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# A Probability Model for Child Mortality in a Family

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## Abstract

*Mortality is one of the third components of Demographic events. Currently in the developing nations the force of child mortality is very high which play a dominant role in determining the growth of population. The aim of present work is to develop a probability model for the child mortality under the certain assumption techniques. The suitability of model tested through observed set of data.*

**Keywords:** Mortality, Probability Model, Estimation Technique.

## Introduction

Mortality is one of the three components of population change, the other two being fertility and migration. Historically, the factor of mortality has played a dominant role in determining the growth of population, the size of which fluctuated in the past mainly in response to variations in mortality. The increase in the population of European countries following the Industrial Revolution in the seventeenth century was mainly due to a decline in the death rates. The developing countries, which are undergoing a typical demographic transition, have also been affected initially by the fall in the death rates. In fact, the single most important contribution of demography has been the revelation of the fact the sharp declines in mortality rates, rather than my rise in the fertility rates, have been responsible for bringing about a rapid growth of population.

The study of child mortality is useful for analysing future demographic condition as well as determining the future prospects of potential changes in mortality condition. Social Scientist, Statistician is studies the child mortality in respect of formulation, implication and evaluation of the future public health programme.

Considerable interest has been shown in the past by several researchers to measure the levels of child mortality. Currently in the Developing Nations, the force of child mortality is still high at the younger ages particularly during the infancy. Infant and child mortality remain disturbingly high in developing countries despite the significant decline in most part of the developed world. The state of the world's children indicated that about 12.9 million children die every year in the developing world (UNICEF 1987). Mortality for infants and child under the age of 5 years are expressed as the number of deaths in a given period. Infant mortality is defined as death during the first year of life and child mortality as that between the first and fifth birthdays. The deaths during childhood suffer from substantial degree of errors. Usually errors occurs due to recall laps which result in omission of events, misplacement of deaths and the distortion of reports on the duration of vital events.

In demography Child Mortality are useful as a sensitive Index of a Nation's Health Condition's and as guided for the structuring of Public Health Programmes. Child Mortality is interrelated to social, cultural, Economic, Physiological and other factor. The high rate of infant and child mortality shows a low- level development of the health programme and also for the Nation's. Infant and child mortality has been of interest o researchers and demographers because of its apparent relationship with fertility and indirect relationship with the acceptance of modern contraceptive method's (Kabir and Amir).

Some effects have been made to estimate to the current levels of child mortality by using data available from the different survey and other specific sources. Hill and Devid[1] have suggested and approach for estimating child mortality for all births which have taken place in last five years before the survey. However, the estimate obtained through this method also suffers from the problem of under reporting Pathak[2] et al. In this Circumstances, some of the earlier studies about child mortality by using model Chauhan[3], Goldblatt[4], Heligman and Pollard[5], Krishna[6], Ronald and Carter[7], Thiele[8], Keyfitz[9] used a hyperbolic function to study the infant and child mortality. Later Arnold[10] used Pareto distribution and Krishnan and Yin[11] applied finite range model for the same.

The main objective of this chapter is to develop probability model for the child mortality pattern, Parameter of the proposed model has been estimated and suitability of model is tested through survey data.

## Probability Model

Let x denote the number of child deaths in a family at the survey point. Then the distribution of x is derived under the following assumption. i. Only those families are considered in which at least one birth prior to the survey has occurred. ii. At the survey point, a family either has experienced a child loss or not. Let $\alpha$ and ($1-\alpha$) be the respective proportions. iii. Out of $\alpha$ proportion of families, let $\beta$ proportion of families in which only one child death has occurred. iv. Remaining $(1-\beta)\alpha$ proportion of families, experiencing multiple child deaths, follows a displaced Poisson distribution with parameter according to the number of child deaths.

