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Inference on Curved Poisson Distribution Using its Statistical Curvature

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Abstract

Statistical Curvature of Bradley Efron helps in comparing curved exponential family of distributions with corresponding exponential family of distribution. The analysis in curved family has been made easy by that concept. But suitable test procedures are unavailable for discrete curved exponent families. In this paper a suitable test procedure for Curved Poisson distribution is obtained

Keywords: Statistical curvature, curved exponential family, curved Poisson distribution, likelihood function..

Introduction

Curved exponential family of distributions plays an important role in Statistics. But in case of inference on such distributions we have to face certain obstacles because curved exponential family does not have good statistical properties as the exponential family. Bradley Efron[1] has compared curved exponential family with exponential family introducing the concept of Statistical Curvature or Efron's Curvature. He has shown in his paper that families with small curvature enjoy good statistical properties of exponential family as exponential family has curvature zero. Moreover it can be shown that if one takes an example of a distribution from exponential family and deals it as a curved exponential family then the statistical curvature of such curved exponential family becomes exactly equal to zero. So, after finding the values of the involved parameter of a curved exponential distribution for which the statistical curvature attains small values, it can be concluded that for such values of the parameter, the test statistic of certain hypothesis is equivalent to that of the corresponding exponential family.

It has been found that some inference procedures are available fragmentally for continuous distributions of curved exponential family. But in case of discrete distributions from curved exponential family we cannot find such procedures for testing purpose.

Using statistical curvature this paper wants to find out a suitable test procedure for Curved Poisson distribution, in which the parameter of Poisson distribution involves another discrete distribution. In this paper Uniform distribution is taken as the involved discrete distribution.

Some preliminary discussions regarding this topic

Definition 2.1 (Curved Exponential Family of Distributions): Let $X = (X_1, X_2,\ldots, X_d)$ have a distribution P_θ, $\theta \in \Theta \subseteq R^q$. Suppose P_θ has a density (pmf) of the form $f(x \mid \theta) = \exp(\sum_{i=1}^{k} \eta_i(\theta) T_i(x) - \Psi(\theta)) h(x)$,

where $k > q$. Then the family $\{P_\theta, \theta \in \Theta\}$ is called curved exponential family[2].

Example 2.1: A set of independently and identically distributed random variables which follow $N(\theta, \theta^2)$, θ is the unknown parameter involved in the distribution[2]. This is an example of a curved exponential distribution of continuous type.

Example 2.2: Suppose X and Y are two random variables such that

$$\left.\begin{matrix} X \sim B(n, p) \\ Y \sim B(m, p^2) \end{matrix}\right\} \succ independently$$

where p is the unknown parameter involved in both the distributions[2]. This is an example of a curved exponential distribution of discrete type.

Definition 2.2 (Statistical Curvature): The concept of mathematical curvature extends to the curved lines in Euclidian k-space, E^k, say, £= {η_θ, $\theta \in \Theta$}, where Θ is the interval of real line. For each θ, η_θ is a vector in E^k whose component wise derivatives with respect to θ is denoted by

$$\dot{\eta}_\theta \equiv \left(\partial / \partial\theta\right)\eta_\theta \text{ and } \ddot{\eta}_\theta \equiv \left(\partial^2 / \partial\theta^2\right)\eta_\theta$$

These derivatives are assumed to exist continuously in neighborhood of a value of θ where it is wished to define the curvature. Let us also suppose that a k×k symmetric non-negative definite matrix Σ_θ is defined continuously in θ.

Let M_θ be a 2×2 matrix, with entries denoted by $\nu_{20}(\theta)$, $\nu_{11}(\theta)$, $\nu_{02}(\theta)$,defined by

$$M_\theta \equiv \begin{pmatrix} \upsilon_{20}(\theta) & \upsilon_{11}(\theta) \\ \upsilon_{11}(\theta) & \upsilon_{02}(\theta) \end{pmatrix} \equiv \begin{pmatrix} \dot{\eta}_\theta{}'\Sigma_\theta \dot{\eta}_\theta & \dot{\eta}_\theta{}'\Sigma_\theta \ddot{\eta}_\theta \\ \ddot{\eta}_\theta{}'\Sigma_\theta \dot{\eta}_\theta & \ddot{\eta}_\theta{}'\Sigma_\theta \ddot{\eta}_\theta \end{pmatrix} \qquad 2.1$$

and let

$$\gamma_\theta \equiv \left(\left|M_\theta\right| / \upsilon_{20}{}^3(\theta)\right)^{1/2} \qquad 2.2$$

