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Differential Transform Method for system of Linear Differential Equations

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Abstract

In this paper, we study Differential Transform Method is applied for solving system of Linear Differential Equations. The approximate solution of the equation is calculated in the form of series with easily computable componant.This Powerful method catches the exact solution. Some linear Differential Equations are solved as numerical examples. The numerical result obtain by DTM are compared with the solution which are obtain by Laplace transform method which are closer to each other.

Keywords: Differential Transform Method, Laplace transform method, linear differential equation.

Introduction

In this paper we apply DTM (one dimensional) on linear differential equation on some example and the result obtained by it are compared with the result obtain by Laplace transform method which are exact solutions. In recent years, Abdel-Halim Hassan I. used differential transform method to solve this type of equations[1]. Arikoglu A applied DTM to obtain numerical solution of differential equations[2]. Ayaz F has used DTM to find the series solution of system of system of differential equations[3]. Bert.W has applied DTM on system of linear equation and analysis its solutions[4]. Chen used DTM to obtain the solutions of nonlinear system of differential equations[5]. Chen C.L. has applied DTM technique for steady nonlinear heat conduction problems[6]. Duan Y used DTM for Burger's equation to obtain the series solution[7]. Using DTM Hassan have find out series solution and that solution compared with decomposition method for linear and nonlinear initial value problems and prove that DTM is reliable tool to find the numerical solutions[8]. Khaled Batiha has been used DTM to obtain the Taylor's series as a solution of linear, nonlinear system of ordinary differential equations[9]. Kuo B has been used to find the numerical solution of the solutions of the free convection Problem[10]. Montri Thongmoon has been used to find the numerical solution of ordinary differential equations[11]. The concept of Differential Transform Method (DTM) was first proposed by Zhou and proves that DTM is an iterative procedure for obtaining analytic Taylor's series solution of differential equations. DTM is very useful to solve equation in ordinary differential equation. It is also applied to solve boundary value problems[12].

The Differential Transform Method

The transformation of the k^{th} derivative of a function with one variable is follows:

$$U(k) = \frac{1}{k!}\left(\frac{d^k u(x)}{dx^k}\right) \; at \; x = x_0 \tag{1}$$

Where $u(x)$ is the original function and $U(k)$ is the transformed function and the differential inverse transformation $U(k)$ is defined by,

$$u(x) = \sum_{k=0}^{k=\infty} U(k)(x - x_0)^k \tag{2}$$

When $x_0 = 0$, the function $u(x)$ defined in (2) is express as

$$u(x) = \sum_{k=0}^{k=\infty} U(k)\, x^k \tag{3}$$

Equation (3) implies that the concept of one dimensional differential transform is almost is same as the one dimensional Taylors series expansion. We use following fundamental theorems on differential transform method

Theorem 1
If $u(x) = \alpha g(x) \pm \beta h(x) \;\; then \;\; U(k) = \alpha G(k) \pm \beta H(k)$

Theorem 2
If $u(x) = x^m \; then \;\; U(K) = \delta(k-m) \; where \; \delta(k-m) = \begin{cases} 1, \; if \; k = m \\ 0\,, \; if \; k \neq m \end{cases}$

Theorem 3
If $u(x) = e^x \; then \;\; U(k) = \dfrac{1}{k!}$

Theorem 4
Ifu (x) = g (x) h (x) then $U(k) = \sum_{l=0}^{k} G(l)H(k-l)$

Theorem 5

$$\text{If } y(x) = y_1(x)\, y_2(x) \; then \; Y(k) = \sum_{k_1=0}^{k} Y_1(k_1)\, Y_2(k-k_1)$$

Theorem 6
If $y(x) = \frac{d^n y_1(x)}{dx^n}$, $then \;\; Y(k) = \frac{(k+n)!}{k!} Y_1(k+n)$

Theorem 7

If $y(x) = e^{\lambda x} then\ Y(k) = \frac{\lambda^k}{k!}$, λ *is constant*

Theorem 8

If $y(x) = \sin(wx + \alpha)\ then\ Y\ (k) = \frac{w^k}{k!}\sin(\frac{k\pi}{2} + \alpha)$, where α, w are constant

Theorem 9

If $y(x) = \cos(wx + \alpha)\ then\ Y\ (k) = \frac{w^k}{k!}\cos(\frac{k\pi}{2} + \alpha)$, where α, w are constant

Numerical Examples

Example-1: Consider the following system of simultaneous linear differential equations

$$\frac{dx}{dt} - 2x + 3y = 0, \quad (4)$$

$$\frac{dy}{dt} + 2x - y = 0, \quad (5)$$

With the condition $x(0) = 8, y(0) = 3$

Taking the differential transform method to equation (4) and (5), using above mentioned theorem we obtain

$$(k+1)X(k+1) = 2X(k) - 3Y(k) \quad (6)$$

$$(k+1)Y(k+1) = Y(k) - 2(x) \quad (7)$$

With initial conditions $X(0) = 8, Y(0) = 3$

Take $k = 0$, we get $X(1) = 7, Y(1) = -13$
Take $k = 1$, we get $X(2) = \frac{53}{2}, Y(2) = -\frac{27}{2}$
Take $k = 2$, we get $X(3) = \frac{187}{6}, Y(2) = -\frac{135}{6}$

