Research Journal of Mathematical and Statistical Sciences ________________________________ISSN 2320–6047
Vol. **1(4),** 6-11, May **(2013)** *Res. J. Mathematical and Statistical Sci.*

Common Fixed Point Results for Sequence of Random Operators

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Available online at: www.isca.in
Received 28th March 2012, revised 10th April 2013, accepted 24th April 2013

Abstract

This paper contents common random fixed point results for random operators in separable Hilbert spaces with the use of implicit relation for three and five co-ordinates. Mathematical Subject Classification: 54H25, 47 H10.

Keywords: Separable Hilbert space, Common random fixed point, Sequence of Random operators.

Introduction

In 1950 study of probabilistic was initiated in The Prague School of probabilistic. After the publication of the survey article of Bharucha-Reid[1], so many research works have been started in this area. Then many interesting random fixed point results and applications have appeared in this area. The study of random fixed points have attracted so much attention of recent literatures in random fixed points are discussed by I. Beg, and N Shahzad, [2,3,4], B.S. Choudhary, and M Ray [5], N.S Papageorgiou, [6], H.K Xu [7], In particular, random iteration schemes leading to random fixed point of random operators have been elaborately discussed in B.S. Choudhary, and M Ray [5], B.S. Choudhary, and A Upadhyay[8], Choudhary, B.S. [9] and V.B Dhagat, A.Sharma, & R.K Bhardwaj [10],.

This paper deals with random common fixed point results for two sequences of random operators in closed subset of separable Hilbert spaces which satisfies implicit condition for three and five co-ordinates.

Preliminaries

Throughout this paper (Ω, Σ) denotes a measurable space, H stands for a separable Hilbert space, and C is non empty subset of H then we recall the following definitions:-

Measurable function: A function $f : \Omega \to C$ is said to be measurable if $f^{-1}(B \cap C) \in \Sigma$ foe every Borel subset B of H.

Random operator: A function $F : \Omega \times C \to C$ is said to be random operator, if $F(.,X) : \Omega \to C$ is measurable for every X $\subset$ C.

Continuous Random operator: A random operator $F : \Omega \times C \to c$ is said to be continuous if for fixed $t \in \Omega, F(t,.) : C \to C$ *is continuous*

Random fixed point: A measurable function $g : \Omega \to C$ is said to be random fixed point of the random operator $F : \Omega \times C \to C, if\ F(t, g(t)) = g(t), \forall t \in \Omega$.

Implicit Relation: Let Φ be the class of all real-valued continuous functions φ: $(R^+)^3 \to R^+$ non -decreasing in the first argument and satisfying the following conditions:
x ≤ φ(y, x+ y, x) or x ≤ φ(y, x, ½(x+ y)) or x ≤ φ(y,, x+ y, x + y), there exists a real number 0<k <1 such that x ≤ k y , for all x, y ≥0.

Similarly for $(R^+)^5$,let Φ be the class of all real-valued continuous functions φ : $(R^+)^5 \to R^+$ non -decreasing in the first argument and satisfying the following conditions for all x, y ≥ 0, x ≤ φ(y, x+ y, x + y, x+ y, y) or x ≤ φ(y, x+ y, x + y, x+ y, x + y) or x ≤ φ(y,1/2(x+ y) ,x+ y, x + y,+ y x) there exists a real number 0<k <1 such that x ≤ k y.

