Research Journal of Mathematical and Statistical Sciences ISSN 2320–6047
Vol. **2(9),** 20-21, September **(2014)** *Res. J. Mathematical and Statistical Sci.*

Heat Conduction and the $\overline{H}$ - Function

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Available online at: www.isca.in, www.isca.me
Received 6th June 2014, revised 10th July 2014, accepted 21th August 2014

Abstract

This paper will put an insight into an application to certain products containing the $\overline{\text{H}}$ *- Function in boundary value problems. The results established in this paper are general in nature and hence encompass several cases of interest.*

Keyword: $\overline{\text{H}}$ - Function, heat conduction, boundary value problem.

Introduction

The $\overline{H}$ - Function is defined and represented in the following manner[1].

$$\overline{H}{}^{m,n}_{p,q}\left[z\right]=\overline{H}{}^{m,n}_{p,q}\left[z\left|\begin{matrix}\left(a_j,\alpha_j;A_j\right)_{1,n}, & \left(a_j,\alpha_j\right)_{n+1,p}\\ \left(b_j,\beta_j;B_j\right)_{1,m}, & \left(b_j,\beta_j\right)_{m+1,q}\end{matrix}\right.\right]$$

$$=\frac{1}{2\pi i}\int_L z^{\xi}\,\overline{\phi}\left(\xi\right)d\xi\left(z\neq 0\right)$$
(1)

Where,

$$\overline{\phi}\left(\xi\right)=\frac{\prod_{j=1}^{m}\Gamma\left(b_j-\beta_j\xi\right)\prod_{j=1}^{n}\left\{\Gamma(1-a_j+\alpha_j\xi)\right\}^{A_j}}{\prod_{j=m+1}^{q}\left\{\Gamma(1-b_j+\beta_j\xi)\right\}^{B_j}\prod_{j=n+1}^{p}\Gamma\left(a_j-\alpha_j\xi\right)}$$

The nature of contour L, sufficient conditions of convergence of defining integral (1) and the behavior and other details about the $\overline{H}$ - Function can be seen in paper[1-5]

The following formulas will be required in our investigation:

$$\int_0^{\pi}(\sin x)^{\rho-1}\cos nx dx=\frac{\pi\cos\frac{n\pi}{2}\Gamma\beta}{2^{\rho-1}\Gamma(\frac{\beta+n+1}{2})(\frac{\rho-n+1}{2})} \tag{2}$$

Main Result

Theorem: Prove that $\int_0^{\pi}(\sin x)^{\rho-1}\cos nx$

$$\overline{H}{}^{m\ n}_{p\ q}\left[z(\sin x)^{\lambda}\left|\begin{matrix}(a_j,\alpha_j;A_j)_{1,n},(a_j,\alpha_j)_{n+1,p}\\(b_j,\beta_j;B_j)_{1,m},(b_j,\beta_j)_{m+1,q}\end{matrix}\right.\right]dx\,\frac{\pi\cos\frac{n\pi}{2}}{2^{\rho-1}}=$$

$$\overline{H}{}^{m\ n}_{p\ q}\left[z^{2-\lambda}\left|\begin{matrix}(1-\rho,\lambda),(a_j,\alpha_j;A_j)_{1,n},(a_j,\alpha_j)_{n+1,p}\\(b_j,\beta_j;B_j)_{1,m},(b_j,\beta_j)_{m+1,q},(\frac{1}{2}-\frac{\rho}{2}+\frac{n}{2},\frac{\lambda}{2})\end{matrix}\right.\right]dx \tag{3}$$

Provided $\text{Re}(\rho)>0,\lambda\geq 0$

$$\left|\arg z\right|<\frac{1}{2}\mu_1\pi$$

where μ_1 is given by equation

$$\mu_1=\sum_{j=1}^{m}\left|B_j\right|+\sum_{j=1}^{q}\left|b_jB_j\right|-\sum_{j=1}^{n}\left|a_jA_j\right|-\sum_{j=n+1}^{q}\left|A_j\right|>0,0<\left|z\right|<\infty$$

Proof of the Theorem: To obtain the result (3) we first express $\overline{\text{H}}$ function occurring on the L.H.S. of equation (3) in terms of contour integral using equation (1) and Interchanging the order of integration and summation then we apply the formula (2) and interpret the resulting contour integral as $\overline{\text{H}}$ function we arrive at the right hand side of (3) after a little simplification.

A Boundary Value Problem

We consider a problem on heat conduction in a square plate under certain boundary conditions. If a square plate has its faces and its edges x = 0 and x = π (0<y<π) insulated. Its edges y = 0 and y = π (0<y<π) are kept at tem

Peratures zero and f(x) respectively, then its steady temperature u(x,y) is given by Churchill[6].

Now, we shall consider the problem of determining $u(x, y)$, where

$$u(x,y) = \frac{a_0}{2\pi} y + \sum_{n=1}^{\infty} a_n \frac{\sinh xy}{\sinh nx} \cos nx \tag{4}$$

$$a_n = \frac{2}{\pi} \int_0^{\pi} f(x) \cos nx dx \tag{5}$$

n = 0, 1, 2…..

We shall consider the problem of determining $u(x, y)$, where

$$u(x,o) = f(x) = (\sin x)^{\rho-1}$$

$$\overline{H}{}_{p\ q}^{m\ n}\left[z(\sin x)^{\lambda} \left| \begin{matrix} (a_j,\alpha_j;A_j)_{1,n},(a_j,\alpha_j)_{n+1,p} \\ (b_j,\beta_j;B_j)_{1,m},(b_j,\beta_j)_{m+1,q} \end{matrix} \right. \right] \tag{6}$$

Solution of the problem

Combining (6) and (7) and making the use of the integral (3), we derive

$$a_n = 2^{2-\rho} \cos(\frac{n\pi}{2})$$

$$\overline{H}{}_{p\ q}^{m\ n}\left[z^{2-\lambda} \left| \begin{matrix} (1-\rho,\lambda),(a_j,\alpha_j;A_j)_{1,n},(a_j,\alpha_j)_{n+1,p} \\ (b_j,\beta_j;B_j)_{1,m},(b_j,\beta_j)_{m+1,q},(\frac{1}{2}-\frac{\rho}{2}-\frac{n}{2},\frac{\lambda}{2}),(\frac{1}{2}-\frac{\rho}{2}+\frac{n}{2},\frac{\lambda}{2}) \end{matrix} \right. \right] \tag{7}$$

Putting the value of a_n from (7) in (4), we get the following required solution of the problem

$$u(x,y) = \frac{a_0}{2\pi} y + \sum_{n=1}^{\infty} \frac{\cos\frac{n\pi}{2}}{2^{\rho-2}} \frac{\sinh xy}{\sinh nx} \cos nx$$

$$\overline{H}{}_{p\ q}^{m\ n}\left[z^{2-\lambda} \left| \begin{matrix} (1-\rho,\lambda),(a_j,\alpha_j,A_j)_{1,n},(a_j,\alpha_j)_{n+1,p} \\ (b_j,\beta_j,B_j)_{1,m},(b_j,\beta_j)_{m+1,q},(\frac{1}{2}-\frac{\rho}{2}-\frac{n}{2},\frac{\lambda}{2}),(\frac{1}{2}-\frac{\rho}{2}+\frac{n}{2},\frac{\lambda}{2}) \end{matrix} \right. \right] \tag{8}$$

Provided the condition stated with (3) are satisfied.

Conclusion

It may be pointed out here that the $\overline{H}$ function is very general in nature and a fruitful nature of $\overline{H}$ has its particular cases a number of important functions can also be obtained but we do not record them here.

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