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Two-dimensional model to describe movement of a car

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Abstract

In this paper, we introduce a two-dimensional model of the car's movement with account curvilinear motion with different speeds, as well as an analytical approach for analyzing this model. This model is a generalization of the models considered earlier in the literature, and the analytical approach used to analyze the introduced model is also more general and allows you to take into account a greater number of factors affecting the movement of the car.

Keywords: Model, movement, analytical approach, analysis of movement.

Introduction

In the framework of a busy city traffic flow, it is necessary to make prognosis of the movement of cars to increase their safety and reduce the route, which leads to faster delivery of products and fuel economy[1-5]. One of the classic steps in choosing a vehicle path is calculating its possible trajectory depending on various parameters. In this paper, we introduce a two-dimensional model, which is more general in comparison with the models proposed in the literature. An analytical approach for analyzing of this model is also introduced. Figure-1 and 2 show the structure of the considered model.

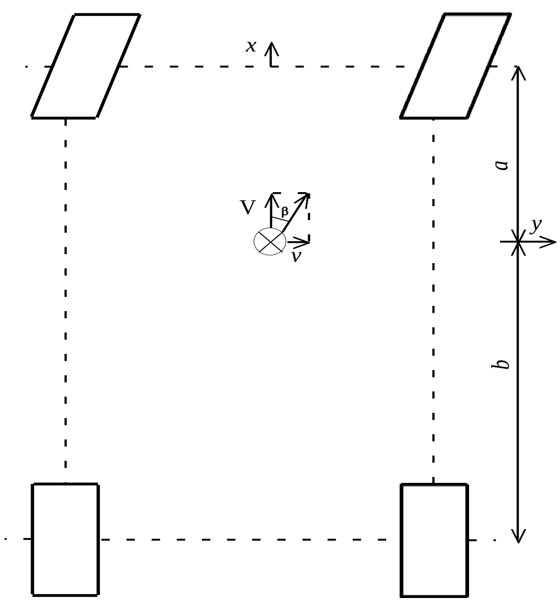

Figure-1: Vehicle lateral angle.

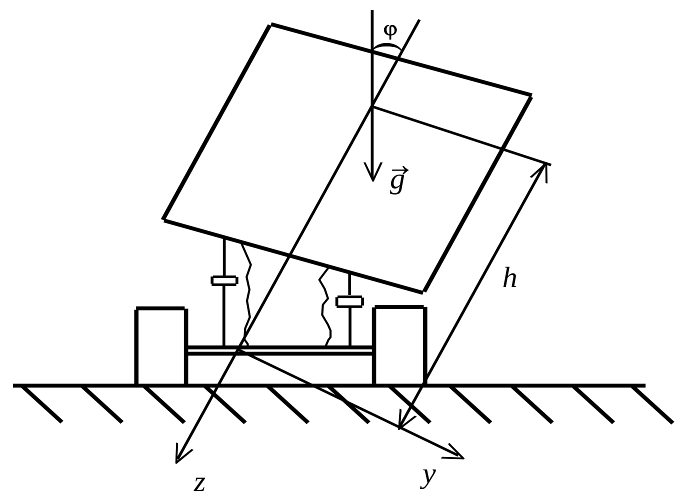

Figure-2: Scheme of the heeling moment in the *yz* plane.

In the Figure-1 and 2, the following notation is introduced: a is the distance from the center of gravity of the vehicle to the front axle; b is the distance from the center of gravity of the car to the rear axle; V is the translational speed of movement; v is the lateral displacement velocity; ω is the angular velocity of rotation of the car relative to the vertical axis z; θ is the angle of rotation of the steered wheels; β is the angle between the longitudinal speed and lateral displacement of the car due to the action of lateral forces; φ is the angle of the side roll of the car; h is the distance from the center of gravity of the unstrung masses to the roll axis; g is the acceleration of gravity; M_n is the sprung mass of the car.

