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Some Fixed Point Theorem for Occasionally Weakly Compatible Mapping in Fuzzy Metric Spaces

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Abstract

This paper presents some common fixed point theorem for occasionally weakly compatible mapping in fuzzy metric space.

Keywords: Common fixed point, Fuzzy metric space, occasionally weakly compatible maps and implicit relation.

Introduction

Zadeh[1] L.A introduced the concept of fuzzy sets in 1965. I. Kramosil and J.Michalek[2] introduced Fuzzy metric space. The notion of compatible maps was introduced by G.Jungck[3] for a pair of self mapping. Application of fuzzy set theory played an important role in all discipline of engineering like industrial, robotics, computer, nuclear, civil, electrical, mechanical etc. Some mathematician A. George and P. Veeramani[4], P.V.Balasubrahmanyam[5], B.E. Rhoades[6], B. Schweizer and A.Sklar[7], B. Singh and S.Jain[8], C.T. Aage and J.N.Salunke[9], I.Altun and Turkoglu[10], M. Grabiec[11], M.Imdadm[12], O. Kaleva and S.Seikkala[13], S. Chauhan and B.D.Pant[14], S. Sharma and B. Deshpande[15], Z. K. Deng[16], S.N. Mishra, N. Sharma and S.L. Singh[17], have done a good job in the research of fuzzy mathematics. M.A. Al-Thagafi and N. Shahzad[18] introduced the concept of occasionally weakly compatible maps. Common fixed point theorems for occasionally weakly compatible maps in fuzzy metric spaces were proved by M.A. Khan and Sumitra[19]. Some fixed point theorems for compatible mappings satisfying an implicit relation proved by V.Popa[20]. In this paper a fixed point theorem has been established the concept of occasionally weakly compatible map which generalized the results of some standard results on fuzzy metric space.

Preliminaries

Definition: If $([0,1],*)$ is an abelian topological monoid with one unit such that $x*y\le z*u,$ when $x\le z$ and $y\le u\ \forall x,y,z,u\in[0,1]$ then the binary operation $*:[0,1]\times[0,1]\to[0,1]$ is called a continuous triangular norm[2].

Definition: I If X is any arbitrary set, $*$ is a continuous triangular- norm and M be a fuzzy set on $X^2\times[0,\infty)$ which satisfies the condition given below, then 3-tuples $(\mathrm{X},\mathrm{M},*)$ is known as fuzzy metric space[2], such that:

Condition: For every a,b in $X \Rightarrow M(\mathrm{a},\mathrm{b},0)=0$,
Condition: For every a,b,c in X such that $M(a,b,t)=M(b,a,t),$
Condition: If $t>0$ and a,b in X then $M(\mathrm{a},\mathrm{b},\mathrm{t})=1$,
Condition: For left continuity satisfies $M(a,b,\cdot):[0,\infty)\to[0,1],$
Condition: $\forall a,b,c\in X$ and $t,s>0;$ $M(a,b,s)*\mathrm{M}(b,c,t)\le M(a,c,s+t).$

Example: Suppose that (X,d) said to be a metric space[4].

Define $x*y=\min\{x,y\}$ and $M_d(a,b,t)=\dfrac{t}{(t+d(\mathrm{a},b))}$ for all $a,b\in X$, $t>0$.

Therefore $(\mathrm{X},\mathrm{M},*)$ is a fuzzy metric space. It is called the fuzzy metric space induced by the metric d denoted as the standard fuzzy metric.

Definition: Suppose $(X,M,*)$ be a fuzzy metric space[4] with continuous triangular norm $*$.
A sequence $\{\mathrm{a}_n\}$ in X is said to be:

Convergent to a in X if for each ∈ >0 and each t > 0, there exists $p_o\in N$ such that $M(a_n,a,t)>1-\varepsilon$ for all $p\ge p_0$, that is $M(a_n,a,t)\to 1$ as $n\to\infty\ \forall t>0,$

Cauchy sequence if for each $\varepsilon > 0$ and $t > 0$, $\exists\ p_0 \in N$ such that $M(p_n, q_m, t) > 1 - \varepsilon\ \forall p, q \ge p_0\ M(a_n, a_m, t) > 1$ as $p, q \to \infty\ \forall t > 0,$

If Cauchy sequence convergent. We say that a fuzzy metric space is completed.

Definition: Suppose that F and G maps from a fuzzy metric space $(X, M, *)$ into itself then mapping F and G are said to compatible[6] if $\lim_{n\to\infty} M(FGa_n, GFa_n, t) = 1\ \forall\ \ t > 0,$ whenever $\{a_n\}$ is a sequence in X such that $\lim_{p\to\infty} Fa_n = \lim_{p\to\infty} Ga_n = c$ for some $c \in X$.

Definition: Suppose that pair (F, G) of self –map in a fuzzy metric space is said to be weak compatible[6] if F and G commute at their coincidence points.
i.e. $\forall\ \ a \in X$ If $Fa = Ga \Rightarrow FGa = GFa.$

Definition: Suppose that ξ denote the sets of all real and continuous function from $\xi : (R^+)^6 \to R$ such that:
ξ is non-increasing in arguments $t_2, t_3, t_4, t_5,\ t_6$.
$\exists\ \ a, b \ge 0$ Such that $\xi(a, b, b, b, b, b) \ge 0 \Rightarrow a \ge b$.