Then γ_θ is "the curvature of £ at θ with respect to inner productΣ_θ". γ_θ (given in 2.2), the statistical curvature of Ғ at θ, is the geometric curvature of £={η_θ : $\theta \in \Theta$ } at θ with respect to the covariance inner product Σ_θ as defined in 2.1 and 2.2.

Here Ғ will stand for the family of densities {$f_\theta(x)$: $\theta \in \Theta$}, our curved exponential family. Statistical Curvature is also known as Efron's Curvature.

Example 2.3: The value of the statistical curvature of the distribution given in example 2.1 is $= \left(\frac{2}{1331\, n}\right)^{1/2}$ for all possible values of θ. That is for all possible values of θ the statistical curvature have very small values. So, here the test procedure for θ is equivalent to that of for exponential Normal distribution.

Example 2.4: The statistical curvature of the distribution given in example 2.2 is given in the following figure:

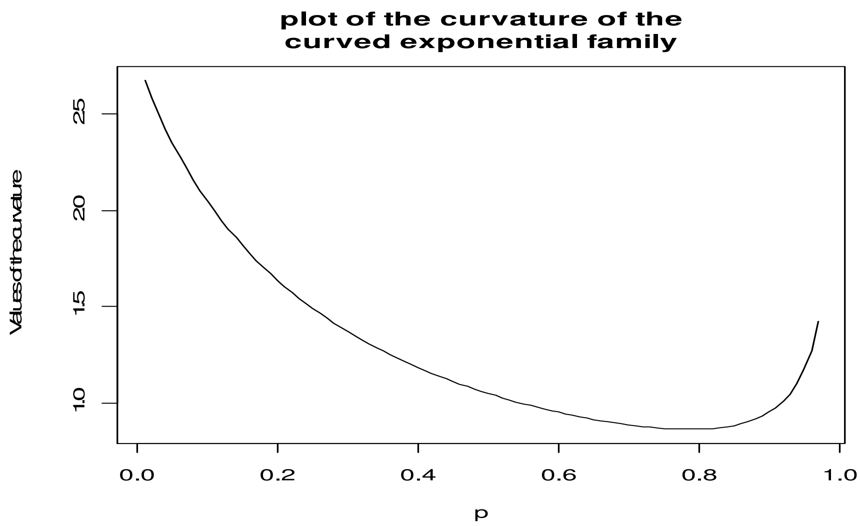

Figure -1
Figure -1, it is seen that for p=0.6 to 0.85 the value of the curvature is small

A suitable test procedure for p in the distribution given in example 2.2 has been tried to be found out by Sanchayita Sadhu and Babulal Seal[3].

Definition 2. 3 (Curved Poisson Distribution): A distribution is defined as Curved Poisson distribution if it is of the following form.

Suppose $Z_i = z_i$, i = 1,2,…,n. X_i are independent Poi(λz_i) variables and $Z_1,Z_2,\ldots,Z_n$ have some joint p.m.f $p(z_1,z_2,\ldots,z_n)$. It is implicitly assumed that each $z_i > 0$ with probability 1. Then the joint p.m.f of $(X_1,X_2,\ldots,X_n,Z_1,Z_2,\ldots,Z_n)$ is

$$f(x_1,x_2,\ldots,x_n,z_1,z_2,\ldots,z_n \mid \lambda) = e^{-\lambda\sum_{i=1}^{n} z_i + (\sum_{i=1}^{n} x_i)\log\lambda} \prod_{i=1}^{n} \frac{z_i^{x_i}}{x_i!} p(z_1,z_2,\ldots,z_n) \quad I x_1,\ldots,x_n \in N_0 \quad I z_1,\ldots,z_n \in N_1 \tag{2.3}$$

N_0 = set of non-negative integers, N_1 = set of positive integers[2].
This paper wants to find out a suitable test procedure for this distribution.

