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Effect of Hall Current on MHD Flow of a Dusty Visco-Elastic Liquid through Porous Medium past an Inclined Plane

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Abstract

The present problem is concerned with the effect of hall current on MHD laminar flow of an unsteady visco-elastic (Kuvshiniski type) liquid through porous medium with uniform distribution of dust particles past an inclined plane under the influence of exponential pressure gradient. The liquid is bounded by a parallel upper surface at a distance h from the plane. Analytical expressions for velocities of liquid and dust particles, temperature and concentration profile are obtained which are in elegant forms. The numerical results are presented graphically for different values of the various parameters entering into the problem.

Keyword: Heat and mass transfer, magnetic field, hall current, porous medium, visco-elastic, dusty liquid.

Introduction

The problem of laminar flow of dusty visco-elastic liquid past an inclined plane has become very important in recent years particularly in the field of industrial and chemical engineering such as latex particles emulsion paints and reinforcing particles in polymer. The study of these problems and rheological aspects of such flows have not received much attention although this has some bearing on the problems of petroleum industry and chemical engineering.

Saffman[1] has expressed a model equation describing the influence of dust particles on the motion of fluids. Several authors[2-5] using equations of Saffman[1] have investigated a number of dusty gas flow problems in different situations. Several other authors[6-9] have contributed in the field of petroleum and chemical engineering. Mandal *et. al.*[10] have considered unsteady flow of dusty visco-elastic (Kuvshiniski type) liquid between two oscillating plates. Chaudhary and Singh[11] have considered the flow of a dusty visco-elastic (Kuvshiniski type) liquid down an inclined plane. Recently, Johari and Gupta[12] have studied MHD flow of a dusty visco-elastic (Kuvshiniski type) liquid past an inclined plane. Varshney *et. al.*[13] have studied on effect of porous medium on MHD flow of a dusty visco-elastic liquid past an inclined plane with mass transfer. Recently, Singh et. al.[14] have studied on Effect of Heat Transfer on MHD Flow of a Dusty Visco-Elastic Liquid Through Porous Medium Past an Inclined Plane,

Mathematical analysis

Consider the laminar flow of an unsteady MHD visco-elastic (Kuvshiniski type) liquid through porous medium and hall current with uniform distribution of dust particles past an inclined plane of inclination θ to the horizontal. We choose the origin of coordinate system at the bottom of the inclined plane. The x-axis is taken opposite to the direction to the flow and along the greatest slop of the plane and y-axis is taken perpendicular to the plane. The magnetic field of uniform strength is applied along to y-axis. Since both the dust and liquid particles move along the greatest slope of the plane and the flow is laminar, the velocity of the both liquid and dust particles can be defined by the following relations:

$$\left.\begin{aligned} &u_1 = u_1(y,t), \quad u_2 = 0, \quad u_3 = 0 \\ &v_1 = v_1(y,t), \quad v_2 = 0, \quad v_3 = 0 \end{aligned}\right\} \tag{1}$$

Where (u_1, u_2, u_3), (v_1, v_2, v_3) are the velocity components of liquid and dust particles respectively.

Following Saffman [1] the equations of motion for the flow of dusty visco-elastic liquid (Kuvshiniski type) through porous medium with hall current are given by:

$$\left(1+\alpha\frac{\partial}{\partial t}\right)\frac{\partial u_1}{\partial t} = -\frac{1}{\rho}\left(1+\alpha\frac{\partial}{\partial t}\right)\frac{\partial p}{\partial x} + \upsilon\frac{\partial^2 u_1}{\partial y^2} + \frac{\kappa N_0}{\rho}\left(1+\alpha\frac{\partial}{\partial t}\right)(v_1 - u_1) - \frac{\sigma B_0^2}{\rho(1+m^2)}u_1 - \frac{\upsilon}{K_0}u_1 - g\sin\theta + g\beta(C_0 - C) + g\beta^*(T_0 - T) \tag{2}$$

$$\frac{1}{\rho}\frac{\partial p}{\partial y} + g\cos\theta = 0 \tag{3}$$

$$-\frac{1}{\rho}\frac{\partial p}{\partial z}=0 \tag{4}$$

$$\frac{\partial v_1}{\partial t}=\frac{\kappa}{m_1}(u_1-v_1) \tag{5}$$

$$\frac{\partial T}{\partial t}=\frac{k}{\rho C_p}\frac{\partial^2 T}{\partial y^2} \tag{6}$$

