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On Some New Integrals Relation of $\overline{H}$ –function

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Abstract

In this paper we derive some new integrals whose integrand contains product of Horn's function and $\overline{H}$-function of one variable.

Keywords: $\overline{H}$-function, Horn's function

Introduction

The $\overline{\boldsymbol{H}}$ –function was introduced by Inayat Hussain[1] and studied by Bushman and Srivastava[2].

The $\overline{\boldsymbol{H}}$ –function is defined and represented in the following manner: $\overline{H}_{p,q}^{m,n}\,[x] = \overline{H}_{p,q}^{m,n}\left[x \left| \begin{matrix} (a_j,\alpha_j;A_j)_{1,n};(a_j,\alpha_j)_{n+1,p} \\ (b_j,\beta_j)_{1,m};(b_j,\beta_j,B_j)_{m+1,q} \end{matrix} \right.\right] = \frac{1}{2\pi\omega}\int_{\mathcal{L}} \overline{\emptyset}(\xi)x^{\xi}\, d\,\xi, (z \neq 0)$ (1)

Where $\overline{\emptyset}\,(\xi) = \frac{\prod_{j=1}^{m}\Gamma(b_j - \beta_j\xi)\ \prod_{j=1}^{n}\{\Gamma(1-a_j+\alpha_j\xi)\}^{A_j}}{\prod_{j=m+1}^{q}\{\Gamma(1-b_j+\beta_j\xi)\}^{B_j}\prod_{j=n+1}^{p}\Gamma(a_j-\alpha_j\xi)}$ (2)

Here a_j (j = 1, . . . , P) and b_j (j = 1,. . . ,Q) are complex parameter α_j = 0, (j =1 , . . . ,P), β_j = 0 (j = 1,. . . ,Q) (not all zero simultaneously) and the exponents, a_j (j = 1 ,. . . ,N) and b_j (j = M+1,. . . ,Q) can take on integer values.

The following sufficient conditions for the absolute convergence of the defining integral for the $\overline{\boldsymbol{H}}$ -function given by Bushman and Srivastava[2].

T = $\sum_{j=1}^{M}\beta_j + \sum_{j=1}^{N}|A_j\alpha_j|$- $\sum_{j=M+1}^{Q}|B_j\beta_j|$- $\sum_{j=N+1}^{P}\alpha_j > 0$ (3)

and $|\arg z| < \frac{1}{2}$ Tπ.

If we use (a, n) = a (a+1) (a+2)………………..(a+n-1) ; (a, 0) = 1

Where n is some positive integer then Horn's function [3] of two variables H_1 and G_1 are define in following

$H_1(\alpha,\beta,\gamma;\delta;u,v) = \sum_{r,s=0}^{\infty}\frac{(\alpha,r-s)(\beta,r+s)(\gamma,s)}{(1,r)(1,s)(\delta,r)}\ u^r\ v^s$, (4)

|x| < r, |y| < s , 4rs = $(s-1)^2$;

$G_1(\alpha,\beta,\gamma;\delta;u,v) = \sum_{r,s=0}^{\infty}\frac{(\alpha,r+s)(\beta,s-r)(\gamma,r-s)}{(1,r)(1,s)}\ u^r\ v^s$, (5)

|x| < r, |y| < s , r + s = 1

Main Result

$$\int_0^{\infty} x^{\rho-1}[\,x+a+(x^2+2ax\,)^{1/2}]^{-\sigma}\,H_1\left[\alpha,\beta,\gamma;u\{x+a+(x^2\ +2ax)^{\frac{1}{2}}\}\right]^{-\mu}$$

$$v[\,x+a+(x^2+2ax\,)^{1/2}]^{-\vartheta}\,\overline{H}_{p,q}^{m,n}\ \left[z\,\{x+a+(x^2\ +2ax)^{\frac{1}{2}}\}^{-\lambda}\right]dx$$

$$= 2\,a^{-\sigma}\left(\frac{a}{2}\right)^{\rho}\ \Gamma(2\rho)\,H_1\left[\alpha,\beta,\gamma;\delta\,;\frac{u}{a^{\mu}},\frac{v}{a^{\vartheta}}\right]\overline{H}_{p+2,q+2}^{m,n+2}\left[za^{-\lambda}\left| \begin{matrix} (-\sigma-\mu r-\vartheta s,\lambda),,(1+\rho-\sigma-\mu r-\vartheta s,\lambda),(a_j,\alpha_j,A_j)_{1,p} \\ (b_j,\beta_j,B_j)_{1,q},(1-\sigma-\mu r-\vartheta s,\lambda),,(-\rho-\sigma-\mu r-\vartheta s,\lambda) \end{matrix}\right.\right], \quad (6)$$

Provided,

$\lambda > 0$, *Re(p,σ,μ,ϑ)* > 0, *Re(ρ) – Re(σ)* - $\lambda\, min_{1\le j\le m} Re\left(\frac{b_j}{\beta_j}\right) < 0$

$$\int_0^\infty x^{\rho-1}[\, x+a+(x^2+2ax\,)^{1/2}]^{-\sigma}\, G_1\left[\alpha,\beta,\gamma;u\{x+a+(x^2\,+2ax)^{\frac{1}{2}}\}\right]^{-\mu}$$
$$v[\, x+a+(x^2+2ax\,)^{1/2}]^{-\vartheta}\, \overline{H}_{p,q}^{m,n}\,\left[z\,\{x+a+(x^2\,+2ax)^{\frac{1}{2}}\}^{\lambda}\right]dx$$
$$= 2\, a^{-\sigma}\left(\frac{a}{2}\right)^{\rho}\, \Gamma(2\rho)\, G_1\left[\alpha,\beta,\gamma;\delta\,;\frac{u}{a^{\mu}},\frac{v}{a^{\vartheta}}\right] \overline{H}_{p+2,q+2}^{m,n+2}\left[za^{\lambda}\left|\begin{matrix}(a_j,\alpha_j,A_j)_{1,p}(1+\rho-\sigma-\mu r-\vartheta s,\lambda)(\sigma+\mu r+\vartheta s,\lambda)_{,,}\\ ,(1+\sigma+\mu r+\vartheta s,\lambda)_{,,}(\sigma-\rho+\mu r+\vartheta s,\lambda)(b_j,\beta_j,B_j)_{1,q}\end{matrix}\right.\right], \tag{7}$$

Provided

$-\lambda > 0$, $Re(p,\sigma,\mu,\vartheta) > 0$, $Re(\rho) - Re(\sigma) + \lambda\, min_{1\le j\le m} Re\left(\frac{b_j}{\beta_j}\right) < 0$

Proof: we get the result (6) if given in the left hand side of inayat hussain $\overline{\mathrm{H}}$ –function in equation (1) eliminate by contour integral, and Horn's H_1 function should be represent in series form given in equation (4). Sequence of integral and compilation should be expressed in the form of internal integral of equation (1)[4].

Similarly we get the result (7), the change of formula (4) by equation (5).

Special cases: On taking $A_j = B_j = 1$ the $\overline{H}$ -function reduces to Fox's H-function and we found that the solution of integral equation (1) in terms of Fox's H-function given in Shrivastava Rajeev and et al[5]. *On taking* $A_j = B_j = 1$ *and* $\alpha_j = \beta_j = c$ *(i = 1,….,p ; j = 1,…..,q), the* $\overline{H}$ *-function reduces to Meijer G-function*

Conclusion

The $\boldsymbol{\overline{H}}$-function is a very general function and has for its particular cases a number of important special functions[1,2,6].

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