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Some results for Monotone and Nondecreasing Operators in Partial Metric spaces

Girish Pandya[1], M.M. Sharma[2] and Sushil Sharma[2]
[1]Bhartiya college, Ujjain MP, INDIA
[2]Govt. Madhav science college,Ujjain, MP, INDIA

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Abstract

In ordered partial metric spaces we introduced some fixed point results, without any type of commutativity of the concerned maps, we established coupled coincidence results. Also we give some results for nondecreasing mappings.

Keywords: Partial metric space;coupled fixed point; coupled coincidence point, nondecreasing mappings..

Introduction

In 1994 the concept of partial metric spaces was introduced by Matthews[1]. This concept is introdused to give a modified version of the Banach contraction principle[2,3]. The existence and uniqueness of a fixed point of different contractive conditions for mappings satisfying on partial metric spaces[4,5] was studied by several authors. In this paper we extend Luong and Thuan,[6] results. O'Regan D, Petruşel A[7] gave some existence results for Fredholm and Volterra type integral equations. In some of their works, the fixed point result is also given for nondecreasing mappings

Preliminaries

Definition-1
A partial metric on a nonempty set X is a function $p : X \times X \to R^+$ such that for all x, y, z $\in$ X

p1. x=y $\Leftrightarrow$ p(x,x)=p(x,y)=p(y,y)
p2. p(x,x) $\leq$ p(x,y)
p3. p(x,y) = p(y,x)
p4. P(x,z) $\leq$ p(x,y)+p(y,z)-p(y,y)

Lemma 1: Let (X,p) be a partial metric space. Then (a) $\{X_n\}$is a Cauchy sequence in (X,p) if and only if it is a Cauchy sequence in the metric space (X,p^s).
(b) (X,p) is complete if and only if the metric space (X,p^s) is complete. Furthermore, $\lim_{n\to\infty} p^s(x_n,x)=0$ if and only if

$$p(x,x)=\lim_{n-\infty} p(x_n,x)=\lim_{n,m-\infty} p(x_n,x_m).$$

Let (X,p) be a partial metric. We endow the product space X×X with the partial metric q defined as follows: for (x,y), (u,v) $\in$ X × X, q ((x,y), (u,v)) = p (x,u) + p (y,v).

A mapping F: X × X $\to$ X is said to be continuous at (x,y) $\in$ X×X if for each $\in>$ 0, there exists □>0 such that $F(B_q((x,y),\delta))\subseteq B_p(F(x,y),\in)$.

Definition-2
Let F : X × X $\to$ X be a mapping in a partially ordered set (X, $\leq$) and F has the mixed monotone property if for any x,y$\in$ X,

$$x_1,x_2\in X,\ x_1\preceq x_2 \Rightarrow F(x_1,y)\preceq F(x_2,y) \text{ and}$$
$$y_1\ y_2\in X,\ y_1\preceq y_2\ \Rightarrow F(x,y_1)\succeq F(x,y_2).$$

Definition-3
The point (x,y) $\in$X×X is a coupled fixed point of F : X × X $\to$ X. if F(x,y) = x and F(y,x) = y.

Definition 4
The mapping F : X × X $\to$ X in partially ordered set $(X,\preceq)$ has the mixed g-monotone property if for any x,y $\in$ X,

$$x_1,x_2\in X,\ gx_1\preceq gx_2 \Rightarrow F(x_1,y)\preceq F(x_2,y) \text{ and}$$
$$y_1\ y_2\in X,\ gy_1\preceq gy_2\ \Rightarrow F(x,y_1)\succeq F(x,y_2).$$

Main results

Theorem-1
Suppose the metric p on partially ordered set $(X,\preceq)$ and (X,d) is a complete partial metric space. Let the mapping F : X × X $\to$ X having the mixed monotone property on X. Let $x_0,y_0\in X$ and $x_o\preceq F(x_o,y_o)$and $y_o\succeq F(y_o,x_o)$.

Suppose $\exists$ $\phi\in\Phi$ and $\psi\in\Psi$ such that

$$\phi(p(F(x,y),F(u,v)))\leq\frac{1}{2}\phi(p(x,u)+p(y,v))-\psi\left(\frac{p(x,u)+p(y,v)}{2}\right) \quad (1)$$

$\forall$ $x,y,u,v\in X$ with $x\succeq u$ and $y\preceq v$. Suppose either (a) The mapping F is continuous or (b) X has the following property:

(i) If $\{x_n\}$ is a non-decreasing sequence such that $\{x_n\} \to x$, then $x_n \leq x, \quad \forall$ n,

(ii) If $\{y_n\}$ is a non-increasing sequence such that $\{y_n\} \to y$ then $y \leq y_n \ \forall$ n.