$$\frac{\partial C}{\partial t}=D\frac{\partial^2 C}{\partial y^2} \tag{7}$$

Where p is the pressure, υ is the kinematic coefficient of viscosity of the gas, α is the coefficient of visco-elasticity of the gas, κ is the Stoke's resistance coefficients, N_0 is the number density of the dust particles which is taken to be constant, ρ is the density of the liquid, m_1 is the mass of a dust particle, β and β^* are the coefficient of volumetric expansion parameter, C_0 and T_0 are concentration and temperature at origin respectively, C and T concentration and temperature of dusty liquid respectively, θ is the inclination to the horizontal, C_p is the specific heat at constant pressure, k is the thermal conductivity, D is the mass diffusivity, m is the hall current parameter and K_0 is the permeability of medium.

$$\left.\begin{array}{ll} t\le 0: & u_1=0=v_1,\quad T=T_0,\quad C=C_0 \quad at \quad y=0 \\ t>0: & u_1=0=v_1,\quad T=T_0,\quad C=C_0 \quad at \quad y=0 \\ & u_1=U,\quad T=T_0-e^{-\lambda^2 t},\quad C=C_0-e^{-\lambda^2 t} \quad at \quad y=h \end{array}\right\} \tag{8}$$

We express the pressure p as

$$p=-\rho g(x\sin\theta+y\cos\theta)-x\rho\phi(t) \tag{9}$$

With the help of equation (9), equations (2), (5) and (6) become:

$$\left(1+\alpha\frac{\partial}{\partial t}\right)\frac{\partial u_1}{\partial t}=F(t)+\upsilon\frac{\partial^2 u_1}{\partial y^2}+\frac{\kappa N_0}{\rho}\left(1+\alpha\frac{\partial}{\partial t}\right)(v_1-u_1) \tag{10}$$
$$-\frac{\sigma B_0^2}{\rho(1+m^2)}u_1-\frac{\upsilon}{K_0}u_1+g\beta C_1+g\beta^* T_1$$

$$\frac{\partial v_1}{\partial t}=\frac{\kappa}{m_1}(u_1-v_1) \tag{11}$$

$$\frac{\partial T_1}{\partial t}=\frac{k}{\rho Cp}\frac{\partial^2 T_1}{\partial y^2} \tag{12}$$

$$\frac{\partial C_1}{\partial t}=D\frac{\partial^2 C_1}{\partial y^2} \tag{13}$$

Where

$C_1=C_0-C,\quad T_1=T_0-T \quad and \quad F(t)=\phi(t)+\alpha\phi'(t)$

Corresponding boundary conditions are:

$$\left.\begin{array}{ll} t\le 0: & u_1=0=v_1,\quad T_1=0,\quad C_1=0 \quad at \quad y=0 \\ t>0: & u_1=0=v_1,\quad T_1=0,\quad C_1=0 \quad at \quad y=0 \\ & u_1=U,\quad T_1=e^{-\lambda^2 t},\quad C_1=e^{-\lambda^2 t} \quad at \quad y=h \end{array}\right\} \tag{14}$$

Let us choose u_1, v_1, T_1, C_1 and $F(t)$ as

$$\left.\begin{array}{l} u_1(y,t)=u(y).e^{-\lambda^2 t} \\ v_1(y,t)=v(y).e^{-\lambda^2 t} \\ T_1(y,t)=T_{11}(y).e^{-\lambda^2 t} \\ C_1(y,t)=C_{11}(y).e^{-\lambda^2 t} \\ F(t)=c.e^{-\lambda^2 t} \end{array}\right\} \tag{15}$$

Substituting the values of u_1, v_1, T_1, C_1 and $F(t)$ in equations (10) to (14), we get:

$$\frac{d^2 u}{dy^2}+\left(A^2-\frac{M_0^2}{(1+m^2)}-\frac{1}{K_0}\right)u=-d-GrT_{11}-GcC_{11} \tag{16}$$

$$v=\frac{k}{k-m_1\lambda^2}u \tag{17}$$

$$T''_{11}+\lambda^2 \Pr T_{11}=0 \tag{18}$$

$$C''_{11}+\lambda^2 ScC_{11}=0 \tag{19}$$

Where

$$A^2 = \frac{\lambda^2}{\upsilon}\left(1-\alpha\lambda^2\right)\left\{1+\frac{Mm_1}{(k-m_1\lambda^2)(1+m^2)}\right\}$$

$$M_0^2 = \frac{\sigma B_0^2}{\mu}, \quad M = \frac{kN_0}{\rho}, \quad d = \frac{c}{v}, \quad Gr = \frac{g\beta^*}{\upsilon},$$

$$Gc = \frac{g\beta}{\upsilon} \quad Sc = \frac{1}{D}, \quad \Pr = \frac{\rho Cp}{k}$$