Lemma: In fuzzy metric space[17] condition if $\forall\ t > 0$ and $a, b \in X$ such that $M(a, b, t) \le M(a, b, qt)$ implies $a = b.$

Lemma: Suppose two maps $F : X \to X, G : X \to X$ are OWC $\exists$ a point a in X implies $Fa = Ga = v$ therefore v is the unique common fixed point of F and G .

Main Results

Theorem: Let $(X, M, *)$ be the complete fuzzy metric space and suppose that F, G, K and L be self mapping of X . Let the pair (F, G) and (K, L) are occasionally weakly compatible maps where $t > 0$ and $q \in (0,1)$ such that:

$$\xi\begin{bmatrix} M(Fx, Ky, qt), M(Ky, Ly, t), M(Fx, Ly, t), \\ M(Fx, Gx, t), M(Gx, Ly, t), M(Ky, Gx, t) \end{bmatrix} \ge 0 \quad \text{(A)}$$

Then F, G, K and L are having a unique common fixed point in X .

Proof: The pair (F, G) and (K, L) are occasionally weakly compatible maps $\exists$ point a and b in X such that $Fa = Ga, FGa = GFa$ and $Kb = Lb, KLb = LKb$.
We show that $Fa = Kb$

We put $x = a$ and $y = b$ in theorem condition (A)

$$\xi\begin{bmatrix} M(Fa, Kb, qt), M(Kb, Lb, t), M(Fa, Lb, t), \\ M(Fa, Ga, t), M(Ga, Lb, t), M(Kb, Ga, t) \end{bmatrix} \ge 0$$

$$\xi\begin{bmatrix} M(Fa, Kb, qt), M(Kb, Kb, t), M(Fa, Kb, t), \\ M(Fa, Fa, t), M(Fa, Kb, t), M(Kb, Fa, t) \end{bmatrix} \ge 0$$

$$\xi\begin{bmatrix} M(Fa, Kb, qt), 1, M(Fa, Kb, t), 1, \\ M(Fa, Kb, t), M(Fa, Kb, t) \end{bmatrix} \ge 0$$

ξ is decreasing in second and fourth argument

$$\xi\begin{bmatrix} M(Fa, Kb, qt), M(Fa, Kb, t), M(Fa, Kb, t), \\ M(Fa, Kb, t), M(Fa, Kb, t), M(Fa, Kb, t) \end{bmatrix} \ge 0$$

By the definition of implicit relation

$$M(Fa, Kb, qt) \ge M(Fa, Kb, t)$$

By using lemma

We get $Fa = Kb$

Therefore $Fa = Kb = Ga = Lb$
Hence $v = Fa = Ga$ is the unique point of coincidence for F and G . Using lemma, F and $G,$ have $v,$ as unique common fixed point.
i.e. $Fv = Gv = v$

Similarly there is unique point $c \in X \Rightarrow c = Kc = Lc$
We show that $v = c$
We put $x = v$ and $y = c$ in theorem condition (A)

$$\xi\begin{bmatrix} M(Fv, Kc, qt), M(Kc, Lc, t), M(Fv, Lc, t), \\ M(Fv, Gv, t), M(Gv, Lc, t), M(Kc, Gv, t) \end{bmatrix} \ge 0$$

$$\xi\begin{bmatrix} M(v, c, qt), M(c, c, t), M(v, c, t), \\ M(v, v, t), M(v, c, t), M(c, v, t) \end{bmatrix} \ge 0$$

$$\xi\left[M(v, c, qt), 1, M(v, c, t), 1, M(v, c, t), M(v, c, t)\right] \ge 0$$

ξ is decreasing second and fourth argument

$$\xi\begin{bmatrix} M(v, c, qt), M(v, c, t), M(v, c, t), \\ M(v, c, t), M(v, c, t), M(v, c, t) \end{bmatrix} \ge 0$$

By the definition of implicit relation

$$M(v,c,\mathrm{qt}) \geq \mathrm{M}(v,c,\mathrm{t})$$

We use lemma

We get $v = c$

Hence v is a unique common fixed point of F, G, K and L in X .

Corollary: Suppose that F, K and G are three self maps on fuzzy metric space $(X, M, *)$ satisfy pair (F, G) and (K, G) occasionally weakly compatible maps $q \in (0,1)$ and for any $a, b \in X$ such that:

$$\xi\begin{bmatrix} M(Fa,\mathrm{Kb},\mathrm{qt}), M(\mathrm{Kb},Gb,t), \mathrm{M}(Fa,Gb,\mathrm{t}), \\ \mathrm{M}(Fa,Ga,\mathrm{t}), M(Ga,Gb,\mathrm{t}), \mathrm{M}(\mathrm{Kb},Ga,\mathrm{t}) \end{bmatrix} \geq 0$$

Then $F, K,$ G have common fixed point in X .

Conclusion

Main results generalized and improve result of Altun and Turkoglu[10]. This result is proved for occasionally weak compatible maps for four self maps on fuzzy metric space. Corollary extended and improve version of theorem. In particular case, If we take two, three and four self maps on fuzzy metric space then fixed point theorem can be easily drive. We conclude that each occasionally weak compatible map is not necessarily compatible.

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