Statistical Curvature of Curved Poisson distribution

Consider the Curved Poisson distribution as given in definition 2.3. Then its p.d.f. is given in equation 2.3.

Let Z_1, Z_2, …, Z_n are independently and identically distributed (i.i.d) as U(0,1) (discrete). Now comparing equation 2.3 with definition 1 i.e. comparing the p.d.f of Curved Poisson distribution with the form of
Curved Exponential distribution, we get

$$\eta_1(\lambda) = -\lambda \qquad and \qquad \eta_2(\lambda) = \log\lambda \tag{3.1}$$

$$T_1(X,Z) = \sum_{i=1}^{n} z_i \qquad and \qquad T_2(X,Z) = \sum_{i=1}^{n} x_i \tag{3.2}$$

$$V(\sum_{i=1}^{n} z_i) = \sum_{i=1}^{n} V(Z_i) = \sum_{i=1}^{n} \frac{n^2-1}{12} = n.\frac{n^2-1}{12} \tag{3.3}$$

$$V(\sum_{i=1}^{n} x_i) = \sum_{i=1}^{n} V(X_i)$$

Now, $V(X) = E(V(X \mid Z)) + V(E(X \mid Z)) = E(\lambda Z) + V(\lambda Z) = \lambda E(Z) + \lambda^2 V(Z)$

$$\therefore V(X) = \lambda\frac{n+1}{2} + \lambda^2\frac{n^2-1}{12} \tag{3.4}$$

$$\therefore \sum_{i=1}^{n} V(X_i) = \lambda\frac{n(n+1)}{2} + \lambda^2\frac{n(n^2-1)}{12} \tag{3.5}$$

Now covariance between T_1 and T_2 is to be found.

$$\text{cov}(\sum_{i=1}^{n} z_i, \sum_{i=1}^{n} x_i) = \sum_{i=1}^{n} \text{cov}(Z_i, X_i)$$

$$\text{cov}(Z_i, X_i) = E(Z_i X_i) - E(Z_i)E(X_i) \tag{3.6}$$

$$E(Z_i X_i) = E_{Z_i} E_{X_i|Z_i}(Z_i X_i \mid Z_i)$$

Now,
$$= E_{Z_i}(\lambda Z_i^2)$$
$$= \lambda[V(Z_i) + E^2(Z_i)]$$
$$= \lambda(\frac{n^2-1}{12} + \frac{(n+1)^2}{4})$$
$$= \lambda\frac{4n^2+6n-2}{12}$$

$$= \frac{\lambda(2n^2+3n-1)}{6} \tag{3.7}$$

$$E(X_i) = E_{Z_i} E_{X_i|Z_i}(X_i \mid Z_i) = \lambda \frac{n+1}{2} \qquad 3.8$$

$\therefore$ From 3.6,

$$\mathrm{cov}(Z_i, X_i) = \frac{\lambda(2n^2+3n-1)}{6} - \lambda \frac{(n+1)^2}{4}$$

$$= \lambda \frac{n^2-1}{12} \qquad 3.9$$

So, $$\sum_{i=1}^{n} \mathrm{cov}(Z_i, X_i) = \lambda \frac{n(n^2-1)}{12} \qquad 3.10$$

Therefore from 3.3, 3.5 and 3.10,

$$\Sigma_\lambda = \begin{pmatrix} \frac{n(n^2-1)}{12} & \frac{\lambda n(n^2-1)}{12} \\ \frac{\lambda n(n^2-1)}{12} & \lambda \frac{n(n+1)}{2} + n\lambda^2 \frac{n^2-1}{12} \end{pmatrix} \qquad 3.11$$

and $$\eta(\lambda) = \begin{pmatrix} -\lambda \\ \log \lambda \end{pmatrix} \qquad 3.12$$

$$\therefore \dot{\eta}(\lambda) = \begin{pmatrix} -1 \\ 1/\lambda \end{pmatrix} \quad \& \quad \ddot{\eta}(\lambda) = \begin{pmatrix} 0 \\ -1/\lambda^2 \end{pmatrix}$$

Using the concept of statistical curvature and 3.11, 3.12 the statistical curvature of Curved Poisson distribution can be found. In this paper statistical curvature of the Curved Poisson distribution for various values of n has been drawn through a programming in statistical software R.

