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Free Transverse Vibrations of Visco- Elastic C-C-C-C Square Plate with Thickness and Temperature variation

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Abstract

The mainstream of visco- elastic materials are receptive to heat and in space technology, highly speed space flights, internal combustion engines, satellites, certain parts of mechanical structures have to man oeuvre under elevated temperatures consequently the state of affairs are thermal sensitive. It is observed that thermal effects are recurrently overlooked in most of the cases so far they have to be taken in to concern. It is significant to study vibration behavior in the presence of thermal gradient due to rising use of modern materials in structural components. Moreover, weight of composite structure is minimized by optimizing its layup and by tapering its thickness. This paper is the study of thermal effect on free transverse vibrations of clamped square plate. Rayleigh-Ritz technique is applied to give a good approximation for the frequency corresponding to the first two modes of vibration.

Keywords: Square plate, frequency, thickness, thermal effect, taper constant.

Introduction

Vibrations are classified in to two ways i.e. first is desirable, controlled, required or wanted vibrations and the other is undesirable, uncontrolled, not required or unwanted vibrations. On the other hand, unwanted vibration causes fatigues. Unwanted vibration can damage electronic components of aerospace system, damage buildings by earthquake, bring tsunami, and contribute to toppling of tall smokestacks, collapse of a suspension bridge in a windstorm. Hence vibrations totally affect our day-to-day life.

In the course of time, the study of vibration of plates has acquired great weight age in the field of research, engineering and space technology. Further, the study of vibration behavior in the presence of thermal gradient of visco-elastic plates is required due to its practical importance in the field of engineering because Machines very repeatedly operate under diverse temperature conditions. In majority of cases the impact of temperature are ignored yet they need to be taken in to consideration. The reason behind this is that during heated up periods structures are exposed to high intensity heat fluxes and the material properties undergo significant changes hence the thermal effect on modulus of elasticity of material cannot be neglected. Most of engineering materials are found to have linear relationship between modulus of elasticity and temperature. Applications of such materials are due to lessening of weight and size, low operating cost and enhancement in efficiency and strength. Further, tapering saves weight by removing preventable weight. Thickness tapering is advantageous since stresses tend to vary appreciably within the structure.

The present paper deal with the vibrations of isotropic square plate is to determine the thermal effect and thickness varying parabolically and linearly in two directional. It is clamped (C-C-C-C) supported on all the four edges.

Formulation of the Problem

Square plate with uniform thickness, where ‘a’ be the length and breadth . Axes ‘x’ and ‘y’ are taken along the edges of the plate. Plate having the four sides clamped (C-C-C-C) boundary conditions:

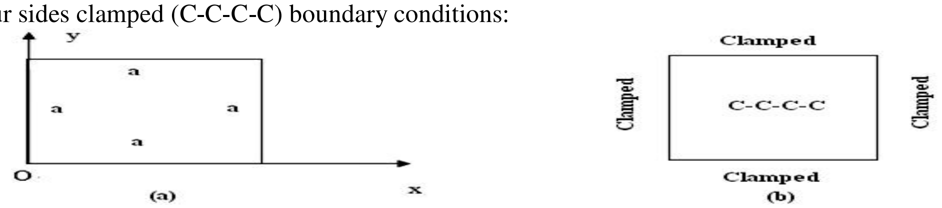


Figure -1
(a) Geometry of the plate (b) boundary conditions

Differential equation of transverse motion of a visco-elastic square plate of variable thickness in Cartesian co-ordinates[1]:

$$\frac{\partial^2 M_x}{\partial x^2} + 2\frac{\partial^2 M_{yx}}{\partial x \partial y} + \frac{\partial^2 M_y}{\partial y^2} = \rho h \frac{\partial^2 w}{\partial t^2} \tag{1}$$

The expression for M_x, M_y, M_{yx} are given by

$$\left.\begin{aligned} M_x &= -\widetilde{D} D_1\left(\frac{\partial^2 w}{\partial x^2} + \vartheta \frac{\partial^2 w}{\partial y^2}\right) \\ M_y &= -\widetilde{D} D_1\left(\frac{\partial^2 w}{\partial y^2} + \vartheta \frac{\partial^2 w}{\partial x^2}\right) \\ M_{yx} &= -\widetilde{D} D_1 (1-\vartheta)\frac{\partial^2 w}{\partial y \partial x} \end{aligned}\right\} \tag{2}$$

where $\widetilde{D}$ is visco-elastic operator.