Under these assumptions the probability distribution of X is given by

$$\left.\begin{array}{ll} P[X=0]=1-\alpha & , K=o \\ P[X=1]=\alpha\beta & , K=1 \\ P[X=k]=\dfrac{(1-\beta)\alpha\left[\theta^{k-1}(e^{\theta}-1)^{-1}\right]}{(k-1)!} & , K=2,3,\ldots.. \end{array}\right\}$$

## Estimation

**Method of Moment:** The present model consists three parameters $\alpha, \beta$ and $\theta$. The parameters $\alpha$, $\beta$ and $\theta$ are estimated by equation gzero$^{th}$, First cell theoretical frequencies to observed frequencies, and theoretical mean to observed mean. Pandey[12].

$$\frac{f_0}{f}=1-\alpha \quad (4.2)$$

$$\frac{f_1}{f}=\alpha\beta \quad (4.3)$$

$$\overline{X}=\alpha\beta+(1-\beta)\alpha\{\theta e^{\theta}(e^{\theta}-1)^{-1}+1\} \quad (4.4)$$

Where,
$f_0$= Number of observations in zero$^{th}$ cell, $f_1$= Number of observation in first cell, f = Total number of observations, $\overline{X}$= Observed mean.

## Maximum Likelihood Method

Consider a sample consisting of N observation of random variable X with probability function (I) in which $f_0$ designates the number of Zero observation; $f_1$ the number of one observation and $f_2$ the number of second observation and the total number of observations. The values chosen as estimates and those which maximise the expression is as follow,

$$P(x_1, x_2, \ldots\ldots\ldots x_f, \alpha, \beta, \theta)=(1-\alpha)^{f_0}(\alpha\beta)^{f_1}\left[(1-\beta)\alpha\theta\left(e^{\theta}-1\right)^{-1}\right]^{f_2}\left[\alpha\left\{1-\beta-(1-\beta)\theta\left(e^{\theta}-1\right)^{-1}\right\}\right]^{f-f_0-f_1-f_2} \quad (4.5)$$

Now, taking logarithmic of above equation and partially differentiating w.r.to $\alpha, \beta$ and $\theta$ in turn, and equating to zero yields the estimating equating:

$$\frac{\partial \log l}{\partial\alpha}=\frac{-f_0}{(1-\alpha)}+\frac{f_1}{\alpha}+\frac{f_2}{\alpha}+\frac{f-f_0-f_1-f_2}{\alpha}=0 \quad (4.6)$$

$$\frac{\partial \log l}{\partial\beta}=\frac{f_1}{\beta}-\frac{f_2}{1-\beta}-\frac{f-f_0-f_1-f_2}{1-\beta}= \quad (4.7)$$

$$\frac{\partial \log l}{\partial\theta}=f_2[-(e^{\theta}-1)^{-1}e^{\theta}+1]-\frac{(f-f_0-f_1-f_2)(e^{\theta}-1)^{-1}[-(e^{\theta}-1)^{-1}e^{\theta}\theta+1]}{[1-\theta(e^{\theta}-1)^{-1}]}=0 \quad (4.8)$$

A solution of equation provides the estimator of $\alpha$ as:

$$\alpha=\frac{f-f_0}{f}$$

$$\theta(e^{\theta}-1)^{-1}=\frac{f_2}{f-f_0-f_1}$$

The second Partial derivative of Log L can be obtained as

$$\frac{\partial^2 \log L}{\partial\alpha^2}=\frac{-f_0}{(1-\alpha)^2}-\frac{f_1}{\alpha^2}-\frac{f-f_0-f_1-f_2}{\alpha^2} \quad (4.9)$$

$$\frac{\partial^2 \log l}{\partial\beta^2}=\frac{f_1}{\beta^2}-\frac{f_2}{(1-\beta)^2}-\frac{f-f_0-f_1-f_2}{(1-\beta)^2} \quad (4.10)$$

(4.1)

In case of $\theta$ we have taken approximation of at one place up to three terms and then partially differentiating of log we obtained as follows

$$\frac{\partial^2 \log L}{\partial\theta^2}=-f_2(e^{\theta}-1)^{-1}\{1--(e^{\theta}-1)^{-1}\}-\frac{f_2}{\theta^2}-\frac{(f-f_0-f_1-f_2)\left[\frac{3\theta^2}{4}+\theta\right]}{\left[\frac{\theta^3}{4}+\frac{\theta^3}{2}\right]^2} \quad (4.11)$$

