\beta_2}{\bar{Y}_2})\varepsilon_4^2-(\frac{\beta_2}{\bar{Y}_2}-\frac{\beta_1}{\bar{Y}_1})\frac{\varepsilon_4\varepsilon_7}{S_x^2}-(\frac{2\beta_2}{\bar{Y}_2}-\frac{\beta_1}{\bar{Y}_1})\frac{\varepsilon_2\varepsilon_4}{\bar{Y}_2}+\frac{\beta_2}{\bar{Y}_2}\frac{\varepsilon_4\varepsilon_6}{S_{xy_2}}-\frac{\beta_1}{\bar{Y}_1}\frac{\varepsilon_4\varepsilon_5}{S_{xy_1}}+\frac{\beta_2}{\bar{Y}_2}\frac{\varepsilon_1\varepsilon_4}{\bar{Y}_1}\}$ (18)

Now, on taking expectation both sides of section (18), we get

Bias ($\boldsymbol{S_{21}}$) = $R[R.B.(Rw)+(\frac{1}{n}-\frac{1}{N})\{(\frac{\beta_2^2}{\bar{Y}_2^2}-\frac{\beta_1}{\bar{Y}_1}\frac{\beta_2}{\bar{Y}_2})S_x^2+(\frac{\beta_1}{\bar{Y}_1}-\frac{2\beta_2}{\bar{Y}_2})\frac{S_{xy_2}}{\bar{Y}_2}+\frac{\beta_2}{\bar{Y}_2}\frac{S_{xy_1}}{\bar{Y}_1}\}+(\frac{1}{n}-\frac{1}{N})\ \{(\frac{\beta_1}{\bar{Y}_1}-\frac{\beta_2}{\bar{Y}_2})\frac{\mu_{30}}{S_x^2}+\frac{\beta_2}{\bar{Y}_2}\frac{\mu_{21(2)}}{S_{xy_2}}\}-\frac{\beta_1}{\bar{Y}_1}\frac{\mu_{21(1)}}{S_{xy_1}}$

Relative Bias ($\boldsymbol{S_{21}}$) = R.B.(Rw) $+(\frac{1}{n}-\frac{1}{N})[\rho_{xy_2}C_{y_2}A+\frac{\mu_{30}}{S_x^3}A-(\frac{\mu_{21(1)}}{\bar{Y}_1}-\frac{\mu_{21(2)}}{\bar{Y}_2})/S_x^2]$ (19)

Now, on squaring and taking expectation both sides after neglecting 3rd and higher terms of section (18), we get

$MSE(\boldsymbol{S_{21}})=E[S21-R]^2$

$=MSE(Rw)-R^2(\frac{1}{n}-\frac{1}{N})A^2$ (20)

Where, $A=\rho_{xy_1}C_{y_1}-\rho_{xy_2}C_{y_2}$, $A_1=\rho_{xy_{1(2)}}C_{y_{1(2)}}-\rho_{xy_{2(2)}}C_{y_{2(2)}}$

$B=\frac{C_{x(2)}}{C_x}$, $C_{x(2)}=\frac{S_{x(2)}}{\bar{X}}$, $C_x=\frac{S_x}{\bar{X}}$, $C_{y_{i(2)}}=S_{y_{i(2)}}/\bar{Y}_i$, $C_{y_i}=S_{y_i}/\bar{Y}_i$ (i = 1,2)

$C_{y_1y_2}=\rho_{y_1y_2}C_{y_1}C_{y_2}$, $C_{y_1y_2(2)}=\rho_{y_1y_2(2)}C_{y_{1(2)}}C_{y_{2(2)}}$,

$C_{xy_i}=\rho_{xy_i}C_xC_{y_i}$, $C_{xy_i(2)}=\rho_{xy_i(2)}C_{x(2)}C_{y_{i(2)}}$ (i =1,2)

$S_{y_1y_2}=\frac{1}{N-1}\sum_{p=1}^{N}(Y_{1p}-\bar{Y}_1)(Y_{2p}-\bar{Y}_2)$

$S_{y_1y_2(2)}=\frac{1}{N_2-1}\sum_{p=1}^{N_2}(Y_{1p(2)}-\bar{Y}_{1(2)})(Y_{2p(2)}-\bar{Y}_{2(2)})$

$S_{xy_i}=\frac{1}{N-1}\sum_{p=1}^{N}(X_p-\bar{X})(Y_{ip}-\bar{Y}_i)$; i = 1,2

$S_{xy_i(2)}=\frac{1}{N_2-1}\sum_{p=1}^{N_2}(X_{p(2)}-\bar{X}_2)(Y_{ip(2)}-\bar{Y}_{i(2)})$; i = 1,2