The boundary conditions are:

$$\left.\begin{array}{llll} y=0, & u=0, & T_{11}=0, & C_{11}=0 \\ y=h, & u=U, & T_{11}=1, & C_{11}=1 \end{array}\right\} \quad (20)$$

Solution of equations (16) to (19) under boundary conditions (20) is given by

$$u(y) = \frac{d}{p^2}\cos(py) + \frac{\sin(py)}{\sin(ph)}\left\{U + \frac{Gr}{p^2-q^2} + \frac{Gc}{p^2-r^2} + \frac{d}{p^2}\left(1-\cos(ph)\right)\right\} \quad (21)$$
$$-\frac{d}{p^2} - \left(\frac{Gr}{p^2-q^2}\right)\frac{\sin(qy)}{\sin(qh)} - \left(\frac{Gc}{p^2-r^2}\right)\frac{\sin(ry)}{\sin(rh)}$$

$$v = \frac{k}{k-m\lambda^2}\left[\frac{d}{p^2}\cos(py) + \frac{\sin(py)}{\sin(ph)}\left\{U + \frac{Gr}{p^2-q^2} + \frac{Gc}{p^2-r^2} + \frac{d}{p^2}\left(1-\cos(ph)\right)\right\}\right. \quad (22)$$
$$\left.\frac{d}{p^2} - \left(\frac{Gr}{p^2-q^2}\right)\frac{\sin(qy)}{\sin(qh)} - \left(\frac{Gc}{p^2-r^2}\right)\frac{\sin(ry)}{\sin(rh)}\right]$$

$$T_{11} = \frac{\sin qy}{\sin qh} \quad (23)$$

$$C_{11} = \frac{\sin ry}{\sin rh} \quad (24)$$

Where

$$p^2 = A^2 - \frac{M_0^2}{(1+m^2)} - \frac{1}{K_0}, \quad q^2 = \lambda^2 \Pr, \quad r^2 = \lambda^2 Sc$$

The equations of velocities of the liquid and dust particle, temperature and concentration profile are expressed as

$$u_1 = \left[\frac{d}{p^2}\cos(py) + \frac{\sin(py)}{\sin(ph)}\left\{U + \frac{Gr}{p^2-q^2} + \frac{Gc}{p^2-r^2} + \frac{d}{p^2}\left(1-\cos(ph)\right)\right\}\right. \quad (25)$$
$$\left.-\frac{d}{p^2} - \left(\frac{Gr}{p^2-q^2}\right)\frac{\sin(qy)}{\sin(qh)} - \left(\frac{Gc}{p^2-r^2}\right)\frac{\sin(ry)}{\sin(rh)}\right].e^{-\lambda^2 t}$$

$$v_1 = \frac{k}{k-m\lambda^2}\left[\frac{d}{p^2}\cos(py) + \frac{\sin(py)}{\sin(ph)}\left\{U + \frac{Gr}{p^2-q^2} + \frac{Gc}{p^2-r^2} + \frac{d}{p^2}\left(1-\cos(ph)\right)\right\}\right. \quad (26)$$
$$\left.\frac{d}{p^2} - \left(\frac{Gr}{p^2-q^2}\right)\frac{\sin(qy)}{\sin(qh)} - \left(\frac{Gc}{p^2-r^2}\right)\frac{\sin(ry)}{\sin(rh)}\right].e^{-\lambda^2 t}$$

$$T_1 = \left[\frac{\sin qy}{\sin qh}\right].e^{-\lambda^2 t} \quad (27)$$

$$C_1 = \left[\frac{\sin ry}{\sin rh}\right].e^{-\lambda^2 t} \quad (28)$$

The skin friction of liquid is expressed as

$$\tau = \left(\frac{\partial u_1}{\partial y}\right)_{y=0} = \left[\frac{p}{\sin(ph)}\left\{U + \frac{Gr}{p^2-q^2} + \frac{Gc}{p^2-r^2} + \frac{d}{p^2}\left(1-\cos(ph)\right)\right\}\right.$$
$$\left.-\left(\frac{Gr}{p^2-q^2}\right)\frac{q}{\sin(qh)} - \left(\frac{Gc}{p^2-r^2}\right)\frac{r}{\sin(rh)}\right].e^{-\lambda^2 t}$$