Then $\exists$ x,y$\in$X such that x = F(x,y) and y = F(y,x), that is, F has a coupled fixed point in X.

Proof

Let $x_o, y_o \in X$ be such that $x_o \leq F(x_o, y_o)$ and $y_o \geq F(y_o, x_o)$ We construct sequences $\{x_n\}$ and $\{y_n\}$ in X as

$x_{n+1} = F(x_n, y_n)$ and $y_{n+1} = F(y_n, x_n)$ $\forall$ $n \geq 0$. (2)

We are to prove that

$$x_n \leq x_{n+1} \qquad \forall\ n \geq 0 \qquad (3)$$

and

$$y_n \geq y_{n+1} \qquad \forall\ n \geq 0 \qquad (4)$$

By using mathematical induction method (3) and (4) hold $\forall$ n ≥ 0. Therefore, $x_0 \leq x_1 \leq x_2 \leq \ldots\ldots\ldots \leq x_n \leq x_{n+1} \leq \ldots\ldots$

and $y_0 \geq y_1 \geq y_2 \geq \ldots\ldots\ldots \geq y_n \geq y_{n+1} \geq \ldots\ldots$

Since $x_n \geq x_{n-1}$ and $y_n \leq y_{n-1}$ using (1) and (2) we get

$$\phi\left(p\left(x_{n+1}, x_n\right)\right) = \phi\left(p\left(F\left(x_n, y_n\right), F\left(x_{n-1}, y_{n-1}\right)\right)\right)$$

$$\leq \frac{1}{2}\phi\left(p\left(x_n, x_{n-1}\right) + p\left(y_n, y_{n-1}\right)\right) - \psi\left(\frac{p\left(x_n, x_{n-1}\right) + p\left(y_n, y_{n-1}\right)}{2}\right)$$

Similarly, since $y_{n-1} \geq y_n$ and $x_{n-1} \leq x_n$, using (1) and (2), we also have

$$\phi\left(p\left(y_n, y_{n+1}\right)\right) = \phi\left(p\left(F\left(y_{n-1}, x_{n-1}\right), F\left(y_n, x_n\right)\right)\right)$$

$$\leq \frac{1}{2}\phi\left(p\left(y_{n-1}, y_n\right) + p\left(x_{n-1}, x_n\right)\right) - \psi\left(\frac{p\left(y_{n-1}, y_n\right) + p\left(x_{n-1}, x_n\right)}{2}\right)$$

so we have

$$\phi\left(p\left(x_{n+1}, x_n\right)\right) + \phi\left(p\left(y_{n+1}, y_n\right)\right) \leq \phi\left(p\left(x_n, x_{n-1}\right) + p\left(y_n, y_{n-1}\right)\right)$$

$$- 2\psi\left(\frac{p\left(x_n, x_{n-1}\right) + p\left(y_n, y_{n-1}\right)}{2}\right)$$

By property (ϕ3), we have

$$\phi\left(p\left(x_{n+1}, x_n\right) + p\left(y_{n+1}, y_n\right)\right) \leq \phi\left(p\left(x_{n+1}, x_n\right)\right) + \phi\left(p\left(y_{n+1}, y_n\right)\right).$$

so we have

$$\phi\left(p\left(x_{n+1}, x_n\right) + p\left(y_{n+1}, y_n\right)\right) \leq \phi\left(p\left(x_n, x_{n-1}\right) + p\left(y_n, y_{n-1}\right)\right)$$

$$- 2\psi\left(\frac{p\left(x_n, x_{n-1}\right) + p\left(y_n, y_{n-1}\right)}{2}\right), \qquad (5)$$

since ψ is a positive function, therefore

$$\phi\left(p\left(x_{n+1}, x_n\right) + p\left(y_{n+1}, y_n\right)\right) \leq \phi\left(p\left(x_n, x_{n-1}\right) + p\left(y_n, y_{n-1}\right)\right).$$

Now we use the fact that ϕ is increasing, we get

$$p\left(x_{n+1}, x_n\right) + p\left(y_{n+1}, y_n\right) \leq p\left(x_n, x_{n-1}\right) + p\left(y_n, y_{n-1}\right)$$