On substitution the values M_x, M_y and M_{yx} from equation (2) in (1) and taking w, as a product of two function, equal to w(x,y,t)=W(x,y)T(t), equation (1) become:

$$\left[\begin{aligned} &D_1\left(\frac{\partial^4 W}{\partial x^4} + 2\frac{\partial^4 W}{\partial x^2 \partial y^2} + \frac{\partial^4 W}{\partial y^4}\right) + 2\frac{\partial D_1}{\partial x}\left(\frac{\partial^3 W}{\partial x^3} + \frac{\partial^3 W}{\partial x \partial y^2}\right) + 2\frac{\partial D_1}{\partial y}\left(\frac{\partial^3 W}{\partial y^3} + \frac{\partial^3 W}{\partial x^2 \partial y}\right) \\ &+ \frac{\partial^2 D_1}{\partial x^2}\left(\frac{\partial^2 W}{\partial x^2} + \vartheta\frac{\partial^2 W}{\partial y^2}\right) + \frac{\partial^2 D_1}{\partial y^2}\left(\frac{\partial^2 W}{\partial y^2} + \vartheta\frac{\partial^2 W}{\partial x^2}\right) + 2(1-\vartheta)\frac{\partial^2 D_1}{\partial x \partial y}\frac{\partial^2 W}{\partial x \partial y} \end{aligned}\right] \Big/ \rho h W = -\frac{\ddot{T}}{\widetilde{D} T} \tag{3}$$

Here dot denote differentiation with respect to t, taking both sides of equation (3) are equal to a constant p^2 (square of frequency), we have

$$\left[\begin{aligned} &D_1(W_{,xxxx} + 2W_{,xxyy}) - 2D_{1,x}(W_{,xxx} + W_{,xyy}) + 2D_{1,y}(W_{,yyy} + W_{,yxx}) + D_{1,xx}(W_{,xx} + \vartheta W_{,yy}) + D_{1,yy}(W_{,yy} + \vartheta W_{,xx}) \\ &+ 2(1-\vartheta) D_{1,xy} W_{,xy} \end{aligned}\right] - \rho h p^2 W = 0 \tag{4}$$

is a differential equation of transverse motion for homogeneous plate of variable thickness. Here, D_1 is the flexural rigidity of plate

i.e. $D_1 = \frac{E h^3}{12(1-\vartheta^2)}$ (5)

and corresponding two-term deflection function is taken as[2]:

$$W = \left[\left(\frac{x}{a}\right)\left(\frac{y}{a}\right)\left(1-\frac{x}{a}\right)(1-y/a)\right]^2 \left[A_1 + A_2\left(\frac{x}{a}\right)\left(\frac{y}{a}\right)\left(1-\frac{x}{a}\right)(1-y/a)\right] \tag{6}$$

Here, we have to consider two dimensional parabolic temperature variations:

$$\tau = \tau_0(1 - x^2/a^2)(1 - y^2/a^2) \tag{7}$$

where τ denotes the temperature excess above the reference temperature at any point on the plate and τ_0 denotes the temperature at any point on the boundary of plate and "a" is the length of a side of square plate. The temperature dependence of the modulus of elasticity for most of engineering materials can be expressed as[3]:

$$E = E_0(1 - \gamma\tau) \tag{8}$$

where, E_0 is the value of the Young's modulus at reference temperature i.e. $\tau = 0$ and λ is the slope of the variation of E with τ. The modulus variation equation (5) become

$$E = E_0[1 - \alpha(1 - x^2/a^2)(1 - y^2/a^2)] \tag{9}$$

where $= \gamma\tau_0 (0 \le \alpha < 1)$, thermal gradient.