The statistical curvatures for various values of n are given in figure-2, figure-3 and figure-4.

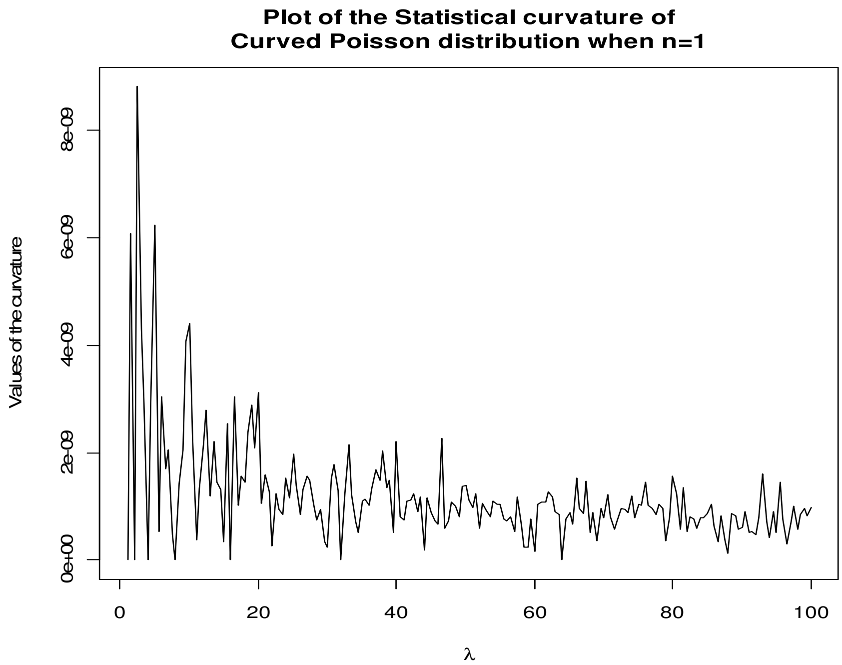

Figure -2

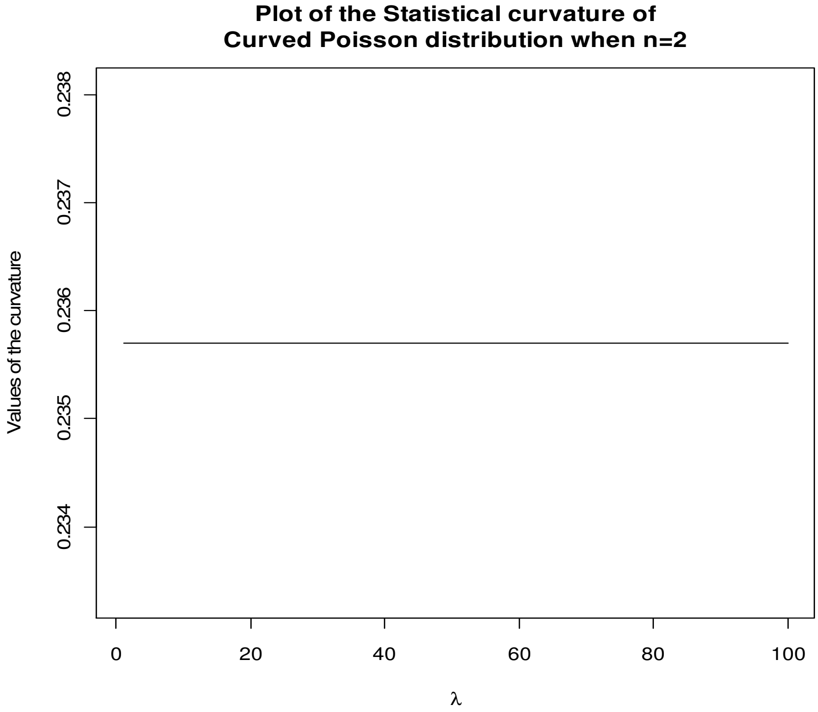

Figure-3

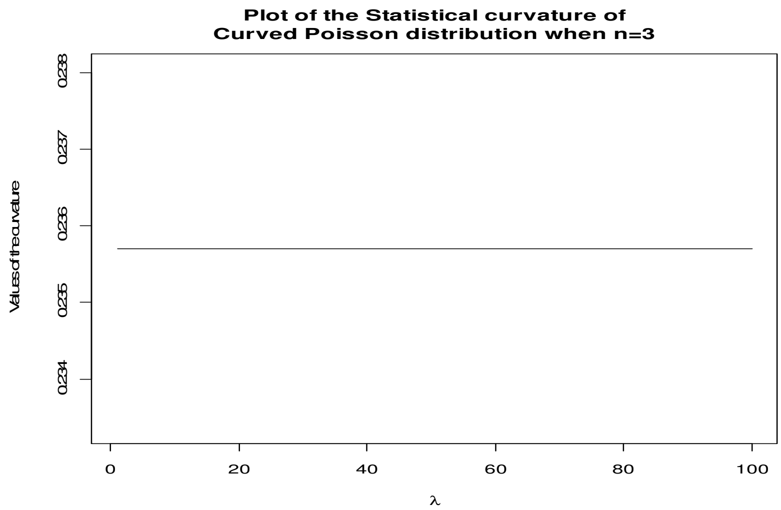

Figure-4

In this way the statistical curvature for this distribution can be found out for different values of n, where n is number of observations. From above pictures, it is seen that for n=1, for different values of parameter λ statistical curvature has different values; in case of other values of n, for different values of λ statistical curvatures have the same value. But in all cases the curvatures assign vary small values.

Inference of the Curved Poisson distribution

There are test procedures for Poisson distribution which does not contain another discrete distribution with its parameter. As test procedure for Poisson distribution which contains another discrete distribution with its parameter are not available, to reach to any decision about λ of the above mentioned distribution, a test procedure should be found out.

To overcome this problem this paper would like to find out a test procedure to draw an inference about the null hypothesis.

