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Common Fixed Point Results in Fuzzy Menger Space

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Abstract

In this paper we establish fixed point theorems in Fuzzy Menger space for weak commutative and weak compatible which satisfying implicit relation.

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Introduction

The notion of probabilistic metric space is introduced by Menger[1] in 1942 and the first result about the existence of a fixed point of a mapping which is defined on a Menger space is obtained by Sehgel and Barucha-Reid. A number of fixed point theorems for single valued and multivalued mappings in menger probabilistic metric space have been considered by many authors[2-9]. In 1998, Jungck[10] introduced the concept weakly compatible maps and proved many theorems in metric space. Recently, Rajesh Shrivastav et. al.[11,12] have proved fixed point theorems for Fuzzy Menger space. In this paper we have proved some fixed point results for weakly commuting mappings and weak compatible mappings in Fuzzy Menger space.

Preliminaries

We required following definitions:

Definition 2.1 A fuzzy probabilistic metric space (FPM space) is an ordered pair $(X,F\alpha)$ consisting of a nonempty set X and a mapping $F\alpha$ from XxX into the collections of all distribution functions $F\alpha\in R$ for all $\alpha. \in [0,1]$. For $x, y \in X$ we denote the distribution function $F\alpha (x,y)$ by $F\alpha(x,y)$ and $F\alpha(x,y)(u)$ is the value of $F\alpha(x,y)$ at u in R. The functions $F\alpha(x,y)$ for all $\alpha. \in [0,1]$ assumed to satisfy the following conditions: i. $F\alpha(x,y)(u) = 1\ \forall\ u > 0$ iff $x = y$, ii. $F\alpha(x,y)(0) = 0\ \forall\ x, y$ in X, iii. $F\alpha(x,y) = F\alpha(y,x)\ \forall\ x, y$ in X, iv. If $F\alpha(x,y)(u) = 1$ and $F\alpha(y,z)(v) = 1$ then $F\alpha(x,z)(u+v) = 1\ \forall\ x, y, z$ in X, v. and $u, v > 0$

Definition 2.2 A commutative, associative and non-decreasing mapping $t: [0,1] \times [0,1] \rightarrow [0,1]$ is a t-norm if and only if $t(a,1)=a$ for all $a\in[0,1]$, $t(0,0)=0$ and $t(c,d) \geq t(a,b)$ for $c \geq a$, $d \geq b$.

Definition 2.3 A Fuzzy Menger space is a triplet $(X,F\alpha,t)$, where $(X,F\alpha)$ is a FPM-space,t is a t-norm and the generalized triangle inequality for all x, y, z in X $u, v> 0$ and $\alpha\in[0,1]$. $F\alpha(x,z)(u+v) \geq t(F\alpha(x,z)(u), F\alpha(y,z)(v))$ The concept of neighborhoods in Fuzzy Menger space is introduced as

Definition 2.4 Let $(X,F\alpha,t)$ be a Fuzzy Menger space. If $x\in X$, $\varepsilon > 0$ and $\lambda \in(0,1)$, then (ε,λ) - neighborhood of x, called $Ux(\varepsilon,\lambda)$, is defined by $Ux(\varepsilon,\lambda) = \{y\in X: F\alpha(x,y)(\varepsilon)>(1-\lambda)\}$ An (ε,λ)-topology in X is the topology induced by the family $\{Ux(\varepsilon,\lambda): x \in X, \varepsilon > 0, \alpha.\in [0,1]$ and $\lambda\in(0,1)\}$ of neighborhood.

Remark: If t is continuous, then Fuzzy Menger space $(X,F\alpha,t)$ is a Housdroff space in (ε,λ)-topology. Let $(X,F\alpha,t)$ be a complete Fuzzy Menger space and $A\subset X$. Then A is called a bounded set if $\lim\inf F\alpha(x,y)(u) =1.$ $u\rightarrow\infty$ $x,y\in A$

Definition 2.5 A sequence $\{xn\}$ in $(X,F\alpha,t)$ is said to be convergent to a point x in X if for every $\varepsilon>0$and $\lambda>0$, there exists an integer $N=N(\varepsilon,\lambda)$ such that $xn \in Ux(\varepsilon,\lambda)$ for all $n \geq N$ or equivalently $F\alpha(xn, x; \varepsilon) > 1-\lambda$ for all $n \geq N$ and $\alpha\in[0,1]$.