Now,

$$\frac{\partial^2 \log L}{\partial\alpha\,\partial\theta}=\frac{\partial^2 \log L}{\partial\theta\,\partial\alpha}=0 \quad (4.12)$$

$$\frac{\partial^2 \log L}{\partial\beta\,\partial\theta}=\frac{\partial^2 \log L}{\partial\theta\,\partial\beta}=0 \quad (4.13)$$

$$\frac{\partial^2 \log L}{\partial\alpha\,\partial\beta}=\frac{\partial^2 \log L}{\partial\beta\,\partial\alpha}=0 \quad (4.14)$$

Using the fact we gate,

$$E(f_0)=f(1-\alpha)$$

$$E(f_1)=f\alpha\beta$$

$$E(f_2)=f(1-\beta)\alpha(e^{\theta}-1)^{-1}\theta$$

$$E(f-f_0-f_1-f_2)=f\propto\{1-\beta-(1-\beta)\theta\}(e^{\theta}-1)^{-1}$$

Where E is denoted for the expectation.

The expected value of second partial derivatives of Log L can be obtained by using three different cases as.

**Case 1**: When $\beta$ is taking known from the method of moment then

$$\emptyset_{11}=E\left(\frac{-\partial^2 \log L}{\partial\alpha^2}\right)/f=\left[\frac{1}{1-\alpha}+\frac{1}{\alpha}\right] \quad (4.15)$$

$$\emptyset_{22}=E\left(\frac{-\partial^2 \log L}{\partial\theta^2}\right)/f=(1-\beta)\alpha\theta e^{\theta}\left(e^{\theta}-1\right)^{-2}\left\{1-e^{\theta}\left(e^{\theta}-1\right)^{-1}\right\}$$

$$+\frac{(1-\beta)\alpha(e^{\theta}-1)^{-1}}{\theta}+\frac{\alpha\left\{1-\beta-(1-\beta)\theta(e^{\theta}-1)^{-1}\right\}\left[\frac{3\theta^2}{4}+\theta\right]}{\left[\frac{\theta^3}{4}+\frac{\theta^2}{2}\right]^2} \quad (4.16)$$

$$\emptyset_{12}=\emptyset_{21}=\frac{E\left[\frac{-\partial^2\log L}{\partial\alpha\,\partial\theta}\right]}{f}=0 \quad (4.17)$$

$$V(\hat{\alpha}) = \frac{1}{f}\left[\frac{\emptyset_{22}}{\emptyset_{11}\emptyset_{22}-\emptyset_{12}{}^2}\right] \quad (4.18a)$$

$$V(\hat{\theta}) = \frac{1}{f}\left[\frac{\emptyset_{11}}{\emptyset_{11}\emptyset_{22}-\emptyset_{21}{}^2}\right] \quad (4.18b)$$

**Case 2:** When θ is taking known from the method of moment then

$$\emptyset_{11}=E\left(\frac{-\partial^2\log L}{\partial\alpha^2}\right)/f=\left[\frac{1}{1-\alpha}+\frac{1}{\alpha}\right] \quad (4.19)$$

$$\emptyset_{22}=E\left(\frac{-\partial^2\log L}{\partial\theta^2}\right)/f=\left[\frac{1}{\beta}+\frac{1}{1-\beta}\right] \quad (4.20)$$

$$\text{And}=\emptyset_{21}=E\left(\frac{-\partial^2\log L}{\partial\alpha\,\partial\theta}\right)/f=0 \quad (4.21)$$

$$V(\hat{\alpha}) = \frac{1}{f}\left[\frac{\emptyset_{22}}{\emptyset_{11}\emptyset_{22}-\emptyset_{12}{}^2}\right] \quad (4.22a)$$

$$V(\hat{\theta}) = \frac{1}{f}\left[\frac{\emptyset_{11}}{\emptyset_{11}\emptyset_{22}-\emptyset_{21}{}^2}\right] \quad (4.22b)$$

**Case 3:** When ∝ is taking known from the method of moment then

$$\emptyset_{11}=E\left(\frac{-\partial^2\log L}{\partial\beta^2}\right)/f=\left[\frac{1}{\beta}+\frac{1}{1-\beta}\right] \quad (4.23)$$

$$\emptyset_{22}=\frac{E\left(\frac{-\partial^2\log L}{\partial\theta^2}\right)}{f}=(1-\beta)\alpha\theta e^{\theta}(e^{\theta}-1)^{-2}\left\{1-e^{\theta}(e^{\theta}-1)^{-1}\right\}+\frac{(1-\beta)\alpha(e^{\theta}-1)^{-1}}{\theta}+\frac{\alpha\left\{1-\beta-(1-\beta)\theta(e^{\theta}-1)^{-1}\right\}\left[\frac{3\theta^2}{4}+\theta\right]}{\left[\frac{\theta^3}{4}+\frac{\theta^2}{2}\right]^2} \quad (4.24)$$