Method of analysis

The translational movement of the car could be described using Newton's second law[6]:

$m\dfrac{d^2\vec{r}}{dt^2}=\vec{F}$, where *m* is the mass of the car; $\vec{r}$ is the radius vector; $\vec{F}$ is the external forces acting on the car. Projections on the coordinate axes of the Newton's second law could be presented in the following form

$$m\frac{d^2x}{dt^2}=F_{tx},\ m\frac{d^2y}{dt^2}=F_{ty}. \tag{1}$$

Here *x* and *y* are the spatial coordinates; *t* is the current time; F_{tx} is the traction force acting on the car along the *x* coordinate; $F_{ty}=\dfrac{\partial F_{ty}}{\partial\beta}\beta+\dfrac{\partial F_{ty}}{\partial\omega}\omega+\dfrac{\partial F_{ty}}{\partial\theta}\theta+\dfrac{\partial F_{ty}}{\partial\varphi}\varphi$ are the force acting on the car along the *y* coordinate; $\partial F_{ty}/\partial\beta$ is the coefficient of lateral force caused by the presence of wheel drive; $\partial F_{ty}/\partial\omega$ is the coefficient of lateral force caused by the rotation of the car relative to the vertical axis passing through its center of gravity; $\partial F_{ty}/\partial\theta$ is the coefficient of lateral force caused by turning steering wheels of a car; $\partial F_{ty}/\partial\varphi$ is the coefficient of lateral force caused by the roll of the car. The rotational motion of the car could be described by the following equation (2)

$$\frac{d\vec{L}}{dt}=\vec{M},\ \vec{L}=I\vec{\omega}. \tag{2}$$

Here $\vec{L}$ is the vehicle moment of the car; $\vec{M}$ is the moment of force turning the car; *I* is the moment of inertia of the car. The magnitude of the moment of force that rotates the car around the z axis is determined using the same ratio F_{ty}: $M_z=\dfrac{\partial M_z}{\partial\beta}\beta+$ $+\dfrac{\partial M_z}{\partial\omega}\omega+\dfrac{\partial M_z}{\partial\theta}\theta+\dfrac{\partial M_z}{\partial\varphi}\varphi$, where $\dfrac{\partial M_z}{\partial\beta}$ is the coefficient of turning torque caused by the presence of a drive wheel; $\partial M_z/\partial\omega$ is the coefficient of turning moment caused by the rotation of the car relative to the vertical axis passing through its center of gravity; $\partial M_z/\partial\theta$ is the coefficient of turning moment caused by the rotation of the car relative to the vertical axis passing through its center of gravity; $\partial M_z/\partial\varphi$ is the torque coefficient caused by vehicle roll; M_z is the projection on the coordinate axis *z*. Similarly, for the projection of the moment of force M_x, one can write:

$M_x=\dfrac{\partial M_x}{\partial p}p+\dfrac{\partial M_x}{\partial\varphi}\varphi$, where *p* is the angular velocity of the roll of the car regarding the axis *x*; $\partial M_x/\partial p$ is the roll damping coefficient; $\partial M_x/\partial\varphi$ is the lateral stiffness. Integration of the right and left parts of Equations (1a) and (2b) on time leads to the following result

$$x=\frac{1}{m}\int_0^t(t-\tau)F_{tx}d\tau,\ y=\frac{1}{m}\int_0^t(t-\tau)F_{ty}d\tau, \tag{1a}$$

$$\theta=\frac{1}{I_x}\int_0^t(t-\tau)M_xd\tau,\ \varphi=\frac{1}{I_z}\int_0^t(t-\tau)M_zd\tau. \tag{2a}$$

The initial values of the coordinates and speeds of the translational motion, as well as the rotation angles and their corresponding speeds, were considered as zero. Substitution of the projections of forces and moments into relations (1a) and (2a) leads to the final result.