From figure-2, figure-3 and figure-4, we see that the value of the curvature is small for all values of λ.

Now, let us demonstrate how to get the test method for such distribution. For example, let us work for getting test for H0: $\lambda_0 = 2$ vs. H_1: $\lambda_0 \neq 2$.

The test procedure is as follows:
Here Xi ~ Poisson (λZ_i), where Z_i ~ U(0,1), X_i's and Z_i's are independently and identically distributed.
Let X_1 takes values 1,2,3,…,k,… and Z_1 ~ U(0,1,…,n).
Then the probability that X_1 takes a particular value k is given by

$$\frac{e^{-\lambda Z_i}(\lambda Z_i)^k}{k!}. \qquad 4.1$$

Considering Z1's probability of taking a particular value, the joint probability becomes

$$\frac{1}{n+1}\frac{e^{-\lambda i}(\lambda i)^k}{k!} \qquad 4.2$$

Let X_1 takes a particular value k and Z_1 takes a particular value m.
∴ $0 \le m \le n$.

Hence for an observation (x_1, z_1), the likelihood function is

$$L(\lambda; x_1, z_1) = \frac{e^{-\lambda z_1}(\lambda z_1)^k}{k!} \times \frac{1}{n+1} \qquad 4.3$$

Let k be the fixed value of X_1 and let m be the fixed value of Z_1. Then from (4.3),

$$\log L(\lambda) = -\lambda m + k\log(\lambda m) + c_1 \qquad 4.4$$

where c_1 is a constant.

$$\therefore \frac{\partial \log L(\lambda)}{\partial \lambda} = -m + k\frac{m}{\lambda m} \qquad 4.5$$

