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An Inventory Model with Quadratic Demand, Constant Deterioration and Salvage Value

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Abstract

In this paper we have studied an inventory model for deteriorating products with demand rate is quadratic function of time .This model is developed to find the total cost of the inventory system. Here the deterioration is considered as constant. The salvage value is used for deteriorated items in the system. Suitable numerical example and sensitivity analysis is also discussed.

Keywords: Inventory, quadratic demand, salvage value, deteriorating items.

Introduction

The demand rate of any product is normally dynamic in nature. This phenomenon is due to price or time or even introducing of new products in the markets. Researchers extensively studied in the different aspects of time-varying demand patterns considering continuous and discrete. Mostly in continuous –time inventory models researchers studied linearly increasing or decreasing demand patterns and exponentially increasing or decreasing demand models. Particularly in time varying demand one may refer the literature review of Goyal and Giri, in realistic terms, the demand need not follow either linear or exponential trend[1]. Bhandari and Sharma have proposed a single period inventory problem with quadratic demand distribution under the influence of marketing policies[2]. In some commodities, due to seasonal variations may follow quadratic function of time [i.e., D(t) = $a + bt + ct^2$; $a \geq 0, b \neq 0, c \neq 0$]. Hence, time-dependent quadratic demand, Khanra and Chaudhuri, explains the accelerated growth/decline in the demand patterns. Depending on the signs of b and c we may explain different types of realistic demand patterns[3]. It is obvious that the demand for spare parts of new aircrafts, chips of advanced computers, etc. scaling very rapidly while the demands for spares of the obsolete aircrafts, computers etc. decrease very rapidly with respect to time. These concepts addressed well in the inventory models with quadratic demand rate. Sana and Chaudhuri (2004) studied a inventory model for stock-review of perishable items with uniform replenishment rate and stock-dependent demand[4]. Ghosh and Chaudhuri discussed an inventory model for a deteriorating item having an instantaneous supply, a quadratic time-varying demand considering shortages. In this model the deterioration rate is considered as Weibull distribution deterioration of two parameters[5]. Kaley McMahon discussed an inventory model of two paremeter weibull demand rate with shortages[6].

Venkateswarlu and Mohan proposed an EOQ model with 2-parameter Weibull deterioration, time dependent quadratic demand and salvage value[7]. Venkateswarlu and Mohan developed an EOQ model for time varying deterioration and price dependent quadratic demand with salvage value[8]. Mohan and Venkateswarlu studied an inventory management models with variable holding cost and salvage value[9]. Mohan and Venkateswarlu proposed an inventory model for, time Dependent quadratic demand with salvage considering deterioration rate is time dependent[10]. Recently, Mohan and Venkateswarlu developed an inventory model with Quadratic Demand, Variable Holding Cost with Salvage value using Weibull distribution deterioration rate[11]. Uttam Kumar KHEDELKAR and Diwahar SHUKLA and Raghovendra Partab Singh CHANDEL, proposed a logarithmic inventory model with shortage for deteriorating items[12]. R Babu Krishnaraj and K. Ramasamy studied an EOQ model for stock-dependent demand with weibull rate of deterioration[13]. Vinod kumar et.al studied an inventory model for deteriorating items with time-dependent demand and time –varying holding cost under partial backlogging[14]. Vikas Sharma and Rekha Rani Chaudhary studied an inventory model for deteriorating items with Weibull Distribution Time Dependent Demand and Shortages[15].

In this paper we consider an inventory model for deteriorating products having a quadratic demand w.r.to time, infinite time horizon and salvage value. Here the rate of deterioration is assumed as constant. The solution of the model, example and sensitivity of the models is discussed at the end.

Assumptions and notations

The following assumptions and notations are used for the development of the model: i. The demand D(t) at time t is assumed to be D(t) = $a + bt + ct^2$, $a \geq 0, b \neq 0, c \neq 0$. Here *a* is the initial rate of demand, *b* is the rate with which the demand rate increases and *c* is the rate with which the change in the rate demand rate itself increases. ii. Replenishment rate is infinite. iii. The deterioration rate, θ is constant. iv. Lead time is zero. v. *I(t)* is the inventory level at time *t. vi.* C, cost per unit. vii. C*i, $0 < i < 1$, the holding cost per unit. viii. *A* is the order cost per unit order is known and constant. ix. The salvage value γC, $0 \le \gamma < 1$ is associated with deteriorated units during a cycle time.