Discussion

In this section, we analyze the coordinates and angles of various parameters considered in the relations (1*a*) and (2*a*). Figure-3 and 4 show the dependences of the *x* coordinate and velocity projection v_x on time for constant values of the forces acting on the vehicle. The dependences of the *y* coordinate, rotation angles, and corresponding velocities on time are similar to those shown in Figure-3 and 4. Figure-5 shows the dependence of the *y* coordinate on the lateral force coefficient. Its dependence on other coefficients, as well as the dependences of the *x* coordinate, rotation angles, and corresponding velocities on the force and moment coefficients are qualitatively similar to those shown in Figure-5 dependencies.

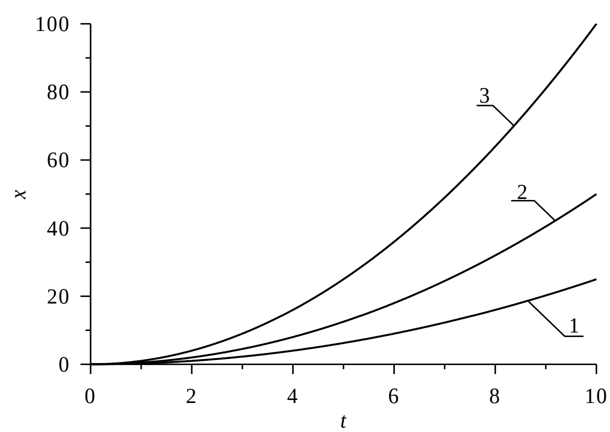

Figure-3: Typical dependences of *x* coordinate on time. Increasing number of curves corresponds to increasing of the corresponding force component.

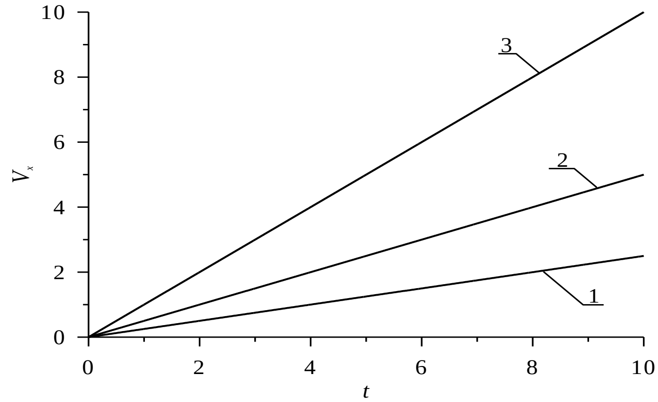

Figure-4: Typical dependences of V_x speed on time. Increasing number of curves corresponds to increasing of the corresponding force component.

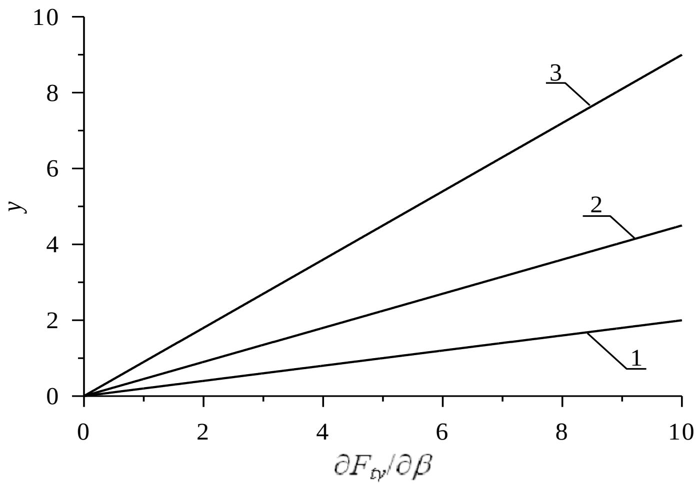

Figure-5: Typical *x* coordinate dependences on lateral force coefficient $\partial F_{ty} / \partial \beta$. Increasing number of curves corresponds to increasing of the corresponding force coefficient.

Conclusion

In this paper, a quantitative approach for analyzing the movement of a car is introduced taking with account curvilinear motion with different speeds. The analysis includes into itself a two-dimensional model of the motion and an analytical approach for its analysis. The analysis of vehicle movement from the parameters.

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