And

$$\emptyset_{12}=\emptyset_{21}=\frac{E\left[\frac{-\partial^2\log L}{\partial\beta\,\partial\theta}\right]}{f}=0 \quad (4.25)$$

$$V(\hat{\alpha}) = \frac{1}{f}\left[\frac{\emptyset_{22}}{\emptyset_{11}\emptyset_{22}-\emptyset_{12}{}^2}\right] \quad (4.26a)$$

$$V(\hat{\theta}) = \frac{1}{f}\left[\frac{\emptyset_{11}}{\emptyset_{11}\emptyset_{22}-\emptyset_{21}{}^2}\right] \quad (4.26b)$$

## Application

The suitability of the proposed model is examined to the study that has been conducted in North-Eastern Libya stretching from Benghazi to Emsaad. From the study area, 7 localities out of 27 have been selected by probability proportional to numbers of families in the localities. The data on fertility and mortality under age 5 along with some other demographic characteristics have been collected from 1,252 couples of childbearing ages of selected localities. About one-third (35.7 percent) of the investigated mothers have lost at least one child. The percentage of multiple child loss mothers is 11.3 and these mothers have given, one and average, 10 or more births. The differential in child loss by fertility level is highly significant. However, child mortality to mothers having lower and differential in child mortality by fertility in north –eastern Libya 325 medium (6 ever born children) fertility is similar. The study indicates that's high parity and high mortality move in the same direction[13] and one set of data has been taken from a Household Sample Survey in Brazil in 1987. Details are given in Sastry[14].

The parameters of the proposed model have been estimated by the method of moment and method of maximum likelihood. The estimated values of different parameters are given in tables 1 to 2 for the child deaths.

The estimated value of α are 0.2683 and 0.3570 for North East Brazil and North East Libya, respectively. It represents that the proportion of families experiencing a child loss was found slightly higher in North East Libya (0.3570), than North East Brazil (0.2683), The estimate of β are 0.6560 and 0.6846 respectively, for North East Brazil and North East Libya. It means that the proportion of families having only one child death was found greater for North East Libya (0.6846) as compared to North East Brazil (0.6560). The estimated values for the probability of success of death θ are 0.9192 and 0.8639 by the method of moment and 0.8945 and 0.7817 by the maximum likelihood, respectively, for the above mentioned countries. The average number of child death per family $\alpha\beta+(1-\beta)\alpha\{\theta e^{\theta}(e^{\theta}-1)^{-1}+1\}$ for North East Brazil and North East Libya were found to be 0.40 and 0.53 respectively .This show that, on an average, the child mortality is high in North East Libya. That exact variances of the estimator obtained by maximum likelihood method are also given.

Changes in levels of mortality may be attributed to socioeconomic factors such as improvements in primary health care services, control of epidemics availability of health care facilities, and with the improvement in economic condition among lower parity women, there is a downward shift in child mortality. However economic condition and mortality move in the same direction among high parity women. Differential impacts of page of female suppose at marriage are observed among mothers of different parity level.

An inflated Poisson distribution for finite range provides a suitable description of child Mortality at micro level, i.e. at the family level (table 1 and table 2). The value of are insignificant at 5 percent level of significance for all set of data. The proposed model fitted satisfactorily and described the pattern of child mortality to several seats of sample data in Indian subcontinents.