Formulation and solution of the model

This paper is developed to determine Total cost (TC) for items having time dependent quadratic demand and constant rate of deterioration with salvage value.

The governing differential equation of the inventory level at time t is as follows:

$$\frac{dQ(t)}{dt} + \theta Q(t) = -(a + bt + ct^2), \qquad 0 \le t \le T \quad (1)$$

The solution of equation (1) with boundary condition $I(T) = 0$, is

$$Q(t) = \begin{cases} \left(\frac{-a}{\theta} + \frac{b}{\theta^2} - \frac{2c}{\theta^3}\right)(1 - e^{\theta(T-t)}) + \frac{b}{\theta}(-t + Te^{\theta(T-t)}) + \\ \frac{c}{\theta}(-t^2 + T^2 e^{\theta(T-t)}) + \frac{2c}{\theta^2}(t - Te^{\theta(T-t)}) \end{cases} \quad (2)$$

The optimum order quantity is given by

$$I(0) = Q = \left(\frac{-a}{\theta} + \frac{b}{\theta^2} - \frac{2c}{\theta^3}\right)(1 - e^{\theta T}) + \frac{b}{\theta}Te^{\theta T} + \frac{c}{\theta}(T^2 e^{\theta T}) - \frac{2c}{\theta^2}(Te^{\theta T})$$

The Total Cost (*TC*) per unit time consists of the following costs: (i) The number of deteriorated units (*NDU*) during one cycle time is given by

$$NDU = Q - \int_0^T D(t)dt \quad (3)$$

where $D(t)=(a+ bt+ct^2)$.

(ii) Carrying cost/holding cost per cycle = $C * i \int_0^T I(t)dt$ (4)

$$= \frac{C*i}{T}\left[\frac{-\left(aT + \frac{bT^2}{2} + \frac{cT^3}{3}\right)}{\theta} - \frac{a}{\theta^2} + \frac{b}{\theta^3} - \frac{2c}{\theta^4} + \left(\frac{(a + bT + cT^2)}{\theta^2} - \frac{b + 2ct}{\theta^3} + \frac{2c}{\theta^4}\right)e^{\theta T}\right]$$

(iii) Cost due to Deterioration = $= \frac{C}{T}\left[\left(\frac{-a}{\theta} + \frac{b}{\theta^2} - \frac{2c}{\theta^3}\right) + \left(\frac{a + bT + cT^2}{\theta} - \frac{b + 2ct}{\theta^2} + \frac{2c}{\theta^3}\right)e^{\theta T}\right] - (aT + \frac{bT^2}{2} + \frac{cT^3}{3})$

(iv) Ordering Cost per cycle = $\frac{A}{T}$

(v) Salvage Value per cycle $= \frac{\gamma C}{T}(Q - (aT + \frac{bT^2}{2} + \frac{cT^3}{3}))$

Total cost (TC) = Ordering cost + Carrying cost + Cost due to Deterioration - Salvage Value

$$= \frac{A}{T} + \frac{C*i}{T}\int_0^T I(t)\,dt + \frac{C}{T}(Q - (aT + \frac{bT^2}{2} + \frac{cT^3}{3})) - \frac{\gamma C}{T}(Q - (aT + \frac{bT^2}{2} + \frac{cT^3}{3}))$$

$$= \begin{cases} \frac{A}{T} + \frac{C*i}{T}\left[\frac{-\left(aT + \frac{bT^2}{2} + \frac{cT^3}{3}\right)}{\theta} - \frac{a}{\theta^2} + \frac{b}{\theta^3} - \frac{2c}{\theta^4} + \left(\frac{(a + bT + cT^2)}{\theta^2} - \frac{b + 2ct}{\theta^3} + \frac{2c}{\theta^4}\right)e^{\theta T}\right] \\ \frac{C}{T}(1-\gamma)\left[\left[\left(\frac{-a}{\theta} + \frac{b}{\theta^2} - \frac{2c}{\theta^3}\right) + \left(\frac{a + bT + cT^2}{\theta} - \frac{b + 2ct}{\theta^2} + \frac{2c}{\theta^3}\right)e^{\theta T}\right] - (aT + \frac{bT^2}{2} + \frac{cT^3}{3})\right] \end{cases} \quad (5)$$