Results

Theorem 3.1: Let C be a non empty subset of Hilbert Space H. Let Ei and Fj sequences of continuous random operators defined on C such that for $\xi \in \Omega$, Ei(ξ, .) and Fj(ξ, .):C→C satisfying conditions (I) and (II)
(I):

$$\| Ei(\xi, x(\xi)) - Fj(\xi, y(\xi)\|^2 \le \phi \begin{bmatrix} \|x(\xi)-y(\xi)\|^2, \{\|x(\xi)-Ei(\xi,x(\xi))\|^2 + \|y(\xi)-Fj(\xi,y(\xi))\|^2\}, \\ \frac{1}{2}\{\|x(\xi)-Fj(\xi,y(\xi))\|^2 + \|y(\xi,-Ei(\xi,x(\xi))\|^2\} \end{bmatrix}$$

(II):

$$\| Ei(\xi, x(\xi)) - Fj(\xi,y(\xi))\|^2 \le \phi \begin{bmatrix} \|x(\xi)-y(\xi)\|^2, \left[\|x(\xi)-Ei(\xi,x(\xi))\|^2 + \|y(\xi)-Fj(\xi,y(\xi))\|^2 \right], \\ \frac{1}{2}[\|x(\xi)-Fj(\xi,y(\xi))\|^2 + \|y(\xi)-Ei(\xi,x(\xi))\|^2], \\ \dfrac{\frac{1}{2}\left[\|x(\xi)-y(\xi)\|^2 + \|x(\xi)-Ei(\xi,x(\xi))\|^2 + \|y(\xi)-Ei(\xi,x(\xi))\|^2\right]}{1+\|x(\xi)-y(\xi)\|^2\|x(\xi)-Fj(\xi,y(\xi))\|^2\|x(\xi)-Ei(\xi,x(\xi))\|^2}, \\ \dfrac{\|y(\xi)-Ei(\xi,x(\xi))\|^2 + \|y(\xi)-Fj(\xi,y(\xi))\|^2}{1+\|y(\xi)-Ei(\xi,x(\xi))\|^2\|y(\xi)-Fj(\xi,y(\xi))\|^2\|x-Fj(\xi,y(\xi,))\|^2} \end{bmatrix}$$

Then the sequences Ei and Fj have the unique common random fixed point.

Proof (I): Let us define $\{g_n\}$ sequence of function as
$g_{2n+1}(\xi) = Ei(\xi, g_{2n}(\xi)), \quad g_{2n+2}(\xi) = Fj(\xi, g_{2n+1})$ for $\xi \in \Omega$, n= 0, 1, 2, 3…… and i,j = 1,2,3…

Then by condition (I)
$\|g_{2n+1}(\xi) - g_{2n}(\xi)\|^2 = \|Ei(\xi, g_{2n}(\xi)) - Fj(\xi, g_{2n-1}(\xi)\|^2$

$$\le \phi \begin{bmatrix} \|g_{2n}(\xi)-g_{2n-1}(\xi)\|^2, \{\|g_{2n}(\xi) - Ei(\xi, g_{2n}(\xi))\|^2 + \|g_{2n-1}(\xi) - Fj(\xi, g_{2n-1}(\xi))\|^2\}, \\ ½\{\|g_{2n}(\xi) - Fj(\xi, g_{2n-1}(\xi))\|^2 + \|g_{2n-1}(\xi) - Ei(\xi, g_{2n}(\xi))\|^2\}] \end{bmatrix}$$

$$= \phi \begin{bmatrix} [\|\{g_{2n}(\xi) - g_{2n-1}(\xi)\|^2, \{\|g_{2n}(\xi) - g_{2n+1}(\xi)\|^2 + \|g_{2n-1}(\xi) - g_{2n}(\xi)\|^2\}, \\ ½\{\|\{g_{2n}(\xi) - g_{2n}(\xi)\|^2 + \|g_{2n-1}(\xi) - g_{2n+1}(\xi)\|^2\}] \end{bmatrix}$$

$$= \phi\left[\|g_{2n}(\xi)-g_{2n-1}(\xi)\|^2, \{\|g_{2n}(\xi)-g_{2n+1}(\xi)\|^2 + \|g_{2n-1}(\xi)-g_{2n}(\xi)\|^2\}, ½\{\|g_{2n-1}(\xi)-g_{2n+1}(\xi)\|^2\}]\right]$$