It is assumed that thickness also varies linearly in x and y directions as shown below:

$$h = h_0(1 + \beta_1 x/a)(1 + \beta_2 y/a) \tag{10}$$

where β_1 and β_2 is taper parameters in x and y directions respectively and $h=h_0$ at x=y=0.
Put the value of *E* and *h* from equation (9) and (10) in the equation (5), one obtain

$$D_1 = \frac{[E_0[1 - \alpha(1 - x^2/a^2)(1 - y^2/a^2)] h_0 (1 + \beta_1 x/a)(1 + \beta_2 y/a)]}{12(1-\vartheta^2)} \tag{11}$$

Solution and Frequency Equation

Rayleigh-Ritz technique is applied to solve the frequency equation. In this method, one requires maximum strain energy must be equal to the maximum kinetic energy[4]. So it is necessary for the problem under consideration that

$$\delta(V^* - T^*) = 0 \qquad (12)$$

for arbitrary variations of W satisfying relevant geometrical boundary conditions.

Since the plate is assumed as clamped at all the four edges, so the boundary conditions are:

$$\left.\begin{array}{ll} W = W_{,x} = 0, & x = 0, a \\ W = W_{,y} = 0, & y = 0, a \end{array}\right\} \qquad (13)$$

Now assuming the non-dimensional variables as

$$X = \frac{x}{a},\ Y = \frac{y}{a},\ \ \overline{W} = \frac{W}{a},\ \ \overline{h} = \frac{h}{a} \qquad (14)$$

The kinetic energy K* and strain energy S* are[5]

$$K^* = \left(\frac{1}{2}\right)\rho p^2 \overline{h_0} a^5 \int_0^1\int_0^1 \left[(1+\beta_1 X)(1+\beta_2 Y)\,\overline{W^2}\right]\, dY dX \qquad (15)$$

and

$$S^* = Q\int_0^1\int_0^1 \left[[1-\alpha(1-X^2)(1-Y^2)][(1+\beta_1 X)(1+\beta_2 Y)]^3 \left\{\left(\overline{W}_{,XX}\right)^2 + \left(\overline{W}_{,YY}\right)^2 + 2\vartheta \overline{W}_{,XX}\overline{W}_{,YY} + 2(1-\vartheta)\left(\overline{W}_{,XY}\right)^2\right\}\right] dY dX \qquad (16)$$

where, $Q = \frac{E_0 {h_0}^3 a^3}{24(1-\vartheta^2)}$

Using equations (15) and (16) in equation (12), one get $(S^{**} - \lambda^2 K^{**}) = 0$ (17)

where, $S^{**} = \int_0^1\int_0^1 \left[[1-\alpha(1-X^2)(1-Y^2)][(1+\beta_1 X)(1+\beta_2 Y)]^3 \left\{\left(\overline{W}_{,XX}\right)^2 + \left(\overline{W}_{,YY}\right)^2 + 2\vartheta \overline{W}_{,XX}\overline{W}_{,YY} + 2(1-\vartheta)\left(\overline{W}_{,XY}\right)^2\right\}\right] dY dX$ (18)

and

$$K^* = \int_0^1\int_0^1 \left[(1+\beta_1 X)(1+\beta_2 Y)\,\overline{W^2}\right]\, dY dX \qquad (19)$$

Here, $\lambda^2 = 12\frac{\rho p^2(1-\vartheta^2)a^2}{E_0 {h_0}^2}$ is a frequency parameter.