Results and Discussion

The velocity profiles for visco-elastic fluid and dust particles are plotted in figure – 1 and 2 having graphs – (I) to (IV) at $\upsilon = 1$, $U = 1$, $t = 0.4$, $\lambda = 2$, $h = 10$, $m_1 = 0.2$, $k = 1$, $\alpha = 0.2$, $K_0 = 2$, $Gr = 2$, $Gc = 3$, $Sc = 0.4$ and different value of Prandtl number Pr, magnetic field parameter M_0 and hall current parameter m. From the graphs of figure – 1 and 2, it is noticed that velocity of visco-elastic liquid and dust particles are sinusoidal in nature. It is also observe that velocity of liquid and dust particles decreases with the increase in magnetic field parameter M_0 and increases with the increase in Pr and hall current parameter m.

The velocity profiles for visco-elastic liquid is plotted in figure – 3 having graphs – (I) to (IV) at $\upsilon = 1$, $U = 1$, $t = 0.4$, $\lambda = 2$, $h = 10$, m = 0.2, $k = 1$, $\alpha = 0.2$, $m_1 = 0.2$, $Gr = 2$, $Gc = 3$, Pr = 0.71, $M_0 = 0.01$, m = 0.1 and different value of Schmidt number Sc and porosity parameter K_0. From the graphs of figure –3, it is also observe that velocity of liquid increases with the increase in porosity parameter K_0 and decreases with the increase in Sc for first half cycle, after it the effect of K_0 and Sc on velocity is reversed for half cycle.

The temperature profile is plotted in figure – 4 having graph – (I) for Pr = 0.71 and graph – (II) for Pr = 0.80 at $\upsilon = 1$, $U = 1$, $t = 0.4$, $\lambda = 2$, $h = 10$, $m_1 = 0.2$, $k = 1$, $\alpha = 0.2$, $m_1 = 0.2$, $K_0 = 2$, $Gr = 2$, $Gc = 3$, $Sc = 0.4$, $M_0 = 0.01$, $m = 0.1$. From figure – 4, it is noticed that temperature is also sinusoidal in nature. It is observed that temperature decreases up to $y = 1.7$ with the increase in Pr, after it temperature increases upto $y = 3.4$ for first half cycle. This effect is reversed for second half cycle.

The concentration profile is plotted in figure –5 having graph – (I) for $Sc = 0.4$ and graph – (II) for $Sc = 0.5$ at $\upsilon = 1$, $U = 1$, $t =$

0.4, $\lambda = 2$, $h = 10$, m = 0.2, $k = 1$, $\alpha = 0.2$, $m_1 = 0.2$, $K_0 = 2$, $Gr = 2$, $Gc = 3$, Pr = 0.71, $M_0 = 0.01$, $m = 0.1$. From figure – 5, it is noticed that concentration is also sinusoidal in nature. It is observed that concentration decreases up to $y = 2.5$ with the increase in Sc, after it concentration increases upto $y = 5.0$ for first half cycle. This effect is reversed for second half cycle.

