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Generating Critical Values of Unit Root Tests

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Abstract

In modeling time series econometrics nonstationarity test is very essential part to make some early decision. In order to test whether the time series is stationary or nonstationary, it is necessary to know the critical values for different sample sizes. Initially we required different critical values for testing unit root. But these critical values are not available in a wide range to us. So we need to generate extent tables of critical values. In this paper, we generated critical values for different sample size using Monte Carlo Simulation for testing unit root.

Keywords: Monte Carlo simulation, DF test, ADF test, critical values.

Introduction

In classical testing approach, critical values are required to test hypothesis. Dickey, D. A., and W. A. Fuller [1, 2], Dickey, D. A.[3], Dickey, D., A., Bell, W. R. and Miller, R, B.[4], Gujarati, D. N.[5], Hamilton, J. D.[6] and Maddala, G. S.[7] reported such values for different tests of testing unit roots, but though reported are very limited. This paper attempts to generate critical values for different sample sizes. Simulated critical values cannot be calculated by exact mathematical form. In this paper we generated simulated critical values of DF and ADF tests for different sample size and compare the simulated critical values with the tabulated values.

Critical Values

Critical values for a test of hypothesis depend upon a test statistic, which is specific to the type of test, and the significance level α, which defines the sensitivity of the test. A value of $\alpha = 0.05$ implies that the null hypothesis is rejected 5% of the time when it is in fact true. The choice of α is somewhat arbitrary, although in practice values of 0.1, 0.05, and 0.01 are common. Critical values are essentially cut-off values that define regions where the test statistic is unlikely to lie; for example, a region where the critical value is exceeded with probability α if the null hypothesis is true. The null hypothesis is rejected if the test statistic lies within this region which is often referred to as the rejection region(s).

Simulated Critical Values

The distribution form of test statistic of the unit root tests look like t-statistic, but its actual distribution differ from t-statistic. So there is no exact distribution pattern of these tests. So we need some simulated critical values. For this case we have written the GAUSS program. By using this program we can generate critical values for any sample sizes. Our simulated critical values are approximately similar to the tabulated critical values and the critical values only for sample size 25, 50, 100, 250, 500, ∞.

Generated Sequence of Observations from Different Models

Monte Carlo Simulation required from generated observations. In this section we demonstrated how to generate sequence of observations for the model $\Delta y_t = \delta y_{t-1} + u_t$ and in the same way we generated observations of the newly proposed distance based model.

Consider the model, $Model\ 1: y_t = \rho y_{t-1} + u_t$, $Model\ 2: y_t = \alpha + \rho y_{t-1} + u_t$, $Model\ 3: y_t = \alpha + \beta t + \rho y_{t-1} + u_t$,
Where $u_t \sim N\left(0, \sigma^2\right)$

In these models, we test the hypothesis $H_0: \rho = 1 \ \ vs \ \ H_1: \rho < 1$

Table-5
Comparison between simulated critical value of Dickey-Fuller and Augmented Dickey-Fuller table

Sample size	t_{nc}				t_c				t_{ct}			
	1%	1%	5%	5%	1%	1%	5%	5%	1%	1%	5%	5%
25	-2.68	-2.63	-1.99	-1.95	-3.88	-3.86	-3.11	-3.09	-4.65	-4.55	-3.64	-3.79
50	-2.64	-2.62	-1.97	-1.97	-3.63	-3.63	-2.98	-2.97	-4.20	-4.14	-3.42	-3.54
100	-2.60	-2.60	-1.95	-1.97	-3.51	-3.55	-2.89	-2.92	-3.59	-3.59	-3.20	-2.91
250	-2.56	-2.60	-1.93	-1.96	-3.45	-3.47	-2.86	-2.88	-3.85	-2.69	-3.06	-2.00
500	-2.56	-2.57	-1.95	-1.94	-3.19	-3.44	-2.54	-2.86	-3.80	-2.48	-3.01	-1.79
∞	-2.57	-2.55	-1.92	-1.94	-3.42	-3.45	-2.86	-2.52	-2.44	-2.48	-1.78	-1.77

From the above tables we have shown that our simulated critical values are approximately same to the critical values given for Dickey-Fuller test. In the following table we generated the critical values of Dickey-Fuller test and Augmented Dickey-Fuller test for different sample sizes and are given in table 6. If the random seed changes then the critical values of Dickey-Fuller test and Augmented Dickey-Fuller test one given in the table.

Table-6
Generated critical value of Dickey-Fuller table for sample size *n*=20, 21,….50