Definition 2.6 A sequence $\{xn\}$in $(X,F\alpha,t)$ is said to be cauchy sequence if for every $\varepsilon > 0$ and $\lambda > 0$, there exists an integer $N=N(\varepsilon,\lambda)$ such that $F\alpha(xn,xm; \varepsilon) > 1-\lambda\ \forall\ n, m \geq N$ for all $\alpha\in[0,1]$.

Definition 2.7 A Fuzzy Menger space (X,Fα,t) with the continuous t-norm is said to be complete if every Cauchy sequence in X converges to a point in X for all α∈[0,1].

Definition 2.8 Let (X,Fα,t) be a Fuzzy Menger space. Two mappings f, g : X→X are said to be weakly comptable if they commute at coincidence point for all α∈[0,1].

Lemma 1 Let {xn} be a sequence in a Fuzzy Menger space (X,Fα,t), where t is continuous and t(p,p) ≥ p for all p∈[0,1], if there exists a constant k(0,1) such that ∀p > 0 and n∈N t(Fα (xn,xn+1; kp)) ≥ t(Fα(xn-1,xn; p)),
for all α∈[0,1] then {xn} is cauchy sequence.

Lemma 2 If (X,d) is a metric space, then the metric d induces, a mapping Fα: XxX→L defined by Fα (p, q) = Hα(x- d(p, q)), p, q ∈ R for all α∈[0,1]. Further if t: [0,1] × [0,1]→ [0,1] is defined by t(a,b) = min{a,b}, then (X,Fα,t) is a Fuzzy Menger space. It is complete if (X,d) is complete.

Definition 2.10: Let (X, Fα,t) be a Fuzzy Menger space. Maps s: X → X and T: X→CB(X), i. s is said to be T weakly commuting at x∈X if ssx ∈ Tsx. ii. are weakly compatible if the commute at their coincidence points, i.e. if sTx = Tsx whenever sx∈Tx.

Main Results

Theorem 1.1: Let $(X, F\alpha, \Delta)$ be a complete Fuzzy Menger space where t is continuous and $t(p,p) \geq p$ for all $p \in [0,1]$. Let A, B, T and S be mappings from X into itself such that
1.1. $S(X) \subset A(X)$ and $T(X) \subset B(X)$
1.2. $AB = BA$, $ST = TS$ weakly commuting
1.3. The pair (S, A) and (T, B) are weakly compatible
1.4. There exists a number $k \in (0,1)$ such that $F_{\alpha(Sx,Ty)}(kp) \geq t\left(\frac{F_{\alpha(Ax,Sx)}(p)}{F_{\alpha(Ax,By)}(p_1)+F_{Sx,By}(p_2)}\right), t\left(\frac{F_{\alpha(By,Ty)}(p)}{F_{\alpha(Sx,By)}(p_1)+F_{Sx,Ty}(p_2)}, t(F_{\alpha(Ax,By)}(p), \frac{t(F_{\alpha(By,Sx)}(\beta p))}{t(F_{\alpha(Sx,By)}(p))}, t\left(F_{\alpha(Ax,Ty)}\left((2-\beta)p\right)\right)\right)\right)$ for all $x, y \in X, \beta \in (0,2)$ and $p > 0; p_1 + p_2 = p$.
Then, A, B, S and T have a unique common fixed point in X.

Proof: Since $S(X) \subset A(X)$ for any $x_0 \in X$ there exists a point $x_1 \in X$ such that $Sx_0 = Ax_1$. Since $T(X) \subset B(X)$ for this point x_1 we can choose a point $x_2 \in X$ such that $Tx_1 = Bx_2$. Inductively we can find a sequence $\{y_n\}$ as follows
$y_{2n} = Sx_{2n} = Ax_{2n+1}$ and $y_{2n+1} = Tx_{2n+1} = Bx_{2n+2}$
For $n = 0, 1, 2, 3$ by (1.4), for all $p > 0$ and $\beta = 1 - q$ with $q \in (0,1)$, we have $F_{\alpha(y_{2n+1},y_{2n+2})}(kp) = F_{\alpha(Sx_{2n+1},Tx_{2n+2})}(kp)$