$\mu_{rs(i)}=\frac{1}{N}\sum_{p=1}^{N}(X_p-\bar{X})^rY_{ip}-\bar{Y}_i)^s$; $0\le(r,s)\le3$ and i = 1,2

Theoretical Comparison

From section (17), the proposed estimator S_{11} is more efficient than estimator Rw iff, $MSE(\boldsymbol{S_{11}})-MSE(Rw)<0$ i.e. $(\frac{1}{n}-\frac{1}{N})A^2-\frac{W2(k-1)}{n}AB[AB-2A_1]>0$

It is obvious, $(\frac{1}{n}-\frac{1}{N})A^2$ be always positive unless $A\neq0$

then, $\frac{W2(k-1)}{n}AB[AB-2A_1]<(\frac{1}{n}-\frac{1}{N})A^2$

or, $B^2-2BA_1/A<f/f^*$

where, $f=(\frac{1}{n}-\frac{1}{N})$ and, $f^*=\frac{W2(k-1)}{n}$

From section(8) and section (17), the proposed estimator S_{11} is more efficient than estimator S_1 iff, $MSE(\boldsymbol{S_{11}})-MSE(\boldsymbol{S_1})<0$

i.e. $-(\frac{1}{n}-\frac{1}{N})A^2+\frac{W2(k-1)}{n}AB[AB-2A_1]<(\frac{l}{n}-\frac{l}{N})[C_x^2+2(C_{xy_1}-C_{xy_2})]+\frac{W2(k-1)}{n}[C_{x(2)}^2+2(C_{xy_1(2)}-C_{xy_2(2)})]$

or, $f^*[(AB-2A_1-C_{x(2)})(AB+C_{x(2)})]<f(A+C_x)^2$

From section (20), the proposed estimator S_{21} is more efficient than estimator Rw iff, MSE(S_{21}) – MSE(Rw) < 0
i.e. $-R^2\left(\frac{1}{n}-\frac{1}{N}\right)A^2 < 0$
It is obvious, $A^2 > 0$ { either, A > 0 or, A < 0

From section(10) and section (20), the proposed estimator S_{21} is more efficient than estimator S_2 iff , MSE(S_{21}) – MSE(S_2) < 0
i.e. $-\left(\frac{1}{n}-\frac{1}{N}\right)A^2 < \left(\frac{1}{n}-\frac{1}{N}\right)[C_x^2 + 2(C_{xy_1} - C_{xy_2})]$
if, $-A^2 < C_x^2 + 2(C_{xy_1} - C_{xy_2})$
or, $(A + C_x)^2 > 0$

An Empirical comparison

For numerical support of above theoretical results, we consider the data which has been used by Khare and Sinha(2007). In this present data, which belong to the data on physical growth of upper socio–economic group of 95 schools going children of Varanasi under an ICMR study, Department of paediatrics, BHU, during 1983-84. The auxiliary and study characters are defined as: y_1 : The height of children in c.m., y_2 : The weight of children in k.g., x : The chest circumference of children in c.m.
the values of parameters related to the study characters y_i (i =1,2) and auxiliary character, when first 25% (i.e. 24 children) units has been considered as non-response units, are given as :

Table-1

N =	95	N_2 =		24		n =	55
$\bar{Y}_1$ =	115.9526	$\bar{Y}_2$ =		19.4968		$\bar{X}$ =	55.8611
C_{y_1} =	0.05146	C_{y_2} =		0.15613		C_x =	0.0586
$C_{Y_{1(2)}}$ =	0.04402	$C_{y_{2(2)}}$ =		0.12075		$C_{x(2)}$ =	0.05402
ρ_{xy_1} =	0.62	$\rho_{y_1y_2}$ =		0.713		ρ_{xy_2} =	0.846
$\rho_{xy_{1(2)}}$ =	0.401	$\rho_{y_1y_{2(2)}}$ =		0.678		$\rho_{xy_{2(2)}}$ =	0.729
$\frac{W2}{Nn}$ =	0.0046	$\left(\frac{1}{n}-\frac{1}{N}\right)$ =		0.0077		R^2 =	35.3699

Table–2
Relative efficiency (in %) of different proposed estimators with respect to Rw for different value of k

Estimator(s)	**1/k**							
	1/5		**1/4**		**1/3**		**1/2**	
	R.E.(%)	**MSE**	**R.E.(%)**	**MSE**	**R.E.(%)**	**MSE**	**R.E.(%)**	**MSE**
Rw	100.00	0.01027	100.00	0.00875	100.00	0.00724	100.00	0.00573
S_1	206.48	0.00497	207.41	0.00422	208.73	0.00347	210.79	0.00272
S_2	128.06	0.00802	134.59	0.00650	145.07	0.00499	164.66	0.00348
S_{11}	221.07	0.00464	226.93	0.00386	235.79	0.00307	250.76	0.00228
S_{21}	136.00	0.00755	145.02	0.00604	160.08	0.00452	190.27	0.00301

Conclusion

After studying the above table, we conclude that the proposed estimators (S_{11} ,S_{21}) are more efficient than (S_1 ,S_2) as well as conventional estimator R_w for all the given values of k.