Set $\delta_n = p\left(x_{n+1}, x_n\right) + p\left(y_{n+1}, y_n\right)$

clearly $\{\delta_n\}$ is decreasing. Therefore,

$$\lim_{n\to\infty} \delta_n = \lim_{n\to\infty}\left[p\left(x_{n+1}, x_n\right) + p\left(y_{n+1}, y_n\right)\right] = \delta. \text{ for } \geq 0$$

It can be shown that $\delta = 0$ as $n \to \infty$ that is,

$$\lim_{n\to\infty} \delta_n = \lim_{n\to\infty}\left[p\left(x_{n+1}, x_n\right) + p\left(y_{n+1}, y_n\right)\right] = 0. \qquad (6)$$

Let $\delta_n^s = p^s\left(x_n, x_{n+1}\right) + p^s\left(y_n, y_{n+1}\right)$ $\forall$ n$\in$N.

By definition of p^s, clearly that $\delta_n^s \leq 2\delta_n$ for all n$\in$N. Using (6), we get

$$\lim_{n\to+\infty} \delta_n^s = \lim_{n\to+\infty} p^s\left(x_n, x_{n+1}\right) + p^s\left(y_n, y_{x+1}\right) = 0$$

Now, we will show that $\{x_n\}$ and $\{y_n\}$ are Cauchy sequences in (X,p). On the contrary we assume that at least one of $\{x_n\}$ and $\{y_n\}$ is not a Cauchy sequence. Then $\exists$ an $\in > 0$ for which we can find subsequences $\{x_{(n)}k\}$, $\{x_{(m)}k\}$ of $\{x_n\}$and $\{y_{n(k)}\}$, $\{y_{m(k)}\}$ of $\{y_n\}$with n(k) > m(k) $\geq$ k such that

$$p^s\left(x_{n(k)}, x_{m(k)}\right) + p^s\left(y_{n(k)}, y_{m(k)}\right) \geq \in.$$

Now we take n(k) > m(k). Then

$$p^s\left(x_{n(k)-1}, x_{m(k)}\right) + p^s\left(y_{n(k)-1}, y_{m(k)}\right) < \in.$$

by triangle inequality, we have

$$\in \leq r_k^s := p^s\left(x_{n(k)}, x_{m(k)}\right) + p^s\left(y_{n(k)}, y_{m(k)}\right)$$

$$\leq p^s\left(x_{n(k)}, x_{m(k)-1}\right) + p^s\left(x_{n(k)-1}, x_{m(k)}\right) + p^s\left(y_{n(k)}, y_{m(k)-1}\right) + p^s\left(y_{n(k)-1}, y_{m(k)}\right)$$

$$\leq p^s\left(x_{n(k)}, x_{n(k)-1}\right) + p^s\left(y_{n(k)}, y_{n(k)-1}\right) + \in.$$

Taking $k \to \infty$ and by (6), we get

$$\lim_{k\to\infty} r_k^s = \lim_{k\to\infty}\left[p^s\left(x_{n(k)}, x_{m(k)}\right) + p^s\left(y_{n(k)}, y_{m(k)}\right)\right] = \in. \qquad (7)$$

By the triangle inequality,

$$r_k^s = p^s\left(x_{n(k)}, x_{m(k)}\right) + p^s\left(y_{n(k)}, y_{m(k)}\right)$$

$$\le p^s(x_{n(k)},x_{n(k)+1})+p^s(x_{n(k)+1},x_{m(k)+1})+p^s(x_{m(k)+1},x_{m(k)})$$
$$+p^s(y_{n(k)},y_{n(k)+1})+p^s(y_{n(k)+1},y_{m(k)+1})+p^s(y_{m(k)+1},y_{m(k)})$$
$$=\delta^s_{n(k)}+\delta^s_{m(k)}+p^s(x_{n(k)+1},x_{m(k)+1})+p^s(y_{n(k)+1},y_{m(k)+1}).$$

By using the properties of ϕ, we have

$$\phi(r_k^s)\le\phi\left(\delta^s_{n(k)}+\delta^s_{m(k)}+p^s(x_{n(k)+1},x_{m(k)+1})+p^s(y_{n(k)+1},y_{m(k)+1})\right)$$
$$\le\phi(\delta^s_{n(k)}+\delta^s_{m(k)})+\phi(p^s(x_{n(k)+1},x_{m(k)+1}))+\phi(p^s(y_{n(k)+1},y_{m(k)+1}))\quad(8)$$