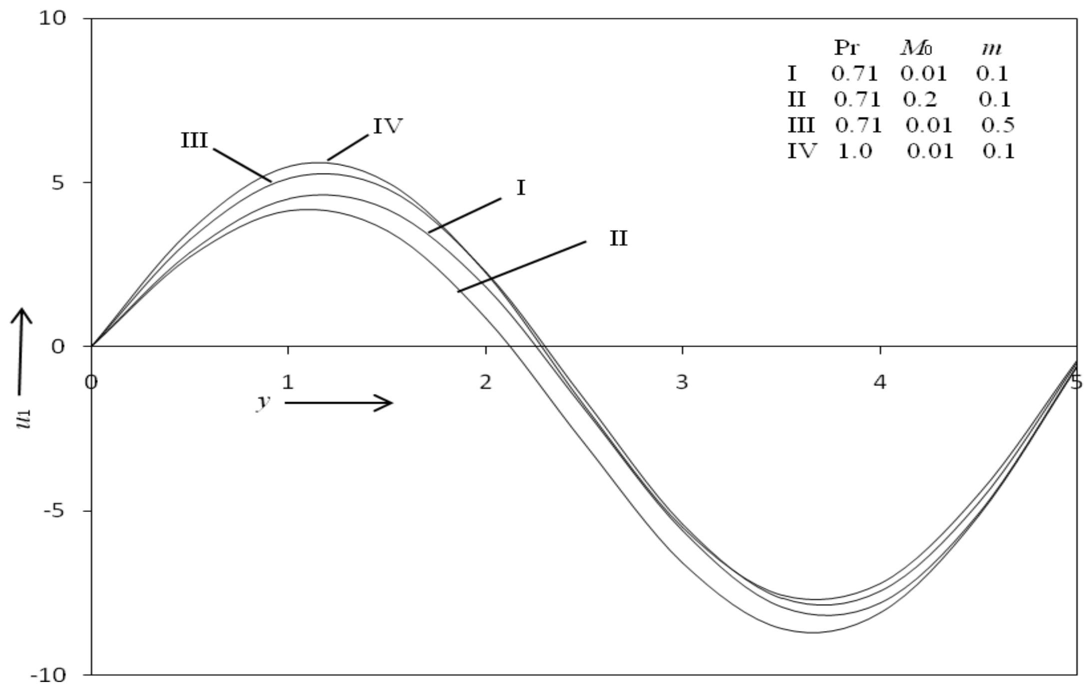

Figure-1
Velocity profile of liquid for different value of Pr, *M* and *m*

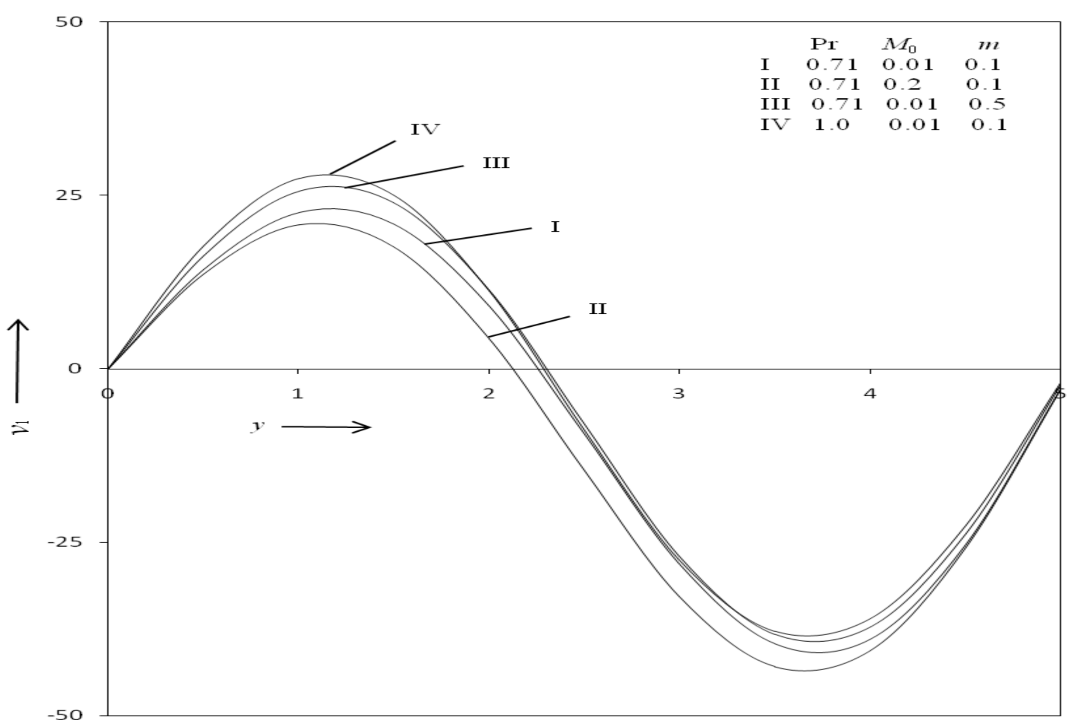

Figure-2
Velocity profile of dust particle for different value of Pr, *M* and *m*