Also, the efficiencies of the above estimators increase when the values of k decrease.

References

1. Singh M.P**.,** On the estimation of ratio and product of population parameters, *Sankhya,* **B,27**, 321-328 **(1965)**

2. Tripathi T.P., contribution of the sampling theory using multivariate information, Ph.D. thesis submitted to *Punjabi University, Patiyala, India,* **(1970)**

3. Tripathi T.P**.,** A general class of estimators for population ratio, *Sankhya*, **42C,1-2,** 63-75 **(1980)**

4. Upadhyaya L.N. and Singh H.P., A class of estimators using auxiliary information for estimating ratio of two finite population means, *Gujarat Stat. Rev*., **12(2)**, 7 -16 **(1985)**

5. Srivastava S., Rani Srivastava, S.R. and Khare B.B., On generalized chain estimator for ratio and product of two population means using auxiliary characters *A.S.R*., **2(1),** 21-29 **(1988)**

6. Srivastava S. Rani, Srivastava S.R. and Khare B.B., Chain ratio-type estimators for ratio of two population means using auxiliary characters. *Commun. Stat. Theory Math.,* **18(10),** 3917-3926 **(1989)**

7. Singh V.K., Singh, Hari P., Singh, Housila P. and Shukla D., A general class of Chain estimators for ratio and product of two means of population, *Commu. Ststist. Theory-Method*, **23(5)**, 1341-1355 **(1994a)**

8. Singh V.K., Singh, Hari P. and Singh, Housila P., Estimation for ratio and product of two population means in two- phase sampling, *Jour. Ststist. Plan, Inference,* **41,** 163-171 **(1994b)**

9. Singh V.P. and Singh H.P**.:** Chain estimators for population ratio in double sampling, *Aligarh Jour. Statat*., **17 and18**, 85-100 **(1997-98)**

10. Hansen M.H. And Hurwitz W.N., The problem of non-response in sample surveys, *Journal of the American Statistical Association*, **41**, 517-529 **(1946)**

11. EL-Badry M.A., A sampling procedure for mailed questionnaires. *Journal of the American Statistical Association,*.**51**,.209-227**(1956)**

12. Cochran W.G., Sampling techniques, Third edition. *John wiley and sons*, new york **(1977)**

13. Rao P.S.R.S, Ratio estimation with sub-sampling the non-respondents, *Meth.* **12(2)**, 217-230 **(1986)**

14. Khare B.B, Pandey S.K. and Srivastava U., An improved class of two phase sampling estimators for population mean using auxiliary character in presence of non-response. *J. Sci. Res. B.H.U.,* **55**, 151-161(**2011)**

15. Khare B.B. and Pandey S.K., A class of estimators for ratio of two population means using auxiliary character in presence of non-response. *J. Sc. Res. B.H.U.,* **50**, 517-529 **(2000)**

16. Khare B.B. and Sinha R.R., Estimation of the ratio of two population means using auxiliary character with unknown population mean in presence of non-response, *Prog. of Maths. B.H.U.,* **36(1, 2)**, 337-348 **(2002)**

17. Khare B.B. and Sinha R.R., Estimation of finite population ratio using two phase sampling in presence of non-response, *Aligarh J. Stat*., **24**, 43-56 **(2004)**

18. Khare B.B. and Sinha R.R., Estimation of the ratio of two populations means using multi-auxiliary characters in the presence of non-response. Published in *"Statistical Techniques in Life Testing, Reliability, Sampling Theory and Quality Control edited by B.N. Pandey, Narosa Publishing House, New Delhi*, 163-171(**2007)**

19. Khare B.B. and Sinha R.R., Improved class of estimators for ratio of two population means with double sampling the non-respondents. *Statistika, Theory Meth*., **49(3),** 75-82 **(2012)**

20. Khare B.B., Srivastava U. and Kumar. K**.,** Chain type estimators for ratio of two population means using auxiliary characters in the presence of non-response, *J. Sci. Res*., B.H.U., **56**, 183-196 **(2012)**

21. Khare B.B., Srivastava U. And Kumar. K., Improved classes of Chain type estimators for ratio of two population means using two auxiliary characters in the presence of non-response, international, *journal of advance statistics and probability*,**1(3)**,.53-63**(2013)**