The necessary condition for total cost to be minimum is $\frac{\partial (TC)}{\partial T} = 0$, i.e.,

$$\frac{\partial(TC)}{\partial T} = \left\{ \begin{array}{l} -\frac{A}{T^2} - \frac{C*i}{T^2}\left[\frac{-\left(aT + \frac{bT^2}{2} + \frac{cT^3}{3}\right)}{\theta} - \frac{a}{\theta^2} + \frac{b}{\theta^3} - \frac{2c}{\theta^4} + \left(\frac{(a+bT+cT^2)}{\theta^2} - \frac{b+2ct}{\theta^3} + \frac{2c}{\theta^4} \right) e^{\theta T} \right] \\ + \frac{C*i}{T}\left[-\left(\frac{a+bT+cT^2}{\theta} \right) + \left(\frac{b+2cT}{\theta^2} - \frac{2c}{\theta^3} \right) e^{\theta T} + \left(\frac{(a+bT+cT^2)}{\theta^2} - \frac{b+2ct}{\theta^3} + \frac{2c}{\theta^4} \right) \theta e^{\theta T} \right] \\ -\frac{C}{T^2}(1-\gamma)\left[\left[\left(\frac{-a}{\theta} + \frac{b}{\theta^2} - \frac{2c}{\theta^3} \right) + \left(\frac{a+bT+cT^2}{\theta} - \frac{b+2ct}{\theta^2} + \frac{2c}{\theta^3} \right) e^{\theta T} \right] - (aT + \frac{bT^2}{2} + \frac{cT^3}{3}) \right] \\ + \frac{C}{T}(1-\gamma)\left[\left(\frac{b+2cT}{\theta} - \frac{2c}{\theta^2} \right) e^{\theta T} + \left(\frac{a+bT+cT^2}{\theta} - \frac{b+2cT}{\theta^2} + \frac{2c}{\theta^3} \right) \theta e^{\theta T} - (a+bT+cT^2) \right] \end{array} \right\} = 0 \quad (6)$$

Solving the above equation using MATHCAD for optimum T, we observed that the existence of the accelerated decline and retarded decline models for T > 0. For such T, *TC* is minimum only if $\frac{\partial^2 (TC)}{\partial T^2} > 0$ for all T>0. Through MATHCAD, we observed that $\frac{\partial^2 (TC)}{\partial T^2} > 0$ for all T>0.

Numerical Example

Model I: (Retarded Decline Model)
We now consider an inventory system with the following parameters:
a = 250, b = 20, c = 3, A = 150,
C= 3, $\theta = 0.1$, i = 0.2 $\gamma = 0.1$

MODEL-I: (a > 0, b > 0 and c < 0)

Table-1
(a > 0, b > 0 and c < 0)

Model Type	T	TC	Q	NDU
Quad. Demand	1.083	266.871	297.188	15.979
Linear Demand	1.074	267.726	295.844	15.809

Comparing the solution of the quadratic model with that of linear model, we noticed the following changes: i. the cycle time increased by 0.84%, ii. the economic lot size is increased by 0.45%, iii. the number of deteriorated units are increased by 1.06%, iv. the total cost is decreased by 0.32%

Thus the changes in economic lot size, cycle time, number of deteriorated units and total cost are not significant in the case of quadratic demand rate for retarded decline models.

Sensitivity of the model: Model I (a > 0, b > 0 and c < 0)

Table-2
Sensitivity for salvage parameter γ

	T	TC	Q	NDU
$\gamma = 0.1$	1.083	266.871	297.188	15.979
$\gamma = 0.15$	1.092	264.648	299.879	16.257
$\gamma = 0.2$	1.101	262.405	302.574	16.537
$\gamma = 0.25$	1.11	260.142	305.272	16.819
$\gamma = 0.3$	1.12	257.857	308.275	17.136