$$= \phi \begin{bmatrix} \|g_{2n}(\xi)-g_{2n-1}(\xi)\|^2, \{\|g_{2n}(\xi)-g_{2n+1}(\xi)\|^2 + \|g_{2n-1}(\xi)-g_{2n}(\xi)\|^2, \\ ½\{\|(g_{2n-1}(\xi)-g_{2n}(\xi) + g_{2n}(\xi)-g_{2n+1}(\xi)\|^2\} \end{bmatrix}$$

$$= \phi \begin{bmatrix} \|\|g_{2n}(\xi)-g_{2n-1}(\xi)\|^2, \{\|g_{2n}(\xi)-g_{2n+1}(\xi)\|^2 + \|g_{2n-1}(\xi)-g_{2n}(\xi)\|^2\}, \\ ½\{2\|(g_{2n-1}(\xi)-g_{2n}(\xi)\|^2 + 2\|g_{2n}(\xi)-g_{2n+1}(\xi)\|^2 - \|g_{2n-1}(\xi)-g_{2n}(\xi)-(g_{2n}(\xi)-g_{2n+1}(\xi))\|^2\}] \end{bmatrix}$$

$$\le \phi \begin{bmatrix} \|\|g_{2n}(\xi)-g_{2n-1}(\xi)\|^2, \{\|g_{2n}(\xi)-g_{2n+1}(\xi)\|^2 + \|g_{2n-1}(\xi)-g_{2n}(\xi)\|^2\}, \\ ½\{2\|(g_{2n-1}(\xi)-g_{2n}(\xi)\|^2 + 2\|g_{2n}(\xi)-g_{2n+1}(\xi)\|^2\} \end{bmatrix}$$

$$= k\|g_{2n-1}(\xi)-g_{2n}(\xi)\|^2$$

Also, $\|g_{2n}(\xi)-g_{2n-1}(\xi)\|^2 \le k\|g_{2n-1}(\xi)-g_{2n-2}(\xi)\|^2$.

Therefore in we get $\|g_n(\xi)-g_{n+1}(\xi)\|^2 \le k\|g_{n-1}(\xi)-g_n(\xi)\|^2$
Since $0 < k < 1$, therefore $\{g_n(\xi)\}$ is a Cauchy sequence and hence it is convergent in H. Since C is closed therefore there exists a measurable function g: C→ C such
that gn (ξ) → g(ξ) as n→ ∞, ∀ $\xi \in \Omega$.

$||g(\xi)-Fj(\xi,g(\xi))||^2 \le ||\, g(\xi)-g_{2n+1}(\xi)+g_{2n+1}(\xi)- Fj(\xi,g(\xi))||^2 \le 2||g(\xi)-g_{2n+1}(\xi)||^2+2||g_{2n+1}(\xi)- Fj(\xi,g(\xi))||^2$ by the use of parallelogram law= $2||g(\xi)-g_{2n+1}(\xi)||^2+2||\, Ei(\xi,g_{2n}(\xi))- Fj(\xi,g(\xi))||^2$

$$= 2||g(\xi)-g_{2n+1}(\xi)||^2 +2\,\phi\,[||\, g_{2n}(\xi)- g(\xi))||^2,\{||g_{2n}(\xi)- Ei(\xi,g_{2n}(\xi))||^2 + ||g(\xi)- Fj(\xi,g(\xi))||^2\},½\{||g_{2n}(\xi)- Fj(\xi,g(\xi))||^2 + ||g(\xi)- Ei(\xi,g_{2n}(\xi))|^2\}]$$

$$= 2||g(\xi)-g_{2n+1}(\xi)||^2 + 2\,\phi\,[||\, g_{2n}(\xi)- g(\xi))||^2,\{||g_{2n}(\xi)- g_{2n+1}(\xi))||^2 + ||g(\xi)- Fj(\xi,g(\xi))||^2\},½\{||g_{2n}(\xi)- Fj(\xi,g(\xi))||^2 + ||g(\xi)- g_{2n+1}(\xi))||^2\}]$$

