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Restricted Testing Procedure and Modified Dickey-Fuller Test

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Abstract

Usual test of testing unit root such as Dickey-Fuller (DF) test, Augmented Dickey-Fuller (ADF) test, Phillips-Perron (PP) test and Kwiatkowski- Phillips-Schmidt-Shin (KPSS) test ignore sign and boundary parameters. Ignorance of these problems may results in unusual estimate and test results. This paper demonstrates the ignorance of sign and boundary of parameters and consequences in estimation and hypothesis test by Monte Carlo simulation. This work proposes a distance-based optimum solution for testing unit root subject to the restriction of boundary and sign problem. Monte Carlo simulation study shows that the proposed one-sided test of testing unit root performs better than the usual tests interms of power property

Keywords: Monte Carlo simulation, DF test, ADF test, PP test.

Introduction

In time series econometrics, nonstationarity test is essential for analyzing the behavior of time series and for advance research. Usually this can be tested by different unit root tests such as Dickey-Fuller (DF) test, Augmented Dickey-Fuller (ADF) test, Phillips-Perron (PP) test and Kwiatkowski- Phillips-Schmidt-Shin (KPSS) test discussed by Dickey, D. A., and W. A. Fuller[1, 2], D. Kwiatkowski, P. C. B. Phillips, P. Schmidt, and Y. Shin[3]., Dickey, D. A., Hasza, D. P., and Fuller, W. A.[4] , Dickey D. A. and Gonzalez-Farias, G.[5] , Majumder, A. K.[6], and Enders, W.[7]. Almost all the unit root tests as well as the estimated model suffer from sign and boundary problems of the parameters. According to the assumption of the Dickey-Fuller test, $|\rho| < 1 \ \ or \ \ -2 < \delta < 0$ of the time series models, such as $y_t = \rho y_{t-1} + u_t$. Any estimated value of δ less than –2 or greater than 0 may results in invalid model. This invalid model cannot be used for making decision regarding nonstationarity. To overcome this situation it is necessary to make suitable restrictions on the parameters, which is greater than zero. Hence, the usual DF test for testing unit root is not always suitable and we need to develop an appropriate one-sided hypothesis test of unit root. But, very few literatures are available on this issue. In this paper, the main attempt to find a suitable estimation and testing procedure for unit root with constraint.

Restricted Estimation and hypothesis testing: In testing restricted alternatives two stage estimation methods are required in distance based techniques; in the first stage we estimate the parameter(s) by usual method, such as, least square method, maximum likelihood method, etc. In the second stage we estimate the constraint parameter(s) by optimizing under the restriction.

For example, we might be interested in testing the following hypotheses,

$$H_0 : \beta_i = 0 \ \ against \ \ H_1 : \beta_i > 0 \tag{1}$$

$$H_0 : \beta_i = \beta_{i0} \ \ against \ \ H_1 : \beta_i < \beta_{i0} \tag{2}$$

Here β_{i0} *is some specific value.*

$$H_0 : \beta_2 + \beta_3 = 1 \ \ against \ \ H_1 : \beta_2 + \beta_3 > 1 \tag{3}$$

$$H_0 : \beta_1 = \beta_2 \ \ or \ \beta_3 - \beta_4 = 0 \ \ against \ \ H_1 : 2\beta_1 + \beta_3 < 2 \tag{4}$$

$$H_0 : \begin{bmatrix} \beta_1 \\ \beta_3 \\ . \\ . \\ . \\ \beta_k \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ . \\ . \\ . \\ 0 \end{bmatrix} \ \ against \ \ H_1 : \begin{bmatrix} \beta_2 \\ \beta_3 \\ . \\ . \\ . \\ \beta_k \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \\ . \\ . \\ . \\ 0 \end{bmatrix} \tag{5}$$

All of the hypotheses are restricted. But the hypothesis (5) is unrestricted. In case of all the hypotheses except the hypothesis (5) the usual two-sided test cannot be applied. Hence we have to develop one-sided or partially one-sided testing procedures.

There are different methods to develop such testing procedures. Among them we are mainly concerned about distance-based approach because our testing approach is developed using this approach whose null hypothesis distribution follows a mixture of corresponding two-sided distribution. The following paragraph illustrates aforesaid problems. For testing strictly one-sided or partially one-sided alternative we make decision about accepting or rejecting the null hypothesis depending on whether, in some sense, the estimated parameter under test is closer to the null value or to an alternative value. We call this approach to hypothesis test a distance-based approach.

In the simple two parameters case, suppose $y_1, y_2, ..., y_n$ being is a random sample from a $N(\mu, \sigma^2)$ population and we are interested in testing $H_0 : \mu = 0$ *against* $H_a : \mu > 0$. If the population variance σ^2 is unknown, then the t-test is uniformly most powerful invariant (UMPI), and the magnitude and sign of test statistic depends on sample mean of observed data. For any negative value of the sample mean $\bar{y}$, we could accept the null hypothesis because the estimated value of μ is closer to H_0 than to any value. On the other hand, for large positive values of the sample mean, we consider the possibility that it could be caused by a positive true mean rather than a zero mean and reject if it is significantly large.

Model and Hypothesis

The unit root test can be applied to the three models
i. $\Delta y_t = \delta y_{t-1} + u_t$, ii. $\Delta y_t = \alpha + \delta y_{t-1} + u_t$, iii. $\Delta y_t = \alpha + \beta t + \delta y_{t-1} + u_t$
where $\delta = (\rho - 1)$ and Δ is the usual first difference operator.