Equating 4.5 to 0, we get

$$\frac{k}{\lambda} = m \quad \Rightarrow \lambda = \frac{k}{m} \qquad 4.6$$

$$\therefore L(\hat{\lambda}) = \frac{e^{-\frac{k}{m}m}\left(\frac{k}{m}m\right)^k}{k!}\frac{1}{n+1}$$
$$= \frac{e^{-k}k^k}{k!}\frac{1}{n+1} \qquad 4.7$$

$$\therefore \frac{L(\lambda_0)}{L(\hat{\lambda})} = \frac{e^{-\lambda_0 m}(\lambda_0 m)^k}{e^{-k}k^k}$$
$$= \left(\frac{\lambda_0 m}{k}\right)^k e^{-(\lambda_0 m + k)} \qquad 4.8$$

So, H_0 will be rejected if 4.8 is less than C, where C is a constant.
Now from 4.8

$$\left(\lambda_0 \frac{m}{k}\right)^k e^{-(\lambda_0 \frac{m}{k}+1)k} = \left(\frac{\lambda_0 m e^{-(\lambda_0 \frac{m}{k}+1)}}{k}\right)^k \quad 4.9$$

$$= \left(te^{-(t+1)}\right)^k \quad 4.10$$

Here $t = \frac{\lambda_0 m}{k}$.

So, if $f(t) = te^{-(t+1)}$ then for a fixed k, f(t) ↑ if $m < k/\lambda_0$ and f(t) ↓ if $m \geq k/\lambda_0$. Hence from 4.8 and 4.10 H_0 will be rejected if 4.10 < C.

Now, $\left(te^{-(t+1)}\right)^k < C$

$$m < \frac{k}{\lambda_0} t_0 \quad if \quad \lambda_0 \frac{m}{k} < 1 \quad 4.11a$$

$$\Leftrightarrow$$

$$m \geq \frac{k}{\lambda_0} t_1 \quad if \quad \lambda_0 \frac{m}{k} \geq 1 \quad 4.11b$$

Here t_0 should be< 1 and t_1 should be>1. 4.12

Therefore for a size α test, for various values of k, the values of t_0 and t_1 to be found out such that

$$\sum_{\left[\frac{k}{\lambda_0} t_0\right]}^{\left[\frac{k}{\lambda_0} t_1\right]} e^{-\lambda_0 m} \frac{(\lambda_0 m)^k}{k!} \frac{1}{n+1} \geq 1-\alpha \quad 4.13$$

Now to achieve this, one way is to consider the following

$$\sum_{m=0}^{\left[\frac{k}{\lambda_0} t_0\right]} e^{-\lambda_0 m} \frac{(\lambda_0 m)^k}{k!} \frac{1}{n+1} \leq \alpha/2 \quad 4.14a$$

and $$\sum_{m=\left[\frac{k}{\lambda_0} t_1\right]}^{n} e^{-\lambda_0 m} \frac{(\lambda_0 m)^k}{k!} \frac{1}{n+1} \leq \alpha/2 \quad 4.14b$$

For calculation purpose inequalities of 4.13a and 4.13b are

$$\sum_{m=0}^{\left[\frac{k}{\lambda_0} t_0\right]} e^{-\lambda_0 m} (\lambda_0 m)^k \leq \frac{\alpha(n+1)k!}{2} \quad 4.15a$$

and $$\sum_{m=\left[\frac{k}{\lambda_0} t_1\right]}^{n} e^{-\lambda_0 m} (\lambda_0 m)^k \leq \frac{\alpha(n+1)k!}{2} \quad 4.15b$$

Considering condition (4.12) and using equations (4.15a) and (4.15b) the values of t_0 and t_1 have been found out taking α=0.01. For this purpose, this paper uses an R program. The following tables will give the values of t_0 and t_1 from which one can reach to the decision.