As $n\to\infty$ $||g(\xi)-Fj(\xi,g(\xi))||^2 =2.k||g(\xi)- Fj(\xi,g(\xi))||^2$
Hence for all Fj (ξ, g (ξ)) = g (ξ). Similarly one can prove Ei (ξ,g(ξ))= g(ξ)

Now, if G: Ω X C →C is a continuous random operation on a non empty subset of a separated Hilbert space H, then for any measurable formula f: Ω →C the function g(ξ)=G(ξ,f(ξ)) is also measurable. Therefore the sequence of measurable functions $\{g_n\}$ converge to measurable function of this fact along with Ei(ξ,g(ξ)) = g(ξ) = Fj(ξ,g(ξ)) shows that g: Ω →C is common random fixed point of Ei and Fj for each i and j .

Uniqueness: Let h: Ω →C be another common random fixed point of Ei and Fj which is also measurable and now by the use of condition (I)
$||g(\xi)-h(\xi)||^2 = ||Ei(\xi ,g(\xi))- Fj(\xi, h(\xi)||^2 \le \phi[||\, g(\xi)- h(\xi)||^2 ,||\, g(\xi) -Ei(\xi , g(\xi))\, ||^2+||h(\xi)- Fj(\xi, g(\xi)||^2\}½\{||g (\xi) - Fj(\xi, h(\xi))||^2+ ||\, h(\xi) - (\xi , g(\xi))\, ||^2\}]$
$= k\, ||\, g (\xi)- h(\xi)||^2$
$\Rightarrow$ g(ξ) = h(ξ) $\forall\, \xi \in \Omega$ (as 0<k <1)

Proof (II): Let us consider sequence $\{g_n\}$ of function defined for $\xi\in\Omega$ and n=0, 1, 2, 3…… as Part (I)
$g_{2n+1}(\xi) = Ei(\xi, g_{2n}(\xi)),\quad g_{2n+2}(\xi) = Fj(\xi, g_{2n+1})$ $||g_{2n+1}(\xi) - g_{2n}(\xi)\, ||^2 = ||Ei(\xi , g_{2n}(\xi))- Fj(\xi,g_{2n-1}(\xi)||^2$

$$\le \phi\left[\begin{array}{c} \|g_{2n}(\xi)-g_{2n-1}(\xi)\|^2,\left[\ \|g_{2n}(\xi)-Eig_{2n}(\xi)\|^2+\|g_{2n-1}(\xi)-Fjg_{2n-1}(\xi)\|^2\ \right], \\ \frac{1}{2}[\|g_{2n}(\xi)-Fjg_{2n-1}(\xi)\|^2+\|g_{2n-1}(\xi)-Eig_{2n}(\xi)\|^2\ ], \\ \dfrac{\frac{1}{2}\left[\|g_{2n}(\xi)-g_{2n-1}(\xi)\|^2+\|g_{2n-1}(\xi)-Eig_{2n}(\xi)\|^2+\|g_{2n}(\xi)-Eig_{2n}(\xi)\|^2\right]}{1+\|g_{2n}(\xi)-g_{2n-1}(\xi)\|^2\,\|g_{2n}(\xi)-Fjg_{2n-1}(\xi)\|^2\,\|g_{2n}(\xi)-Eig_{2n}(\xi)\|^2}, \\ \dfrac{\|g_{2n-1}(\xi)-Fjg_{2n-1}(\xi)\|^2+\|g_{2n}(\xi)-Fjg_{2n-1}(\xi)\|^2}{1+\|g_{2n-1}(\xi)-Eig_{2n}(\xi)\|^2\,\|g_{2n-1}(\xi)-Fjg_{2n-1}(\xi)\|^2\,\|g_{2n}(\xi)-Fjg_{2n-1}(\xi)\|^2} \end{array}\right]$$