$$\geq t\left(\frac{F_{\alpha(Ax_{2n+1},Sx_{2n+1})}(p)}{t(F_{\alpha(Ax_{2n+1},Bx_{2n+2})}(p_1)+F_{\alpha(Sx_{2n+1},Bx_{2n+2})}(p_2))}, \frac{t(F_{\alpha(Bx_{2n+2},Tx_{2n+2})}(p)}{t(F_{\alpha(Sx_{2n+1},Bx_{2n+2})}(p_1)+F_{\alpha(Sx_{2n+1},Tx_{2n+2})}(p_2))}, t(F_{\alpha(Ax_{2n+1},Bx_{2n+2})}\right.$$

$$\left.(p), \frac{t(F_{\alpha(Bx_{2n+2},Sx_{2n+1})}(\beta p))}{t(F_{\alpha(Sx_{2n+1},Bx_{2n+2})}(p))}, t\left(F_{\alpha(Ax_{2n+1},Tx_{2n+2})}(2p-\beta p)\right)\right)\right) \geq t\left(\frac{F_{\alpha(y_{2n},y_{2n+1})}(p)}{t(F_{\alpha(y_{2n},y_{2n+1})}(p_1) + F_{\alpha(y_{2n+1},y_{2n+1})}(p_2))},\right.$$

$$\left.\frac{t(\ F_{\alpha(y_{2n+1},y_{2n+2})}(p)}{t(F_{\alpha(y_{2n+1},y_{2n+1})}(p_1) + F_{\alpha(y_{2n+1},y_{2n+2})}(p_2))}, t(F_{\alpha(y_{2n},y_{2n+1})}(p), \frac{t(F_{\alpha(y_{2n+1},y_{2n+1})}(\beta p))}{t(F_{\alpha(y_{2n+1},y_{2n+1})}(p))}, t\left(F_{\alpha(y_{2n},y_{2n+2})}((1+q)p)\right)\right)\right)$$

$$\geq t\left(\frac{F_{\alpha(y_{2n},y_{2n+1})}(p)}{t(F_{\alpha(y_{2n},y_{2n+1})}(p))}, t(\frac{F_{\alpha(y_{2n+1},y_{2n+2})}(p)}{F_{\alpha(y_{2n+1},y_{2n+2})}(p)}), t(F_{\alpha(y_{2n},y_{2n+1})}(p), \frac{t(F_{\alpha(y_{2n+1},y_{2n+1}}((1-q)p))}{t(F_{\alpha(y_{2n+1},y_{2n+1})}(p))}, t\left(F_{\alpha(y_{2n},y_{2n+2})}((1+q)p)\right)\right)\right)$$

$$\geq t\left(1,1, t(F_{\alpha(y_{2n},y_{2n+1})}(p), t(1, t\left(F_{\alpha(y_{2n},y_{2n+1})}(p), F_{\alpha(y_{2n+1},y_{2n+2})}(qp)\right)\right)\right) \geq t\left(F_{\alpha(y_{2n},y_{2n+1})}(p), F_{\alpha(y_{2n+1},y_{2n+2})}(qp)\right)$$

$$F_{\alpha(y_{2n+1},y_{2n+2})}(kp) \geq t\left(F_{\alpha(y_{2n},y_{2n+1})}(p), F_{\alpha(y_{2n+1},y_{2n+2})}(qp)\right)$$

Since t is continuous and the distribution function is left continuous, making $q \to 1$ we have

$F_{\alpha(y_{2n+1},y_{2n+2})}(kp) \geq t\left(F_{\alpha(y_{2n},y_{2n+1})}(p), F_{\alpha(y_{2n+1},y_{2n+2})}(p)\right)$ Similarly

$F_{\alpha(y_{2n+2},y_{2n+3})}(kp) \geq t\left(F_{\alpha(y_{2n+1},y_{2n+2})}(p), F_{\alpha(y_{2n+2},y_{2n+3})}(p)\right)$

Therefore $F_{\alpha(y_n,y_{n+1})}(kp) \geq t\left(F_{\alpha(y_{n-1},y_n)}(p), F_{\alpha(y_n,y_{n+1})}(p)\right)$ for all $n \in N$

Consequently $F_{\alpha(y_n,y_{n+1})}(p) \geq t\left(F_{\alpha(y_{n-1},y_n)}(k^{-1}p), F_{\alpha(y_n,y_{n+1})}(k^{-1}p)\right)$ for all $n \in N$