Tables to find the values of t_0 corresponding to different values of k and n:

Table-1
Table to find the values of t_0 corresponding to different values of k and n=1

When n=1				
Values of k	**Values of R.H.S. of (4.15a)**	**Values of m (= kt_0/λ_0) for which L.H.S. is just > R.H.S.**	**Values of L.H.S. which is just > R.H.S.**	**Values of t_0 (= $m\lambda_0/k$)**
0	0.01	0	1	0
1	0.01	0.01	0.019	0.02
2	0.02	0.05	0.02027	0.05
3	0.06	0.14	0.07	0.0933
4	0.24	0.26	0.269	0.13
5	1.2	0.41	1.34	0.164
6	7.2	0.58	7.7668	0.1933
7	50.4	0.77	53.164	0.22
8	403.2	0.98	428.929	0.245
9	3628.8	1.2	3777.693	0.267
10	36288	1.43	36602.72	0.286
11	399168	1.68	411733.8	0.30545
12	4790016	1.93	4799841	0.3217
13	62270208	2.2	64074714	0.338
14	871782912	2.48	887863695	0.353

One can proceed in this way for more values of k until t_0 is less than 1 (from 4.12)

Table-2
Table to find the values of t_0 corresponding to different values of k and n=2

When n=2				
Values of k	**Values of R.H.S. of (4.15a)**	**Values of m (= kt_0/λ_0) for which L.H.S. is just > R.H.S.**	**Values of L.H.S. which is just > R.H.S.**	**Values of t_0 (= $m\lambda_0/k$)**
0	0.015	0	1	0
1	0.015	0.01	0.019	0.02
2	0.03	0.06	0.033	0.06
3	0.09	0.15	0.09	0.1
4	0.36	0.28	0.375	0.14
5	1.8	0.44	1.94	0.176
6	10.8	0.62	11.52	0.207
7	75.6	0.82	80.35	0.234
8	604.8	1.02	613.3	0.255
9	5443.2	1.26	5519.946	0.28
10	54432	1.5	54522.99	0.3
11	598752	1.76	621890.6	0.32
12	7185024	2.02	7362665	0.337
13	93405312	2.29	95218507	0.352
14	1307674368	2.57	1339455685	0.367

One can proceed in this way for more values of k until t_0 is less than 1 (from 4.12)

Table-3
Table to find the values of t_0 corresponding to different values of k and n=3

When n=3				
Values of k	**Values of R.H.S. of (4.15a)**	**Values of m (= kt_0/λ_0) for which L.H.S. is just > R.H.S.**	**Values of L.H.S. which is just > R.H.S.**	**Values of t_0 (= $m\lambda_0/k$)**
0	0.02	0	1	0
1	0.02	0.01	0.0196	0.02
2	0.04	0.07	0.05	0.07
3	0.12	0.17	0.142	0.113
4	0.48	0.3	0.51	0.15
5	2.4	0.46	2.445	0.184
6	14.4	0.65	15.19	0.217
7	100.8	0.85	101.49	0.243
8	806.4	1.08	858.75	0.27
9	7257.6	1.31	7442.62	0.291
10	72576	1.56	75285.12	0.312
11	798336	1.82	833695.1	0.331
12	9580032	2.08	9653827	0.347
13	124540416	2.36	127667909	0.363
14	1743565824	2.64	1762388776	0.377

One can proceed in this way for more values of k until t_0 is less than 1 (from 4.12), In a similar manner tables can be formed for other values of n., Tables to find the values of t_1 corresponding to different values of k and n:

Table-4
Table to find the values of t_1 corresponding to different values of k and n=1

When n=1				
Values of k	**Values of R.H.S. of (4.15a)**	**Values of m (= kt_1/λ_0) for which L.H.S. is just > R.H.S.**	**Values of L.H.S. which is just > R.H.S.**	**Values of t_1 (= $m\lambda_0/k$)**
0	0.01	1	0.135	∞
1	0.01	1	0.27	2
2	0.02	1	0.541	1

In this case we can not proceed farther because for other values of k the values of t_1 will be less than 1. But t_1 can not be <1 by 4.12.

Table-5
Table to find the values of t_1 corresponding to different values of k and n=2

When n=2				
Values of k	**Values of R.H.S. of (4.15a)**	**Values of m (= kt_1/λ_0) for which L.H.S. is just > R.H.S.**	**Values of L.H.S. which is just > R.H.S.**	**Values of t_1 (= $m\lambda_0/k$)**
0	0.015	2	0.018	∞
1	0.015	2	0.073	4
2	0.03	1.99	0.5890	1.99
3	0.09	2	1.17	1.33
4	0.36	2	4.688	1

In this case we cannot proceed farther because for other values of k the values of t_1 will be less than 1. But t_1 cannot be <1 by 4.12.