$$= \phi\left[\begin{array}{c} \|g_{2n}(\xi)-g_{2n-1}(\xi)\|^2,\left[\ \|g_{2n}(\xi)-g_{2n+1}(\xi)\|^2+\|g_{2n-1}(\xi)-g_{2n}(\xi)\|^2\ \right],\frac{1}{2}\|g_{2n-1}(\xi)-g_{2n+1}(\xi)\|^2, \\ \dfrac{\frac{1}{2}\left[\|g_{2n}(\xi)-g_{2n-1}(\xi)\|^2+\|g_{2n-1}(\xi)-g_{2n+1}(\xi)\|^2+\|g_{2n}(\xi)-g_{2n+1}(\xi)\|^2\right]}{1+\|g_{2n}(\xi)-g_{2n-1}(\xi)\|^2\,\|g_{2n}(\xi)-g_{2n}(\xi)\|^2\,\|g_{2n}(\xi)-g_{2n+1}(\xi)\|^2}, \\ \dfrac{\|g_{2n-1}(\xi)-g_{2n}(\xi)\|^2+\|g_{2n}(\xi)-g_{2n}(\xi)\|^2}{1+\|g_{2n-1}(\xi)-g_{2n+1}(\xi)\|^2\,\|g_{2n-1}(\xi)-g_{2n}(\xi)\|^2\,\|g_{2n}(\xi)-g_{2n}(\xi)\|^2} \end{array}\right]$$

$$= \phi\begin{bmatrix} \|g_{2n}-g_{2n-1}\|^2, \|g_{2n}-g_{2n+1}\|^2+\|g_{2n-1}-g_{2n}\|^2, \frac{1}{2}\|g_{2n-1}-g_{2n+1}\|^2, \\ \frac{1}{2}(\|g_{2n}-g_{2n-1}\|^2+\|g_{2n}-g_{2n-1}\|^2+\|g_{2n}-g_{2n+1}\|^2), \|g_{2n-1}-g_{2n}\|^2 \end{bmatrix} \quad \text{3.II (a)}$$

Now

$$\|g_{2n-1}(\xi)-g_{2n+1}(\xi)\|^2 = \|[g_{2n-1}(\xi)-g_{2n}(\xi)]+[g_{2n}(\xi)-g_{2n+1}(\xi)]\|^2$$

By using parallelogram law we can write

$$\|x+y\|^2+\|x-y\|^2 = 2\|x\|^2+2\|y\|^2 \Rightarrow \|x+y\|^2 = 2\|x\|^2+2\|y\|^2-\|x-y\|^2, \forall x, y\,\varepsilon\, C$$

$$\|[g_{2n-1}(\xi)-g_{2n}(\xi)]+[g_{2n}(\xi)-g_{2n+1}(\xi)]\|^2$$

$$= 2\|g_{2n-1}(\xi)-g_{2n}(\xi)\|^2+2\|g_{2n}(\xi)-g_{2n+1}(\xi)\|^2-\|[g_{2n-1}(\xi)-g_{2n}(\xi)]-[g_{2n}(\xi)-g_{2n+1}(\xi)]\|^2$$

$$\le 2\|g_{2n-1}(\xi)-g_{2n}(\xi)\|^2+2\|g_{2n}(\xi)-g_{2n+1}(\xi)\|^2$$

On putting this in 3.II (a), we get

$$\|g_{2n+1}(\xi)-g_{2n}(\xi)\|^2 \le \phi\begin{bmatrix} \|g_{2n}(\xi)-g_{2n-1}(\xi)\|^2, \|g_{2n-1}(\xi)-g_{2n}(\xi)\|^2+\|g_{2n}(\xi)-g_{2n+1}(\xi)\|^2, \\ \|g_{2n-1}(\xi)-g_{2n}(\xi)\|^2+\|g_{2n}(\xi)-g_{2n+1}(\xi)\|^2, \\ \|g_{2n-1}(\xi)-g_{2n}(\xi)\|^2+\|g_{2n}(\xi)-g_{2n+1}(\xi)\|^2, \|g_{2n-1}(\xi)-g_{2n}(\xi)\|^2 \end{bmatrix}$$

$$\|g_{2n}(\xi)-g_{2n+1}(\xi)\|^2 \le k\,\|g_{2n}(\xi)-g_{2n-1}(\xi)\|^2 \quad \text{-----3.II (b)}$$