Sample Size	t_{nc}		t_c		t_{ct}	
	1%	5%	1%	5%	1%	5%
20	-2.7912	-2.0226	-4.0478	-3.2178	-4.9185	-3.9973
21	-2.7665	-2.0098	-4.0097	-3.1894	-4.8272	-3.9415
22	-2.7297	-1.9791	-4.0058	-3.1775	-4.8232	-3.9051
23	-2.6966	-1.9916	-3.9049	-3.1309	-4.7512	-3.8622
24	-2.706	-1.9847	-3.9295	-3.1464	-4.6634	-3.861
25	-2.6897	-1.9969	-3.8827	-3.116	-4.6587	-3.8197
26	-2.7172	-1.9965	-3.8807	-3.1139	-4.6291	-3.8218
27	-2.7062	-1.9946	-3.8324	-3.1094	-4.5906	-3.7762
28	-2.7473	-2.0093	-3.8396	-3.0871	-4.5686	-3.7781
29	-2.6972	-1.9848	-3.8591	-3.0923	-4.5575	-3.7526
30	-2.6686	-1.9804	-3.8206	-3.0739	-4.5386	-3.7457
31	-2.649	-1.9858	-3.7788	-3.0644	-4.5209	-3.7165
32	-2.6653	-1.9681	-3.8078	-3.0674	-4.4785	-3.7226
33	-2.6852	-1.9725	-3.7743	-3.0568	-4.4334	-3.7095
34	-2.675	-1.974	-3.7538	-3.0445	-4.4181	-3.6853
35	-2.643	-1.9802	-3.7424	-3.0409	-4.4289	-3.6766
36	-2.667	-1.9784	-3.7169	-3.0314	-4.3974	-3.6628
37	-2.6771	-1.9835	-3.7302	-3.0295	-4.3919	-3.641
38	-2.6765	-1.9747	-3.7555	-3.0369	-4.3479	-3.6536
39	-2.6617	-1.9822	-3.6732	-3.0116	-4.3409	-3.6236
40	-2.659	-1.9707	-3.725	-3.0205	-4.3336	-3.6127
41	-2.6347	-1.9634	-3.6809	-3.0105	-4.3077	-3.5997
42	-2.6679	-1.9741	-3.6974	-3.0096	-4.3046	-3.5969
43	-2.6503	-1.9751	-3.6763	-2.9995	-4.2747	-3.5766
44	-2.6871	-1.9878	-3.6781	-2.9939	-4.2793	-3.5727
45	-2.6663	-1.9591	-3.6595	-2.999	-4.2519	-3.5519
46	-2.6463	-1.9883	-3.6642	-2.988	-4.2502	-3.5507
47	-2.6313	-1.9812	-3.6698	-2.9931	-4.2612	-3.5529
48	-2.6331	-1.9557	-3.6638	-2.9875	-4.2087	-3.536
49	-2.6386	-1.9548	-3.6617	-2.9953	-4.2248	-3.5267
50	-2.6468	-1.9752	-3.6317	-2.9813	-4.2083	-3.5101

Table-8
Generated critical value of Dickey-Fuller table for sample size *n*=20, 21,….50 (changing the random seed)

Sample Size	t_{nc}		t_c		t_{ct}	
	1%	5%	1%	5%	1%	5%
20	-2.7641	-2.0099	-3.9205	-3.1107	-4.6271	-3.7599
21	-2.7318	-1.9992	-3.9208	-3.1109	-4.6058	-3.7328
22	-2.7329	-2.0027	-3.8645	-3.0865	-4.5545	-3.7118
23	-2.7158	-1.9863	-3.8685	-3.086	-4.4909	-3.6963
24	-2.7058	-1.9977	-3.8372	-3.0573	-4.4834	-3.6604
25	-2.7106	-1.9811	-3.8238	-3.0635	-4.4378	-3.6526
26	-2.7621	-1.9875	-3.8398	-3.0668	-4.4201	-3.6525
27	-2.7232	-1.9984	-3.8146	-3.065	-4.4138	-3.6529
28	-2.6708	-1.9825	-3.7855	-3.0363	-4.3888	-3.6236
29	-2.6567	-1.9712	-3.7934	-3.0363	-4.4006	-3.6253
30	-2.6777	-1.9873	-3.7384	-3.0147	-4.364	-3.5997
31	-2.7	-1.9723	-3.7272	-3.0068	-4.3663	-3.6113
32	-2.7142	-1.9903	-3.7263	-3.0043	-4.3215	-3.5805
33	-2.6855	-1.9771	-3.7394	-3.0088	-4.3329	-3.5892
34	-2.6424	-1.9739	-3.7024	-3.0046	-4.3263	-3.5788
35	-2.6606	-1.9568	-3.7097	-2.9918	-4.2657	-3.5595
36	-2.6761	-1.9675	-3.6794	-2.9844	-4.2703	-3.5464
37	-2.6483	-1.9734	-3.6593	-2.9725	-4.27	-3.548
38	-2.6522	-1.9779	-3.6907	-2.9786	-4.2221	-3.5344
39	-2.6696	-1.9737	-3.6812	-2.9932	-4.2224	-3.5178
40	-2.6451	-1.9704	-3.6131	-2.9706	-4.2036	-3.4999
41	-2.6629	-1.981	-3.6445	-2.9846	-4.2229	-3.4986
42	-2.6542	-1.9888	-3.6271	-2.9833	-4.1835	-3.4939
43	-2.6633	-1.9774	-3.6383	-2.9645	-4.2152	-3.4987
44	-2.6297	-1.9619	-3.5997	-2.9555	-4.1727	-3.4823
45	-2.6518	-1.9746	-3.6142	-2.9619	-4.1564	-3.4858
46	-2.6519	-1.9915	-3.6212	-2.9501	-4.1405	-3.4816
47	-2.6488	-1.9596	-3.6307	-2.9607	-4.1375	-3.4689
48	-2.6349	-1.9674	-3.6104	-2.9602	-4.187	-3.4906
49	-2.64	-1.9718	-3.61	-2.9572	-4.1062	-3.4497
50	-2.6414	-1.9591	-3.6118	-2.9543	-4.1126	-3.4502

Conclusion

Unit root test is potentially a serious problem for time series data. Most of the unit root tests have estimation and testing problems. Hence the test of hypothesis becomes invalid and gives serious misleading conclusions about the statistical significant for the estimated coefficients as well as the model. There are different types of unit root tests for testing unit root. But these exiting tests have a number of drawbacks. In this paper we introduce some concept of simulated critical values. Also we discuss how to generate critical values of Dickey-Fuller type statistics and finally we generated simulated critical values for a wide range sample sizes. These tables can be used to tests unit roots for different sizes. Considered in this paper, the program is written in GAUSS code and is available to the author.

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