The approximation solution when $n = 3$ (number of terms) using equation (3) is given by

$$x(t) = \sum_{k=0}^{k=3} X(k)t^k$$

$$y(t) = \sum_{k=0}^{k=3} Y(k)t^k$$

$$x(t) = 8 + 7t + \frac{53}{2}t^2 + \frac{187}{6}t^3$$

$$y(t) = 3 - 13t - \frac{27}{2}t^2 - \frac{135}{6}t^3$$

Using the Laplace transform method, the exact solution of example1 are
$x(t) = 5e^{-t} + 3e^{4t}$, $y(t) = 5e^{-t} - 2e^{4t}$

Example-2: Consider the following system of simultaneous linear differential equations

$$\frac{dx}{dt} - y += e^t, \quad (8)$$

$$\frac{dy}{dt} + x = sint, \quad (9)$$

With the conditions $x(0) = 1, y(0) = 0$

Taking the differential transform method to equation (8) and (9) using above mentioned theorem we obtain

$$(k+1)X(k+1) = \frac{1}{k!} + Y(k) \quad (10)$$

$$(k+1)Y(k+1) = \left[\frac{1}{k!}sin\left(\frac{k\pi}{2}\right) - X(k)\right] \quad (11)$$

With initial conditions $X(0) = 1, Y(0) = 0$

Take$k = 0$, we get $X(1) = 1, Y(1) = -1$
Take$k = 1$, we get $X(2) = \frac{-1}{4}, Y(2) = 0$
Take$k = 2$, we get $X(3) = \frac{1}{6}, Y(3) = \frac{1}{12}$
Take$k = 3$, we get $X(4) = \frac{1}{16}, Y(4) = \frac{-1}{12}$

The approximation solution when $n = 4$ (number of terms) using equation (3) is given by

$$x(t) = \sum_{k=0}^{k=4} X(k)t^k$$

$$y(t) = \sum_{k=0}^{k=4} Y(k)t^k$$

Putting all table values in equation (10), (11) we get

$$x(t) = 1 + t - \frac{1}{4}t^2 + \frac{1}{6}t^3 + \frac{1}{16}t^4$$

$$y(t) = -t + \frac{1}{12}t^3 - \frac{1}{12}t^4$$

Using the Laplace transform method, the exact solution of example 2 are

$x(t) = \frac{1}{2}e^t + \frac{1}{2}cost + sint - tcost$,$y(t) = \frac{1}{2}tsint - \frac{1}{2}e^t + cost - sint$

Example-3: Consider the following non homogenous differential equations

$$\frac{dx(t)}{dt} = z(t) - cost \quad (12)$$

$$\frac{dy(t)}{dt} = z(t) - e^t \quad (13)$$

$$\frac{dz(t)}{dt} = x(t) - y(t) \quad (14)$$

With the initial conditions $x(0) = 1, y(0) = 0, z(0) = 2$ by applying differential transformation and above theorem, we obtain

$$X(k+1) = \frac{1}{k+1}\left[Z(k) - \frac{1}{k!}cos\left(\frac{\pi k}{2}\right)\right]$$
$$Y(k+1) = \frac{1}{k+1}\left[Z(k) - \frac{1}{k!}\right]$$
$$Z(k+1) = \frac{1}{k+1}[X(k) - Y(k)]$$

with the initial condition $\mathrm{X}(0) = 1, Y(0) = 0, Z(0) = 2$

put $k = 0$ we get,
$Z(1) = 1, Y(1) = 1, X(1) = 1$

put $k = 1$ we get,
$Z(2) = 0, Y(2) = 0, X(2) = \frac{1}{2}$

put $k = 2$ we get,
$Z(3) = \frac{1}{6}, Y(3) = \frac{-1}{6}, X(3) = \frac{1}{6}$

put $k = 3$ we get,
$Z(4) = \frac{1}{120}, Y(4) = \frac{1}{120}, X(4) = \frac{1}{24}$

The approximation solution when $n = 3$ (number of terms) using equation (3) is given by

$$x(t) = \sum_{k=0}^{k=3} X(k)t^k$$
$$y(t) = \sum_{k=0}^{k=3} Y(k)t^k$$
$$z(t) = \sum_{k=0}^{k=3} Z(k)t^k$$

$$\mathrm{x(t)} = 1 + \mathrm{t} + \frac{1}{2}\mathrm{t}^2 + \frac{1}{6}\mathrm{t}^3 + \frac{1}{12}\mathrm{t}^4$$

$$\mathrm{y(t)} = \mathrm{t} - \frac{1}{12}\mathrm{t}^3$$

$$\mathrm{z(t)} = 2 + \mathrm{t} + \frac{1}{4}\mathrm{t}^2 + \frac{1}{6}\mathrm{t}^3 + \frac{1}{12}\mathrm{t}^4 \qquad (15)$$

Using the Laplace transform method, the exact solution of this example

$$x(t) = e^t$$
$$y(t) = sint$$
$$z(t) = e^t + cost \qquad (16)$$

From (15) and (16) are we say that numerical solution of example3 which are closed form to exact solution.

Conclusion

We apply Differential Transform Method to linear differential equations on some examples and the results obtained by it are in the Taylor's series form. These numerical examples have proved good results. A specific advantage of this method over any purely numerical method is that it offers a smooth, functional form of the solution over a time step. All the calculations in the method are very easy It may be concluded that DTM is very powerful and efficient in finding analytical as well as numerical solutions for wide classes of linear differential equations.

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