Similarly we can find for

$$\|g_{2n}(\xi)-g_{2n-1}(\xi)\|^2 \le k\,\|g_{2n-1}(\xi)-g_{2n-2}(\xi)\|^2\ \forall\,\xi\,\varepsilon\,\Omega \text{------------ 3.II(c)}$$

Equations 3.II (b) and 3.II(c) jointly implies that

$$\|g_n(\xi)-g_{n+1}(\xi)\|^2 \le k\,\|g_{n-1}(\xi)-g_n(\xi)\|^2\ \forall\,\xi\,\varepsilon\,\Omega \text{------- 3.II (d)}$$

It is clear that $g_n(\xi)$ is a Cauchy sequence and hence it is convergent in the Hilbert spaces H.

So $g_n(\xi) \to g(\xi)\, as\, n \to \infty$ …………….3.II(e).

Since C is closed and $g: C \to C$, so for $\xi\,\varepsilon\,\Omega$

$$\|g(\xi)-Ei(\xi, g(\xi))\|^2 = \|(g(\xi)-g_{2n}(\xi))+(g_{2n}(\xi)-Ei(\xi, g(\xi))\|^2$$

$$\le 2\|g(\xi)-g_{2n}(\xi)\|^2+2\|g_{2n}(\xi)-Ei(\xi, g(\xi)\|^2\ by\ parale\log ram\ law$$

$$= 2\|g(\xi)-g_{2n}(\xi)\|^2+2\|Fj(\xi, g_{2n-1}(\xi))-Ei(\xi, g(\xi)\|^2$$

$$== 2\|g(\xi)-g_{2n}(\xi)\|^2+2\|Ei(\xi, g(\xi)-Fj(\xi, g_{2n-1}(\xi))\|^2$$

$$= 2\|g(\xi) - g_{2n}(\xi)\|^2 + 2\begin{bmatrix} \|g(\xi) - g_{2n-1}(\xi)\|^2, \left[\|g(\xi) - Eig(\xi)\|^2 + \|g_{2n-1}(\xi) - Fjg_{2n-1}(\xi)\|^2\right], \\ \frac{1}{2}\left[\|g(\xi) - Fjg_{2n-1}(\xi)\|^2 + \|g_{2n-1}(\xi) - Eig(\xi)\|^2\right], \\ \dfrac{\frac{1}{2}\left[\|g(\xi) - g_{2n-1}(\xi)\|^2 + \|g(\xi) - Eig(\xi)\|^2 + \|g_{2n-1}(\xi) - Eig(\xi)\|^2\right]}{1 + \|g(\xi) - g_{2n-1}(\xi)\|^2 \|g(\xi) - Eig(\xi)\|^2 \|g_{2n-1}(\xi) - Eig(\xi)\|^2}, \\ \dfrac{\left[\|g_{2n-1}(\xi) - Eig(\xi)\|^2 + \|g_{2n-1}(\xi) - Fjg_{2n-1}(\xi)\|^2\right]}{1 + \|g_{2n-1}(\xi) - Eig(\xi)\|^2 \|g_{2n-1}(\xi) - Fjg_{2n-1}(\xi)\|^2 \|g\ (\xi) - Fjg_{2n-1}(\xi)\|^2} \end{bmatrix} \text{........3.}II(f)$$