Equation (19) consists two unknown constants i.e. A_1 and A_2 arising due to the substitution of W. These two constants are to be determined as follows[6]:

$$\frac{\partial(S^{**} - \lambda^2 K^{**})}{\partial A_n}, = 0 \quad , n=1, 2 \qquad (20)$$

On simplifying (20), we get

$$c_{n1}A_1 + c_{n2}A_2 = 0 \quad , n=1, 2 \qquad (21)$$

where bn_1, bn_2 (n=1,2) involve parametric constant and the frequency parameter.

For a non-trivial solution, the determinant of the coefficient of equation (21) must be zero. So one gets, the frequency equation as

$$\begin{vmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{vmatrix} \qquad (22)$$

With the help of equation (22), one can obtains a quadratic equation in λ^2 from which the two values of λ^2 can found. These two values represent the two modes of vibration of frequency i.e. λ_1 (Mode1) and λ_2 (Mode2) for different values of taper constant and thermal gradient for a clamped plate.

Results and Discussion

All calculations are carried out with the help of software i.e. MATLAB. Computation has been done to obtain first two modes of frequency of square plate:

Figure 2: It is clearly seen that value of frequency decreases as thermal gradient ‘α’ increases from 0.0 to 1.0 for $\beta_1 = \beta_2 = 0.0$ and $\beta_1 = \beta_2 = 0.6$ for both modes of vibrations. Also, note that frequency increases fast as taper parameters (β_1) increase from 0.0 to 0.4 and 0.6 respectively.

Figure 3: Value of frequency increases with the increment in taper parameter β_1 from 0.0 to 1.0 for following cases:
i) α= 0.3, β_2= 0.2.
ii) α= 0.6, β_2= 0.4.
Interesting to note that frequency increases with the increment in β_1 from 0.0 to 1.0.