Table-3
Sensitivity of the deterioration parameter θ

	T	TC	Q	NDU
$\theta = 0.1$	1.083	266.871	297.188	15.979
$\theta = 0.12$	1.046	275.667	289.264	17.967
$\theta = 0.14$	1.013	284.235	282.217	19.745
$\theta = 0.16$	0.982	292.592	275.487	21.291
$\theta = 0.18$	0.953	300.756	269.111	22.644

Table-4
Sensitive analysis of the parameters γ and θ

		T	TC	Q	NDU
$\theta = 0.1$	$\gamma = 0.1$	1.083	266.871	297.188	15.979
$\theta = 0.12$	$\gamma = 0.15$	1.056	273.078	292.301	18.327
$\theta = 0.14$	$\gamma = 0.2$	1.033	278.326	288.386	20.568
$\theta = 0.16$	$\gamma = 0.25$	1.015	282.662	285.824	22.817
$\theta = 0.18$	$\gamma = 0.3$	1.000	286.12	284.053	25.053

From tables 3–4, it is observed that the total cost of the system increases with the increases in the value of the parameter θ while the same total cost decreases as the salvage parameter increases from γ = 0.1 to γ=0.3. However the rate of change in total cost is not so significant.

Numerical Example

Model – II: Accelerated Decline Model
To illustrate the effectiveness of the model, we consider the following values for the parameters:
a = 250, b = 20, c = 3, A = 150,
C= 3, θ = 0.1, i = 0.2, $\gamma = 0.1$

MODEL-I1: (a > 0, b < 0 and c < 0)

Table-5
(a > 0, b < 0 and c < 0)

Model Type	T	TC	Q	NDU
Quad. Demand	1.227	250.901	307.992	18.144
Linear Demand	1.207	252.136	304.92	17.739

We now compare the solution of the quadratic model with that of linear model, we observed the following changes: i. the cycle time increased by 1.66%, ii. the economic lot size is increased by 1.01%, iii. the number of deteriorated units increased by 2.28%, iv. the total cost is decreased by only 0.49%.

The accelerated decline model also behaves like retarded decline model but the rate of change is slightly more than the retarded decline model. The deteriorated units are increased slightly which does not yield more salvage value and as a result the total cost of the system is decreased marginally.

Sensitivity of the model: MODEL-II: (a > 0, b < 0 and c < 0)

Table-6
Sensitivity for salvage parameter γ

	T	TC	Q	NDU
γ = 0.1	1.227	250.901	307.992	18.144
γ = 0.15	1.238	248.673	310.739	18.463
γ = 0.2	1.25	246.426	313.735	18.813
γ = 0.25	1.262	244.158	316.731	19.167
γ = 0.3	1.275	241.868	319.974	19.553

Table-7
Sensitivity of the parameter θ

	T	TC	Q	NDU
θ= 0.1	1.227	250.901	307.992	18.144
θ= 0.12	1.178	259.819	299.257	20.268
θ= 0.14	1.134	268.498	291.295	22.113
θ= 0.16	1.094	276.956	283.942	23.719
θ= 0.18	1.057	285.213	277.002	25.106

Table-8
Sensitive analysis of the parameter γ and θ

		T	TC	Q	NDU
θ= 0.1	γ = 0.1	1.227	250.901	307.992	18.144
θ= 0.12	γ = 0.15	1.19	257.226	302.329	20.675
θ= 0.14	γ = 0.2	1.16	262.585	298.107	23.124
θ= 0.16	γ = 0.25	1.135	267.024	294.918	25.512
θ= 0.18	γ = 0.3	1.115	270.58	292.851	27.92

From tables 6 –8, we infer the following observations: The total cost for this model also decreases with the increase in salvage parameter γ. When the deterioration parameter θ increases the total cost of the system also increases. But the rate of change in either case is very small. But when both are θ and γ are changed, the total cost is increased which shows that salvage parameter is not so significant to determine the total cost of the system.

Conclusion

In this paper we have developed deterministic inventory model for constant deteriorating items when the demand is a quadratic function of time. The optimum total cost and total order quantity has been calculated in each case. It is found that the retarded decline and accelerated decline have shown good results which will be useful to describe a realistic situation for any product. Inventory models for constant deterioration rate together with salvage value have been formulated. It is found that the existence of retarded decline and accelerated decline models in this case. The behavior of these models is almost similar.

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