Making $n \to \infty$ and by the help of 3.II (e)

$$\|g(\xi) - Ei(\xi, g(\xi)\|^2 \le \|g(\xi) - Eig(\xi)\|$$

$$So\,\forall\, \xi\, \varepsilon\, \Omega, Ei(\xi, g(\xi)) = g(\xi)$$ ----- 3.II (g)

Similarly we can prove that $Fj(\xi, g(\xi)) = g(\xi)$ -------- 3.II (h)

Again, for a nonempty subset C of a separable spaces H there exists $A : \xi \times C \to C$ random operator then for any measurable function $f : \Omega \to C$, *the function* $h(\xi) = A(\xi, f(\xi))$ is also measurable. It follows from the construction of $\{g_n\}$ and the above considerations that $\{g_n\}$ is sequence of measurable functions it follows that g is also a measurable function. From 3.II (g) and 3.II (h) shows that $g : \Omega \to C$ is a common fixed point of Ei and Fj.

Uniqueness: Let $h : \Omega \to C$ be another random common random fixed point of Ei and Fj that is for $\xi\, \varepsilon\, \Omega$, $Ei(\xi, h(\xi)) = h(\xi), Fj(\xi, h(\xi)) = h(\xi)$ ---- 3.II (i)

$$\|g(\xi) - h(\xi)\|^2 = \|Ei(\xi, g(\xi)) - Fj(\xi, h(\xi))\|^2$$

$$\le \phi\begin{bmatrix} \|g(\xi) - h(\xi)\|^2, \left[\|g(\xi) - Eig(\xi)\|^2 + \|h(\xi) - Fjh(\xi)\|^2\right], \frac{1}{2}[\|g(\xi) - Fjh(\xi)\|^2 + \|h(\xi) - Eig(\xi)\|^2], \\ \dfrac{\frac{1}{2}\left[\|g(\xi) - h(\xi)\|^2 + \|g(\xi) - Eig(\xi)\|^2 + \|h(\xi) - Eig(\xi)\|^2\right]}{1 + \|g(\xi) - h(\xi)\|^2 \|g(\xi) - Fjh(\xi)\|^2 \|g(\xi) - Eig(\xi)\|^2}, \dfrac{\|h(\xi) - Eig(\xi)\|^2 + \|h(\xi) - Fjh(\xi)\|^2}{1 + \|h(\xi) - Eig(\xi)\|^2 \|h(\xi) - Fjh(\xi)\| \|g(\xi) - Fjh(\xi)\|^2} \end{bmatrix} = \|h(\xi) - g(\xi)\|^2$$

which is a contradiction therefore $g(\xi) = h(\xi)$.

Remark: Our results are generalization of V. B. Dhagat and et.al. As the convergent mappings of sequences also satisfy both conditions and therefore we can obtained random common fixed point results

Example

Let H = R , $\Omega = [0,1]$ and Σ be the sigma algebra of Lebesgue's measurable subset of [0,1]. Let C = $[0, \infty)$ and define a mapping d: $(\Omega x H) x (\Omega x H) \to C$ by

d (x,y) = $|x(\omega) - y(\omega)|$. Define random operator Ei, Fj: $\Omega x H \to H$ as

$Ei(\xi,x) = (1-\xi^2)x^i$ and $Fj(\xi,x) = (1-\xi^2)(x/2)^j$. Also sequence of mapping $g_n : \Omega \to C$ is defined by $g_n(\xi) = (1 - \xi^2)^{1+1/n}$ for every $\xi \in \Omega$ and n $\in$N. Define measurable mapping $g : \Omega \to C$ as $g(\xi) = 1-\xi^2$ for every $\xi \in \Omega$ which is fixed point of Ei and Fj.

Conclusion

We ensure the unique fixed point of sequences of continuous random operators for implicit relation analogue of a plane contractive. Also, we provide measurable sequence of function which converse to measurable function to ensure the existence of unique common fixed point.

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