Now we take $r_k=p(x_{n(k)},x_{m(k)})+p(y_{n(k)},y_{m(k)}).$

Then by definition of r_k^s,

$$r_k^s=p^s(x_{n(k)},x_{m(k)})+p^s(y_{n(k)},y_{m(k)})$$
$$=2p(x_{n(k)},x_{m(k)})-p(x_{n(k)},x_{n(k)})-p(x_{m(k)},x_{m(k)})$$
$$+2p(y_{n(k)},y_{m(k)})-p(y_{n(k)},y_{n(k)})-p(y_{m(k)},y_{m(k)})$$
$$=2r_k-p(x_{n(k)},x_{n(k)})-p(x_{m(k)},x_{m(k)})$$
$$-p(y_{n(k)},y_{n(k)})-p(y_{m(k)},y_{m(k)}).$$

By property (p2) and (6), we have

$$\lim_{k\to+\infty}p(x_{n(k)},x_{n(k)})=\lim_{k\to+\infty}p(x_{m(k)},x_{m(k)})$$
$$=\lim_{k\to+\infty}p(y_{n(k)},y_{n(k)})$$
$$=\lim_{k\to+\infty}p(y_{m(k)},y_{m(k)})=0.$$

Therefore, taking k→+ ∞ and using (7), we get

$$=\lim_{k\to+\infty}r_k=\frac{\in}{2}.$$

Since $x_{n(k)}\le x_{m(k)}$ *and* $y_{n(k)}\le y_{m(k)}$, we have

$$\phi(p^s(x_{m(k)+1},x_{m(k)+1}))\le\phi(2p(x_{n(k)+1},x_{m(k)+1}))$$
$$\le 2\phi(p(x_{n(k)+1},x_{m(k)+1}))$$
$$=2\phi(p(F(x_{n(k)},y_{n(k)})),\ p(F(x_{m(k)},y_{m(k)})))$$
$$\le\phi(p(x_{n(k)},x_{m(k)})+p(y_{n(k)},y_{m(k)}))$$
$$-2\psi\left(\frac{p(x_{n(k)},x_{m(k)})+p(y_{n(k)},y_{m(k)})}{2}\right)$$
$$=\phi(r_k)-2\psi\left(\frac{r_k}{2}\right).$$

that is $\phi(p^s(x_{m(k)+1},x_{m(k)+1}))\le\phi(r_k)-2\psi\left(\frac{r_k}{2}\right).$

Similarly, $\phi(p^s(y_{n(k)+1},y_{m(k)+1}))\le\phi(r_k)-2\psi\left(\frac{r_k}{2}\right).$

So adding we get

$$\phi(p^s(x_{n(k)+1},x_{m(k)+1}))+\phi(p^s(y_{n(k)+1},y_{m(k)+1}))\le 2\phi(r_k)-4\psi\left(\frac{r_k}{2}\right).$$

Thus, from (8), we have

$$\phi(r_k^s)\le\phi(\delta^s_{n(k)}+\delta^s_{m(k)})+2\phi(r_k)-4\psi\left(\frac{r_k}{2}\right).$$

Now using the properties of ϕ and ψ and letting k→+∞ , we have

$$\phi(\in)\le\phi(0)+2\phi\left(\frac{\in}{2}\right)-4\lim_{k\to+\infty}\psi\left(\frac{r^k}{2}\right)\le\phi(\in)-4\lim_{\frac{r_k}{2}\to\lambda}\psi\left(\frac{r_k}{2}\right)$$
$$\le\phi(\in)-4\lim_{\frac{r_k}{2}\to\lambda}\psi(t)<\phi(\in).$$

which is a contradiction. So, $\{x_n\}$ and $\{y_n\}$ are Cauchy sequences in the complete metric space (X,p^s).Thus, by lemma 1 there are x,y∈X such that

$$\lim_{n\to+\infty}p^s(x_n,x)=\lim_{n\to+\infty}p^s(y_n,y)=0\qquad(9)$$

which implies that $\lim_{n\to+\infty}F(x_n,y_n)=\lim_{n\to+\infty}x_n=x,$

$$\lim_{n\to+\infty}F(y_n,x_n)=\lim_{n\to+\infty}y_n=y,\qquad(10)$$

Using (6),Lemma 1 and the property (p2), we have

$$p(x,x)=\lim_{n\to+\infty}p(x_n,x)=\lim_{n\to+\infty}p(x_n,x_n)=0,$$
$$p(y,y)=\lim_{n\to+\infty}p(y_n,y)=\lim_{n\to+\infty}p(y_n,y_n)=0,\ (11)$$