Table-6
Table to find the values of t_1 corresponding to different values of k and n=3

When n=3				
Values of k	**Values of R.H.S. of (4.15a)**	**Values of m (= kt_1/λ_0) for which L.H.S. is just > R.H.S.**	**Values of L.H.S. which is just > R.H.S.**	**Values of t_1 (= $m\lambda_0/k$)**
0	0.02	2.99	0.09	∞
1	0.02	2.99	0.0299	5.98
2	0.04	2.99	0.1797	2.99
3	0.12	3	0.535	2
4	0.48	3	3.212	1.5
5	2.4	3	19.27	1.2
6	14.4	3	115.65	1

In this case we cannot proceed farther because for other values of k the values of t_1 will be less than 1. But t_1 cannot be <1 by 4.12. [L.H.S. = Left Hand Side; R.H.S. = Right Hand Side], Hence from tables (Table-1 to Table-6) one can easily get the values of t_0 and t_1 for different values of k and n. And hence decision regarding 4.13 can be taken. The following subsection contains the corresponding R program.

R program used for finding values of t_0 and t_1

```
n=2;alpha=0.01  # Taking n=2; the same program will run for n=1, n=3 and so on.
k=seq(0,14,by=1) # For computation purpose k=0 to 14 has been taken. More values can be taken.
product=0
for(i in 1:26)
{
 product[i]=(factorial(k[i]))*(n+1)*(alpha/2)
 }
product                          # product symbolises the R.H.S of (4.15a) and (4.15b) for different values of k
lamda.not=2
mm=k/lamda.not                   #Here mm= k/λ0 for different values of k.

## PROGRAM FOR (4.15a) i.e. TO FIND THE VALUES OF t0 ##
#### PROGRAM TO BE DONE TAKING k=0 ####
m=0
m[1]=0
s1=(exp((-lamda.not)*m[1]))*(lamda.not*m[1])^k[1]
 # [In this program s1 is the expression given under summation in the L.H.S. of (4.15a) for k=0]
#### PROGRAM TO BE DONE MANY TIMES CHANGING THE VALUES OF "k" ####
s=0
s[1]=(exp((-lamda.not)*m[1]))*(lamda.not*m[1])^k[15]
for(i in 2:300){
 m[2]=0.01
 s[i]=(exp((-lamda.not)*m[i]))*(lamda.not*m[i])^k[15]
 m[i+1]=m[i]+0.01
 }
product[15];mm[15]
sum=0
sum[1]=s[1]
for(i in 2:300){
 sum[i]=sum[i-1]+s[i]
}
sum
## PROGRAM FOR (4.15b) i.e. TO FIND THE VALUES OF t1 ##
#### PROGRAM TO BE DONE TAKING k=0 ####

m1=0        # Here m1 is equivalent to m (as used in (4.15b))
m1[1]=n
```

s2=(exp((-lamda.not)*m1[1]))*(lamda.not*m1[1])^k[1] *# if necessary, this line has to be run many times for different values of m, by 'for loop'*
 # [In this program s2 is the expression given under summation in the L.H.S. of (4.15a) for k=0]

```
#### PROGRAM TO BE DONE MANY TIMES CHANGING THE VALUES OF "k" ####
ss=0
ss[1]=(exp((-lamda.not)*m1[1]))*(lamda.not*m1[1])^k[5]
for(i in 2:100)
{
m1[2]=m1[1]-0.01
ss[i]=(exp((-lamda.not)*m1[i]))*(lamda.not*m1[i])^k[5]
m1[i+1]=m1[i]-0.01
}
product[5];mm[5]
sum1=0
sum1[1]=ss[1]
for(i in 2:100)
{
sum1[i]=sum1[i-1]+ss[i]
}
sum1
```

Conclusion

In this paper a test procedure for Curved Poisson distribution has been found out. Here Uniform distribution has been used as the distribution involved in Poisson distribution. One may use other discrete distributions in place of Uniform distribution also and plenty of scopes are there for doing further research. This paper hopes the test procedure will help to take decision about a data that follow Curved Poisson distribution.

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