Repeated application of this inequality will imply that
$F_{\alpha(y_n,y_{n+1})}(p) \geq t\left(F_{\alpha(y_{n-1},y_n)}(k^{-1}p), F_{\alpha(y_n,y_{n+1})}(k^{-1}p)\right) \geq \cdots \ldots \ldots \ldots \geq t\left(F_{\alpha(y_{n-1},y_n)}(k^{-1}p), F_{\alpha(y_n,y_{n+1})}(k^{-i}p)\right), i \in N$
Since $F_{\alpha(y_n,y_{n+1})}(k^{-i}p) \to 1$ as $i \to \infty$, it follows that $F_{\alpha(y_n,y_{n+1})}(p) \geq t\left(F_{\alpha(y_{n-1},y_n)}(k^{-1}p)\right)$ for all $n \in N$

Consequently $F_{\alpha(y_n,y_{n+1})}(kp) \geq t\left(F_{\alpha(y_{n-1},y_n)}(p)\right)$ for all $n \in N$

Therefore by lemma [4], $\{y_n\}$ is a Cauchy sequence in X. Since X is complete, $\{y_n\}$converges to a point $z \in X$.

Since $\{Sx_{2n}\}, \{Tx_{2n+1}\}, \{Ax_{2n+1}\}$ and $\{Bx_{2n+2}\}$ are subsequences of $\{y_n\}$, they also converge to the point z, i.e. as $n \to \infty$, $Sx_{2n}, Tx_{2n+1}, Ax_{2n+1}\ Bx_{2n+2} \to z$.

Since$S(X) \subset A(X)$, there exists a point $u \in X$ such that $Au = z$. By putting $x = u$ and $y = 2n - 1$ with $\beta = 1$ in (1.4)we have,
$F_{\alpha(Su,Tx_{2n-1})}(kp) \geq t\left(\frac{F_{\alpha(Au,Su)}(p)}{t(F_{\alpha(Au,Bx_{2n-1})}(p_1)+F_{\alpha(Su,Bx_{2n-1})}(p_2))}, \frac{t(F_{\alpha(Bx_{2n-1},Tx_{2n-1})}(p)}{t(F_{\alpha(Su,Bx_{2n-1})}(p_1)+F_{\alpha(Su,Tx_{2n-1})}(p_2))}, t(F_{\alpha(Au,Bx_{2n-1})}(p)\right.$
$\left., \frac{t(F_{\alpha(Bx_{2n-1},Su)}(\beta p))}{t(F_{\alpha(Su,Bx_{2n-1})}(p))}, t\left(F_{\alpha(Au,Tx_{2n-1})}((2-\beta)p)\right)\right)\right)$

Proceeding limit as $n \to \infty$, we have $F_{\alpha(Su,z)}(kp)$
$\geq t\left(\frac{F_{\alpha(z,Su)}(p)}{t(F_{\alpha(z,z)}(p_1)+F_{\alpha(Su,z)}(p_2))}, \frac{t(F_{\alpha(z,z)}(p)}{t(F_{\alpha(Su,z)}(p_1)+F_{\alpha(Su,z)}(p_2))}, t(F_{\alpha(z,z)}(p), \frac{t(F_{\alpha(z,Su)}((1-q)p))}{t(F_{\alpha(Su,z)}(p))}, t\left(F_{\alpha(z,z)}((1+q)p)\right)\right)\right)\right)$
$\geq t\left(\frac{F_{\alpha(z,Su)}(p)}{t(F_{\alpha(z,Su)}(p))}, \frac{t(F_{\alpha(z,z)}(p)}{t(F_{\alpha(z,z)}(p)}, t(F_{\alpha(z,z)}(p), \frac{t(F_{\alpha(z,Su)}(p))}{t(F_{\alpha(Su,z)}(p))}, t\left(F_{\alpha(z,z)}(p)\right)\right)\right)\right) \geq t\left(F_{\alpha(z,Su)}(p), t(1, t(1,1,t(1)))\right)$