Suppose the condition (a) holds. Since F is continuous at (x,y), for any ∈>0, ∃ δ>0 such that if (u,v)∈X×X with $v((x,y),(u,v))<v((x,y),(x,y))+\delta=\delta$ that is

$$p(x,u)+p(y,v)<p(x,x)+p(y,y)+\delta=\delta,$$

since p(x,x) = p(y,y) = 0. Then

$$p(F(x,y),F(u,v))<p(F(x,y),F(x,y))+\frac{\in}{2}.$$

Since $\lim_{n\to+\infty}p(x_n,x)=\lim_{n\to+\infty}p(y_n,y)=0,$ for $\eta=\min\left(\frac{\delta}{2},\frac{\in}{2}\right)>0,$ there exist $n_0,m_0\in N$ such that, for $n\ge n_0$, $m\ge m_0$, $p(x_n,x)<\eta$ and $p(y_m,y)<\eta$.

Then for $n\in N$, $n\ge\max(n_0,m_0)$, we have $p(x_n,x)+p(y_n,y)<2\eta<\delta$ so we get

$$p\left(F(x,y), F(x_n,y_n)\right) < p\left(F(x,y), F(x,y)\right) + \frac{\in}{2}.$$

Further, for any $n \geq \max(n_0, m_0)$,

$$\begin{aligned} p(F(x,y), x) &\leq p\left(F(x,y), x_{n+1}\right) + p\left(x_{n+1}, x\right) \\ &= p\left(F(x,y), F(x_n,y_n)\right) + p\left(x_{n+1}, x\right) \\ &\leq p\left(F(x,y), F(x,y)\right) + \frac{\in}{2} + \eta \\ &\leq p\left(F(x,y), F(x,y)\right) + \in. \end{aligned} \tag{12}$$

Putting $p(x,x) = p(y,y) = 0$ in (1), we get $p(F(x,y), F(x,y)) = 0$. Hence, for any $\in > 0$, by (12), we get $p(F(x,y), x) < \in$.

Hence $F(x,y) = x$. Similarly it can be shown that $F(y,x) = y$.
Assuming that (b) holds. By (3), (9) and (10), we have $\{x_n\}$ is a non-decreasing sequence, $x_n \to x$ and $\{y_n\}$ is a non-increasing sequence, $y_n \to y$ as $n \to \infty$ Hence, by (b), we have $\forall\ n \geq 0$,
$x_n \leq x$ *and* $y_n \leq y$

Using (p4), we get

$$p(x,F(x,y)) \leq p(x,x_{n+1}) + p(x_{n+1}, F(x,y)) = p(x,x_{n+1}) + p(F(x_n,y_n), F(x,y)).$$

Therefore,

$$\phi\left(p(x,F(x,y))\right) \leq \phi\left(p(x,x_{n+1})\right) + \phi\left(p(F(x_n,y_n)\, F(x,y))\right)$$
$$\leq \phi\left(p(x, x_{n+1})\right) + \frac{1}{2}\phi\left(p(x_n,x) + p(y_n,y)\right) - \psi\left(\frac{p(x_n,x)+p(y_n,y)}{2}\right)$$

In the above inequality ,we take limit as $n \to \infty$, using (12) and (11) and the properties of ϕ and ψ, we get $\phi(p(x, F(x,y)))=0$, which implies $p(x,F(x,y))=0$. Hence, $x=F(x,y)$. Similarly, it can be shown that $y=F(y,x)$ .Thus F has a coupled fixed point.

Implicit Relation

Let T be the set of all continuous functions $T : {}^6+ \to$ where $+$ is set of the nonnegative real numbers, and satisfying

T_1 : $T(t_1 \ldots, t_6)$ is decreasing in variables $t_2, \ldots, t_6$;
T_2 : A right continuous function $f : {}_+ \to {}_+$ is exist, $f(0)=0$, $f(t) < t$ for $t > 0$, such that for $u \geq 0$, $T(u, v, u, v, 0, u + v) \leq 0$ or $T(u, v, 0, 0, v, v) \leq 0 \Longrightarrow u \leq f(v)$;
T_3: $T(u, 0, u, 0, 0,u) > 0$, $\forall\ u > 0$.

Lemma 2

Let for every $t > 0$ there be a right continuous function f: ${}_+ \to {}_+$ such that $f(t) < t$, then the n-times repeated composition of f with itself is zero as $n \to \infty$. That is
$\lim_{n\to\infty} f^n(t) = 0$.