**Table-1**
**Distribution of Observed and Expected Number of Families, according to the Number of Child Deaths in North East Brazil**

| Number of child dead | Observed number of family | Expected | |
|---|---|---|---|
| | | Method of Moment (Expected no. of families) | Method of Maximum Likelihood (Expected no. of families) |
| 0 | 769 | 769.0167 | 769.0167 |
| 1 | 185 | 184.9810 | 184.9810 |
| 2 | 60 | 59.3751 | 60.0056 |
| 3 | 26 | 27.1018 | 26.8357 |
| 4 | 9 | 8.2471 | 8.0020 |
| 5 | 1 | | |
| 6 | 1 | 2.2783 | 2.1572 |
| 7 | 0 | | |
| Total | 1051 | 1051.0000 | 1051.0000 |
| $\hat{\alpha}$ | | 0.2683 | 0 .2683 |
| $\hat{\beta}$ | | 0.6560 | 0.6560 |
| $\hat{\theta}$ | | 0.9129 | 0.8945 |
| $\chi^2$ | | 0.1545 | 0.1620 |
| d.f | | 3 | 3 |

**Information about variance of the estimated parameters for Table-1**

| Case 1: When β is taking from the method of moment then the variance of α and θ will be; | |
|---|---|
| $\emptyset_{11}$=5.4639<br>$\emptyset_{22}$=8.0301<br>$\emptyset_{11}\emptyset_{22}$=43.8757 | $V(\hat{\alpha})$=0.00017412<br>$V(\hat{\theta})$=0.000118459 |
| Case 2: When θ is taking from the method of moment then the variance of α and β will be; | |
| $\emptyset_{11}$=5.4639<br>$\emptyset_{22}$=4.4314<br>$\emptyset_{11}\emptyset_{22}$=24.2127 | $V(\hat{\alpha})$=0.0001742<br>$V(\hat{\beta})$=0.00021474 |
| Case 3: When α is taking from the method of moment then the variance of β and θ will be; | |
| $\emptyset_{11}$=1.1889<br>$\emptyset_{22}$=8.0301<br>$\emptyset_{11}\emptyset_{22}$=9.5496 | $V(\hat{\beta})$=0.00021474<br>$V(\hat{\theta})$=0.000118487 |

**Table-2**
**Distribution of Observed and Expected Number of Families, according to the Number of Child Deaths in North East Libya**

| Number of child dead | Observed number of family | Expected | |
|---|---|---|---|
| | | Method of Moment (Expected no. of families) | Method of Maximum Likelihood (Expected no. of families) |
| 0 | 805 | 805.0360 | 769.0167 |
| 1 | 306 | 305.9916 | 184.9810 |
| 2 | 93 | 88.7422 | 60.0056 |
| 3 | 36 | 38.3322 | 26.8357 |
| 4 | 7 | 11.0384 | 8.0020 |
| 5 | 2 | | |
| 6 | 1 | 2.8596 | 2.1572 |
| 7 | 2 | | |
| Total | 1252 | 1252.0000 | 1051.0000 |
| $\hat{\alpha}$ | | 0.3570 | 0.3570 |
| $\hat{\beta}$ | | 0.6846 | 0.6846 |
| $\hat{\theta}$ | | 0.8639 | 0.7817 |
| $\chi^2$ | | 3.4327 | 2.1916 |
| d.f | | 3 | 3 |

**Information about variance of the estimated parameters for Table-2**

| | |
|---|---|
| Case 1: When β is taking from the method of moment then the variance of α and θ will be; | |
| $\emptyset_{11}$=4.3563<br>$\emptyset_{22}$=.4552<br>$\emptyset_{11}\emptyset_{22}$=1.9829 | $V(\hat{\alpha})$=0.000183348<br>$V(\hat{\theta})$=0.0001754739 |
| Case 2: When θ is taking from the method of moment then the variance of α and β will be; | |
| $\emptyset_{11}$=4.3563<br>$\emptyset_{22}$=4.6312<br>$\emptyset_{11}\emptyset_{22}$=20.1749 | $V(\hat{\alpha})$=0.000183348<br>$V(\hat{\beta})$=0.000172462 |
| Case 3: When α is taking from the method of moment then the variance of β and θ will be; | |
| $\emptyset_{11}$=1.6533<br>$\emptyset_{22}$=0.4552<br>$\emptyset_{11}\emptyset_{22}$=0.7526 | $V(\hat{\beta})$=0. 0001724662<br>$V(\hat{\theta})$=0.0001754739 |

## Conclusion

The proposed inflated typed model under various real life assumptions is very important tool to study the pattern of child mortality in a family specially in under develop countries an less develop countries, which also become a vital indicator of the medical and child care facilities for these type of countries.

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