Consequently $F_{\alpha(Su,z)}(p) \geq F_{\alpha(Su,z)}(k^{-1}p) \geq \cdots \ldots \ldots \ldots \geq F_{\alpha(Su,z)}(k^{-j}p)$

which tends to 1 and j tends to ∞ ($j \in N$) Therefore $Su = z$ and thus $Au = Su = z$.
Since$T(X) \subset B(X)$, there exists a point $v \in X$ such that $Bv = z$. Then by putting $x = u$ and $y = v$ with $\beta = 1$ in (1.4.) we have
$F_{\alpha(Su,Tv)}(kp) \geq t\left(\frac{F_{\alpha(Au,Su)}(p)}{t(F_{\alpha(Au,Bv)}(p_1)+F_{\alpha(Su,Bv)}(p_2))}, \frac{t(F_{\alpha(Bv,Tv)}(p)}{t(F_{\alpha(Su,Bv)}(p_1)+F_{\alpha(Su,Tv)}(p_2)}, t(F_{\alpha(Au,Bv)}(p), \frac{t(F_{\alpha(Bv,Su)}(\beta p))}{t(F_{\alpha(Su,Bv)}(p))}, t\left(F_{\alpha(Au,Tv)}((2-\beta)p)\right)\right)\right)\right)$
$\geq t\left(\frac{F_{\alpha(Au,Su)}(p)}{t(F_{\alpha(Au,Bv)}(p_1)+F_{\alpha(Su,Bv)}(p_2))}, \frac{t(F_{\alpha(Bv,Tv)}(p)}{t(F_{\alpha(Su,Bv)}(p_1)+F_{\alpha(Su,Tv)}(p_2)}, t(F_{\alpha(Au,Bv)}(p), \frac{t(F_{\alpha(Bv,Su)}(\beta p))}{t(F_{\alpha(Su,Bv)}(p))}, t\left(F_{\alpha(Au,Tv)}((2-\beta)p)\right)\right)\right)\right)$

Using above we have we have $F_{\alpha(Su,Tv)}(kp) \geq t\left(\frac{F_{\alpha(z,z)}(p)}{t(F_{\alpha(z,z)}(p_1)+F_{\alpha(z,z)}(p_2))}, \frac{t(F_{\alpha(Bv,Tv)}(p)}{t(F_{\alpha(z,z)}(p_1)+F_{\alpha(z,Tv)}(p_2)}, t(F_{\alpha(z,z)}(p),\right.$
$\left.\frac{t(F_{\alpha(z,z)}((1-q)p))}{t(F_{\alpha(z,z)}(p))}, t\left(F_{\alpha(z,Tv)}((1+q)p)\right)\right)\right)\right)$
$F_{\alpha(Su,Tv)}(kp) \geq t\left(\frac{F_{\alpha(z,z)}(p)}{F_{\alpha(z,z)}(p)}, \frac{F_{\alpha(z,Tv)}(p)}{F_{\alpha(z,Tv)}(p)}, t(F_{\alpha(z,z)}(p), \frac{t(F_{\alpha(z,z)}((1-q)p))}{t(F_{\alpha(z,z)}(p))}, t\left(F_{\alpha(z,Tv)}((1+q)p)\right)\right)\right)\right)$
$F_{\alpha(z,Tv)}(kp) \geq t\left(F_{\alpha(z,Tv)}(p)\right)$

As above we have Tv = z. Therefore Au = Su = Tv = Bv = z … … (a)

Since pair of maps S and A are weakly compatible, then Su = Au implies S(A)u = (A)Su, i.e. Sz = Az. Now to show that z is a fixed point of S so by putting x = z and y = x_{2n} with β = 1 in (1.4)

$F_{\alpha(Sz,Tx_{2n})}(kp) \geq t\left(\frac{F_{\alpha(Az,Sz)}(p)}{t(F_{\alpha(Az,Bx_{2n})}(p_1)+F_{\alpha(Sz,Bx_{2n})}(p_2))}, \frac{t(F_{\alpha(Bx_{2n},Tx_{2n})}(p)}{t(F_{\alpha(Sz,Bx_{2n})}(p_1)+F_{\alpha(Sz,Tx_{2n})}(p_2))}, t(F_{\alpha(Az,Bx_{2n})}(p), \frac{t(F_{\alpha(Bx_{2n},Sz)}(p)}{t(F_{\alpha(Sz,Bx_{2n})}(p))}, t\left(F_{\alpha(Az,Tx_{2n})}(p)\right)\right)\right)$