Theorem 2.

Suppose that there is a metric d on X. Where $(X, \leq)$ is a partially ordered set such that (X,d) is a complete metric space. Suppose $F:X \to X$ is a nondecreasing mapping such that $\forall$ x, $y \in X$ with $y \leq x$, $T(d(Fx,Fy), d(x,y), d(x,Fx), d(y,Fy), d(x,Fy), d(y,Fx)) \leq 0$, (13)

where $T \in$ T. Also F is continuous, or if $\{x_n\} \subset X$ is a nondecreasing sequence with $x_n \to x$ in x, then $x_n \leq x\ \forall n$ hold. If there exists an $x_0 \in X$ with $x_0 \leq F(x_0)$, then F has a fixed point.

Proof.

There is nothing to prove If $Fx_0 = x_0$, so suppose $x_0 \# Fx_0$. Now let $x_n=Fx_{n-1}$ for $n \in \{1,2,\ldots\}$. Notice that, since $x_0 \preceq Fx_0$ and F is nondecreasing, we have $x_0 \leq x_1 \leq x_1 \leq \ldots\ldots\ldots \leq x_n \leq x_{n+1} \leq \ldots\ldots.$

Now since $x_{n-1} \leq x_n$ by inequality (13) **,**we have $T(d(Fx_n, Fx_{n-1}), d(x_n,x_{n-1}), d(x_n, Fx_n), d(x_{n-1}, Fx_{n-1}), d(x_n, Fx_{n-1}), d(x_{n-1}, Fx_n)) \leq 0$ so
$T(d(x_{n+1}, x_n), d(x_n,x_{n-1}), d(x_n, x_{n+1}), d(x_{n-1}, x_n), 0, d(x_{n-1}, x_{n+1})) \leq 0$

By using T_1, we have $T(d(x_{n+1}, x_n), d(x_n,x_{n-1}), d(x_n, x_{n+1}), d(x_{n-1}, x_n), 0, d(x_{n-1}, x_n) + d(x_n, x_{n+1})) \leq 0$, Using T_2 a right continuous function f: ${}_+ \to {}_+$ is exist, $f(0) = 0$, $f(t) < t$, for $t>0$ such that for all $n \in \{1,2,\ldots.\}$, $d(x_{n+1}, x_n) \leq f(d(x_n, x_{n-1}))$.

If we continue this procedure, we can have
$d(x_{n+1}, x_n) \leq f^n(d(x_1, x_0))$, (14)
and so by lemma 2
$\lim\limits_{n\to\infty} d(x_{n+1}, x_n)=0.$

Now this can be easily prove that $\{x_n\}$ is a Cauchy sequence in X, so $\exists$ an $x \in X$ with $\lim_{n\to\infty} x_n = x.$ then clearly $x = Fx$. Suppose $d(x, Fx)>0$. Now since $\lim_{n\to\infty} x_n = x$, then $x_n \preceq x$ for all n. Using the inequality (13)**,** we have $T(d(Fx, Fx_n), d(x,x_n), d(x, x_n), d(x,Fx), d(x_n,Fx_n), d(x, Fx_n), d(x_n, Fx)) \leq 0$,
so letting $n \to \infty$ from the last inequality, we have
$T(d(Fx, x), 0, d(x, Fx), 0, 0, d(x,Fx)) \leq 0$,

which is a contradiction to T_3. Thus $d(x, Fx)$ and so $x = Fx$.

Corollary

Suppose that there is a metric d on partially ordered set $(X, \leq)$ and (X,d) is a complete metric space and $F : X \to X$ is a nondecreasing mapping such that $\forall$ x, $y \in X$ with $y \leq x$,
$d(Fx, Fy) \leq a \max\{d(x,y), d(x,Fx), d(y,Fy)\} + (1-a)[a d(x, Fy) + b d(y,Fx)]$, where $0 \leq a < 1$, $0 \leq a < 1/2$, $0 \leq b < 1/2$.
Also F is continuous or if $\{x_n\} \subset X$ is a nondecreasing sequence with $x_n \to x$ in X, then $x_n \leq x \forall n$ hold. If there exists an $x_0 \in X$ with $x_0 \preceq F(x_0)$, then F has a fixed point.

Conclusion

In view of above results It is very much clear that we extend some fixed point results in partial metric space having the mixed monotone property and for nondecreasing mappings. This is the first effort in the existing literature.

Our results contain so many results in the existing literature and will be helpful for the workers in the field.

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