Now we have to test the null hypothesis
$H_0 : \delta = 0$ i.e., the time series is nonstationary, against alternative. $H_a : \delta < 0$ i.e., the time series is stationary.
Or, equivalently
$H_0 : \rho = 1$, against, $H_a : \rho \le 1$.

The usual Dickey-Fuller (DF) test: The usual Dickey-Fuller test examines the condition that the model has a unit root and differencing helps to remove this unit root. Dickey-Fuller suggested that under the null hypothesis the estimated coefficient of y_{t-1} in the model $y_t = \rho y_{t-1} + \varepsilon_t$ follows the tau (τ) statistic, must be compared to critical values tabulated by Dickey-Fuller.

In the above three models, the null hypothesis is that $\delta = 0$ implies that $\rho = 1$. That is the time series is non-stationary it means that it has unit root. The alternative hypothesis is that δ is less than 0 implies that the time series is stationary and no decision is made on the basis of alternative hypothesis.

Proposed Dickey-Fuller (DF) test: The DF test can be applied to the three models
i. $\Delta y_t = \delta y_{t-1} + u_t$, ii. $\Delta y_t = \alpha + \delta y_{t-1} + u_t$, iii. $\Delta y_t = \alpha + \beta t + \delta y_{t-1} + u_t$

All of the models are valid when $|\rho| < 1$ *i.e.,* $-2 < \delta < 0$. So models are invalid when ρ values are different from ρ values are specified above. In constructing DF test ignores this restriction. Ignorance of these two restrictions may result in three different problems: i. Estimated parameters values will be overestimated or under estimated, ii. At the same time the test statistic based on this estimates loss power due to ignoring of these restrictions. iii. Very often ignorance of this sign problems result in invalid estimated models.

Decisions based on these models are quite unacceptable. In the following table we report the proportion of time, we may ignorance of sign problem provide invalid estimated models by Monte Carlo Simulation.

We generated each model under H_0 100 times with sample sizes $20, 50, 100, 200, 500$ .We assumes that the true values of α and β for the last two models as 1 and 2, respectively. , we observe that estimation are invariant with the choice of α and β. The proportion of time usual and our proposed (optimized) estimation procedure provides invalid estimated equations.

Table-1
The proportion of time the estimated δ values are invalid for the usual and optimized procedure

Model	Sample size n	True value of δ	Proportion of time $\hat{\delta}$ value is positive	True value of δ	Proportion of time $\hat{\delta}$ value is positive	Proportion of time optimized $\hat{\delta}$ value is positive
$\Delta y_t = \delta y_{t-1} + u_t$	20	0	37	-2	35	All of the values are zero
	50		35		37	
	100		35		28	
	200		32		30	
	500		30		22	
$\Delta y_t = \alpha + \delta y_{t-1} + u_t$	20		30		33	
	50		38		33	
	100		42		34	
	200		55		31	
	500		38		40	
$\Delta y_t = \alpha + \beta t + \delta y_{t-1} + u_t$	20		87		43	
	50		75		41	
	100		55		41	
	200		34		32	
	500		45		32	

Since the parameters $\delta < 0,\ \infty < \alpha < \beta < \infty$, we estimate the model by minimizing the error sum of squares for three models are:

i. $Model\ 1: \sum_{t=0}^{T} u_t^2 = \sum_{t=0}^{T} (\Delta y_t - \delta y_{t-1})^2$, ii. $Model\ 2: \sum_{t=0}^{T} u_t^2 = \sum_{t=0}^{T} (\Delta y_t - \alpha - \delta y_{t-1})^2$, iii. $Model\ 3: \sum_{t=0}^{T} u_t^2 = \sum_{t=0}^{T} (\Delta y_t - \alpha - \beta t - \delta y_{t-1})^2$

Where $-2 < \delta < 0,\ \infty < \alpha < \beta < \infty$.
We can also estimate the ADF model by minimizing the error sum of squares

$$\sum_{t=0}^{T} u_t^2 = \sum_{t=0}^{T} \left(\Delta y_t - \alpha - \beta t - \delta y_{t-1} - \sum_{j=1}^{p} \gamma_j \Delta y_{t-j} \right)^2$$

Where $-2 < \delta < 0,\ \infty < \alpha < \beta < \infty$.
For these models the τ -statistic does not follow the usual τ distribution. In these case the τ -statistic follows the weighted mixture τ distribution.(See Mujumder,1999). Our proposed τ -statistic is:

$$\tilde{\tau} = \frac{\tilde{\delta}}{SE(\tilde{\delta})}$$

where $\tilde{\tau}$ is the optimized τ -statistic and $\tilde{\delta}$ is the optimized estimated parameter.

Sometimes the decision based invalid and valid proposed (optimized) procedure may be similar but the estimated coefficients are likely to be different. The following simulation results support our argument.

Results

The usual DF test is not totally suitable for testing the hypothesis mentioned above. This is because the hypothesis is strictly one-sided. For testing this hypothesis, we are interested to develop a testing procedure using distance-based approach called distance-based DF test. The test statistic of our proposed DF test follows weighted mixture of τ distribution whereas the test statistic of usual DF test follows τ distribution.

Conclusion

Most of the unit root tests have estimation and testing problems. Hence the test of hypothesis becomes invalid and gives serious misleading conclusions about the statistical significance of the estimated coefficients as well as the model. There are different types of unit root tests for testing unit root. But these exiting tests have a number of drawbacks. Our main objective is to develop a distance-based one-sided testing approach for testing unit root in different unit root tests. The usual method of testing unit root is DF test considering three models. As the coefficient of δ is lies between –2 to 0, we found that our proposed DF test for testing unit root gives better result than the usual DF test.

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