Using above, we have $F_{\alpha(Sz,z)}(kp), \geq t\left(\frac{F_{\alpha(Sz,Sz)}(p)}{t(F_{\alpha(Sz,z)}(p)+F_{\alpha(Sz,z)}(p_2))}, \frac{t(F_{\alpha(z,z)}(p)}{t(F_{\alpha(Sz,z)}(p)+F_{\alpha(Sz,z)}(p_2))},\right.$ $\left. t(F_{\alpha(Sz,z)}(p), \frac{t(F_{\alpha(z,Sz)}((1-q)p))}{t(F_{\alpha(Sz,z)}(p))}, t\left(F_{\alpha(Sz,z)}((1+q)p)\right)\right)\right), \geq t\left(\frac{F_{\alpha(Sz,Sz)}(p)}{F_{\alpha(Sz,Sz)}(p)}, \frac{F_{\alpha(z,z)}(p)}{F_{\alpha(z,z)}(p)}, t(F_{\alpha(Sz,z)}(p), \frac{t(F_{\alpha(z,Sz)}(p))}{t(F_{\alpha(Sz,z)}(p))}, t\left(F_{\alpha(Sz,z)}(p)\right)\right)\right)$

Thus we have Sz = z. Hence Sz = z = Az .

Similarly, pair of maps T and B is weakly compatible and by (a) implies T(Bv) = B(Tv), i.e. Tz = Bz Now we show that z is a fixed point of T so by putting $x = x_{2n}$ and $y = z$ with $\beta = 1$ in (1.4.)

$F_{\alpha(Sx_{2n},Tz)}(kp) \geq t\left(\frac{F_{\alpha(Ax_{2n},Sx_{2n})}(p)}{t(F_{\alpha(Ax_{2n},Bz)}(p_1)+F_{\alpha(Sx_{2n},Bz)}(p_2))}, \frac{t(F_{\alpha(Bz,Tz)}(p)}{t(F_{\alpha(Sx_{2n},Bz)}(p_1)+F_{\alpha(Sx_{2n},Tz)}(p_2))},\right.$ $\left. t(F_{\alpha(Ax_{2n},Bz)}(p), \frac{t(F_{\alpha(Bz,Sx_{2n})}(p))}{t(F_{\alpha(Sx_{2n},Bz)}(p))}, t\left(F_{\alpha(Ax_{2n},Tz)}(p)\right)\right)\right) \geq t\left(\frac{F_{\alpha(Ax_{2n},Sx_{2n})}(p)}{t(F_{\alpha(Ax_{2n},Sx_{2n})}(p))}, \frac{F_{\alpha(Tz,Tz)}(p)}{F_{\alpha(Tz,Tz)}(p)}, t(F_{\alpha(Ax_{2n},Bz)}(p), \frac{t(F_{\alpha(Bz,Sx_{2n})}(p))}{t(F_{\alpha(Sx_{2n},Bz)}(p))}, t\left(F_{\alpha(Ax_{2n},Tz)}(p)\right)\right)\right)$

Proceeding limit as $n \to \infty$, we have $F_{\alpha(Sx_{2n},Tz)}(kp) \geq t\left(\frac{F_{\alpha(z,z)}(p)}{t(F_{\alpha(z,z)}(p))}, \frac{F_{\alpha(Tz,Tz)}(p)}{F_{\alpha(Tz,Tz)}(p)}, t(F_{\alpha(z,Tz)}(p), \frac{t(F_{\alpha(Tz,z)}(p))}{t(F_{\alpha(z,Tz)}(p))}, t\left(F_{\alpha(z,Tz)}(p)\right)\right)\right)$

Thus we have $Tz = z$. Hence $Tz = z = Bz$.
By combining the above results, we have $Sz = Az = Tz = Bz = z$. That is z is a common fixed point of S, T, A and B.
For uniqueness, let w $(w \neq z)$ be another common fixed point of S, T, A and B and $\beta = 1$, then by (1.4.), we write