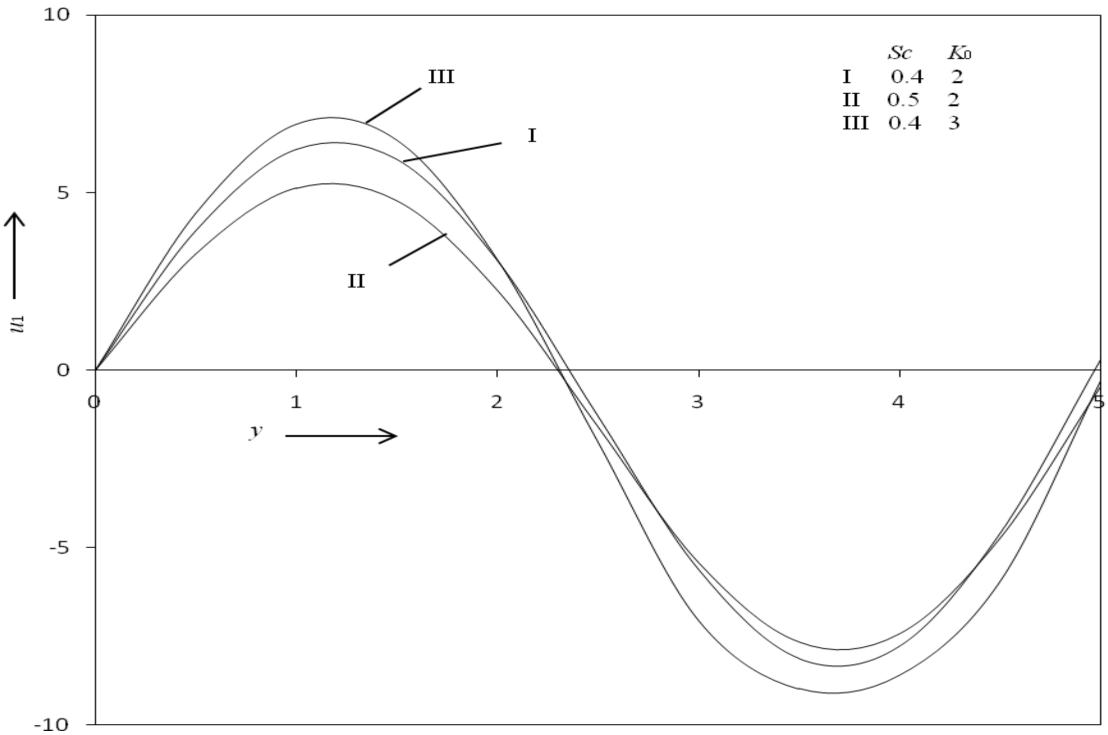

Figure-3
Velocity profile of liquid for different value of *Sc* and *K*0

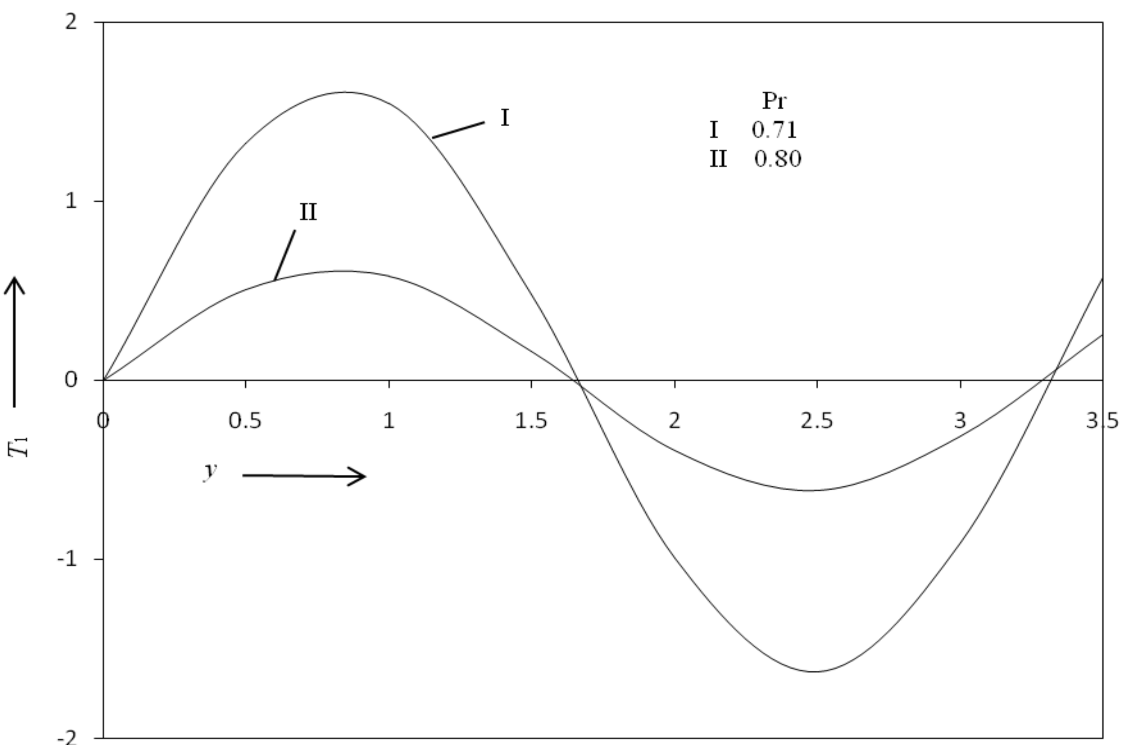

Figure-4
The temprature profile of liquid for different value of Pr

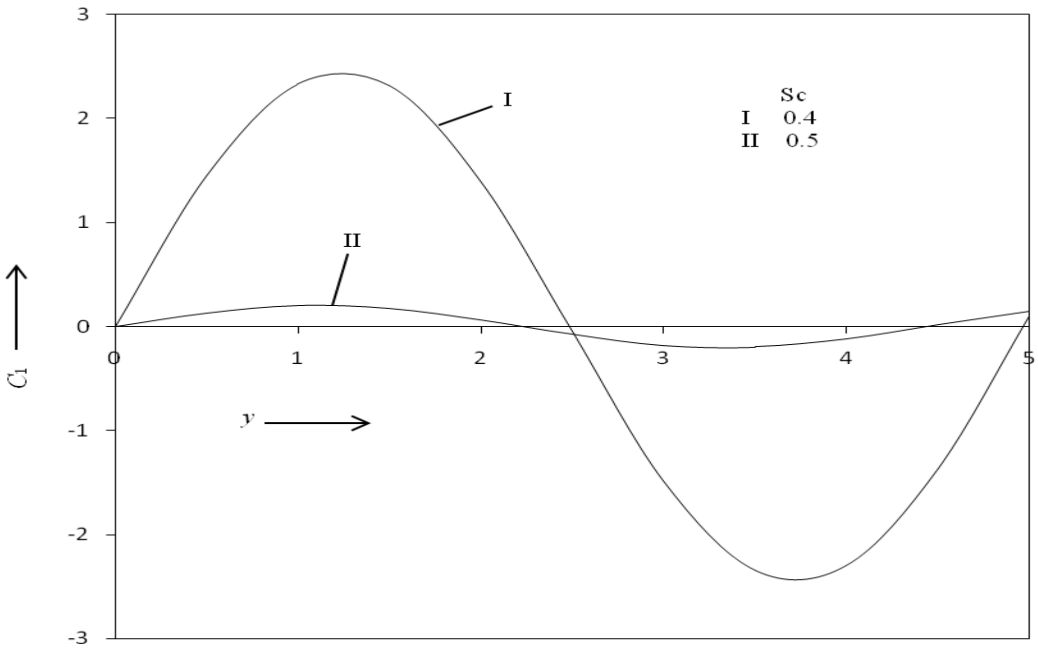

Figure-5
The concentration profile of liquid for different value of *Sc*

Conclusions

The theoretical solution for the effect of hall current on MHD laminar flow of an unsteady visco-elastic (Kuvshiniski type) liquid through porous medium with uniform distribution of dust particles past an inclined plane under the influence of exponential pressure gradient. The liquid is bounded by a parallel upper surface at a distance h from the plane. The study concludes the following results: i. The velocity of liquid and dust particles decreases with the increase in magnetic field parameter M_0 and increases with the increase in Pr and hall current parameter m. ii. The velocity of liquid increases with the increase in porosity parameter K_0 and decreases with the increase in Sc for first half cycle, after it the effect of K_0 and Sc on velocity is reversed for half cycle. iii. The temperature decreases up to y = 1.7 with the increase in Pr, after it temperature increases upto y = 3.4 for first half cycle. This effect is reversed for second half cycle. iv. The concentration decreases up to y = 2.5 with the increase in Sc, after it concentration increases upto y = 5.0 for first half cycle. This effect is reversed for second half cycle.

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