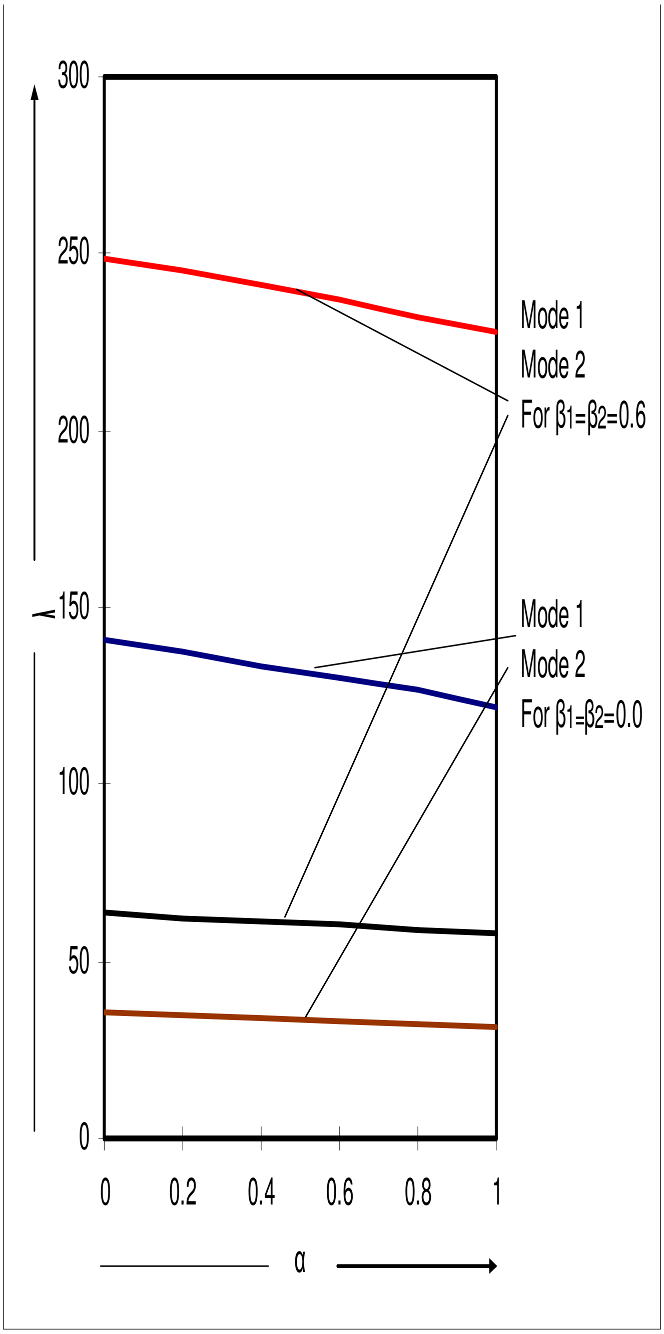
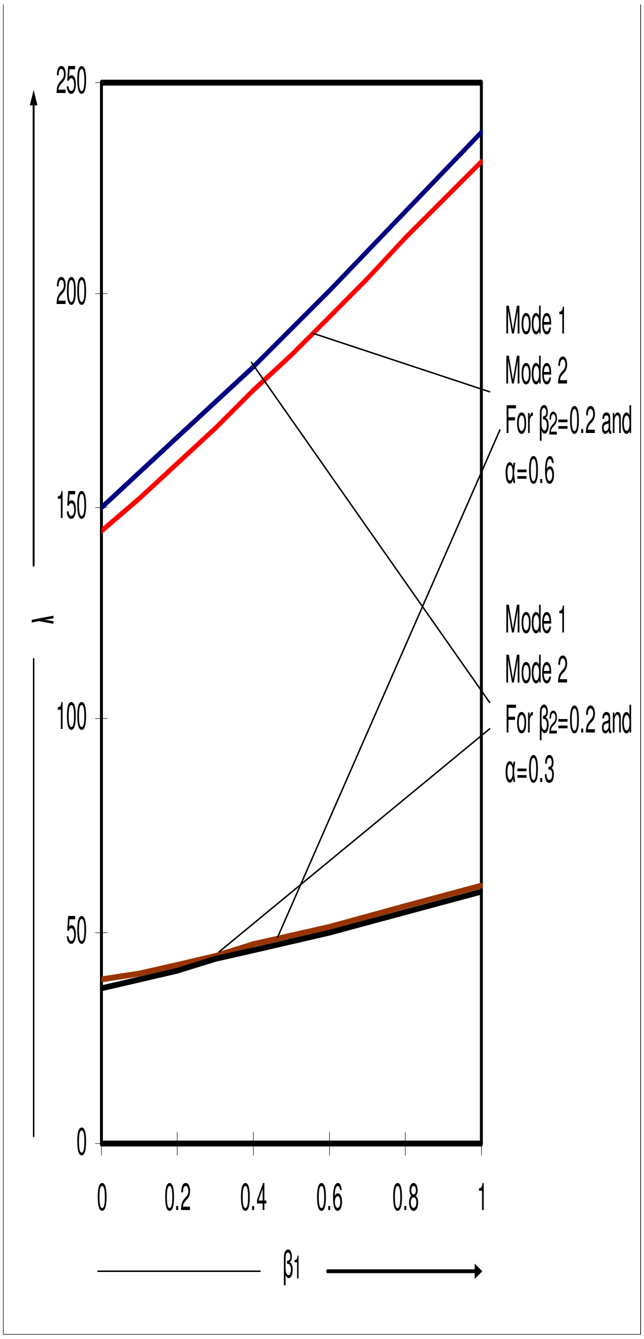


Figure-2
Frequency vs. Thermal gradient



Figure–3
Frequency vs. Taper Parameter

Conclusion

Motive is to provide such kind of a mathematical design so that scientist can perceive their potential in mechanical engineering field and increase strength, durability and efficiency of mechanical design and structuring with a practical approach .Actually this is the need of the hour to develop more but authentic mathematical model for the help of mechanical engineers/researchers/practitioners .Therefore mechanical engineers and technocrats are advised to study and get the practical importance of the present paper and to provide much better structure and machines with more safety and economy.

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