$F_{Sz,Tw}(kp) \geq t\left(\frac{F_{\alpha(Az,Sz)}(p)}{t(F_{\alpha(Az,Bw)}(p_1)+F_{\alpha(Sz,Bw)}(p_2))}, \frac{t(F_{\alpha(Bw,Tw)}(p)}{t(F_{\alpha(Sz,Bw)}(p_1)+F_{\alpha(Sz,Tw)}(p_2))}, t(F_{\alpha(Az,Bw)}(p), \frac{t(F_{\alpha(Bw,Sz)}(\beta p))}{t(F_{\alpha(Sz,Bw)}(p))}, t\left(F_{\alpha(Az,Tw)}((2-\beta)p)\right)\right)\right)$
$\geq t\left(\frac{F_{\alpha(Az,Sz)}(p)}{F_{\alpha(Az,Sz)}(p)}, \frac{F_{\alpha(Bw,Tw)}(p)}{F_{\alpha(Bw,Tw)}(p)}, t(F_{\alpha(Az,Bw)}(p) \frac{t(F_{\alpha(Bw,Sz)}(\beta p))}{t(F_{\alpha(Sz,Bw)}(p))}, t\left(F_{\alpha(Az,Tw)}(p)\right)\right)\right)$

It follows that $F_{\alpha(z,w)}(kp) \geq t\left(\frac{F_{\alpha(z,z)}(p)}{F_{\alpha(z,z)}(p)}, t(\frac{F_{\alpha(w,w)}(p)}{F_{\alpha(w,w)}(p)}, t(F_{\alpha(z,w)}(p), \frac{t(F_{\alpha(w,z)}(\beta p))}{t(F_{\alpha(z,w)}(p))}, t\left(F_{\alpha(z,w)}((2-\beta)p)\right)\right)\right)$
$\geq t\left(1, t\left(1, t(F_{\alpha(z,w)}(p), 1, t\left(F_{\alpha(z,w)}(p)\right)\right)\right)\right)$
$\geq F_{\alpha(z,w)}(p)$
Thus we have $z = w$. This completes the proof of the theorem.

Corollary 1.2: Let $(X, F\alpha, t)$ be a complete Menger space where Δ is continuous and $t(p,p) \geq p$ for all $p \in [0,1]$. Let T and S be mappings from X into itself such that
1.2.1. ST = TS weakly commuting
1.2.2. There exists a number $k \in (0,1)$ such that $F_{\alpha(Sx,Ty)}(kp) \geq t\left(\frac{F_{\alpha(x,Sx)}(p)}{F_{\alpha(x,y)}(p_1)+F_{\alpha(Sx,y)}(p_2)}, t(\frac{F_{\alpha(y,Ty)}(p)}{F_{\alpha(x,y)}(p_1)+F_{\alpha(x,Ty)}(p_2)}, t(F_{\alpha(x,y)}(p), \frac{t(F_{\alpha(y,Sx)}(\beta p))}{t(F_{\alpha(Sx,y)}(p))}, t(F_{\alpha(x,Ty)}((2-\beta)p)))\right)$
for all $x, y \in X, \beta \in (0,2)$ and $p > 0$; $p_1 + p_2 = p$ Then S and T have a unique common fixed point in X.
Proof: Put A = B = I in the proof of theorem 1.1.1.

Corollary 1.3: Let (X, F_α, t) be a complete Menger space where t is continuous and $t(p,p) \geq p$ for all $p \in [0,1]$. Let B and S be mappings from X into itself such that

1.3.1. $S(X) \subset B(X)$
1.3.2. The pair (S, B) is weakly compatible
1.3.3. There exists a number $k \in (0,1)$ such that

$$F_{\alpha(Sx,Sy)}(kp) \geq t\left(\frac{F_{\alpha(Bx,Sx)}(p)}{F_{\alpha(Bx,By)}(p_1) + F_{\alpha(Sx,By)}(p)}, t(\frac{F_{\alpha(By,Sy)}(p)}{F_{\alpha(Bx,Sy)}(p_1) + F_{\alpha(Sx,By)}(p)}, t(F_{\alpha(Bx,By)}(p), \frac{t(F_{\alpha(By,Sx)}(\beta p))}{t(F_{\alpha(Sx,By)}(p))}, t(F_{\alpha(Bx,Sy)}((2-\beta)p))))\right)$$

for all $x, y \in X, \beta \in (0,2)$ and $p > 0; p_1 + p_2 = p$. Then S and B have a unique common fixed point in X.
Proof: Put $T = S$ and $A = B$ in the proof of theorem 1.1.

Conclusion

Clearly $(X, F\alpha, \Delta)$ be a complete Fuzzy Menger space where t is continuous and $t(p,p) \geq p$ for all $p \in [0,1]$.

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