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Retailer's profit maximization Model for Weibull deteriorating items with Permissible Delay on Payments and Shortages

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Abstract

In this paper, we have formulated an Inventory model for deteriorating items with Weibull distribution deterioration rate with two parameters. We have considered the demand of the item as time dependent and linear. Shortages are not considered. A certain fixed period is provided to retailer by the supplier for resolving the account and that condition is defined as permissible delay in payment. There are two cases are viewed first is permissible credit period is less than to Inventory cycle length or second is permissible credit period is greater than and equal to Inventory cycle length. The designed model optimizes retailer's order quantity by maximizing his total profits. The retailer can gain interest on the revenue created during this period. The numerical solution of the model is obtained to verify the optimal solution. The model is solved analytically by maximizing the total profit.

Keywords: Inventory, deteriorating items, weibull distribution with two parameters, time dependent linear Demand, permissible delay

Introduction

The main objective of inventory control is to minimize the total inventory cost or maximize the total profit. Inventory control is required to determine the optimal stock and optimal time of replenishment of the inventory to meet the next demand. Deterioration of physical goods is one of the important factors in any inventory and production system. The deteriorating items have received much attention of several researches in the recent year because most of the physical goods undergo decay or deterioration over time. Commodities such as fruits, vegetables and food stuffs from depletion by direct spoilage wile kept in store. Ghare and Schrader[1] were first who presented an economic order quantity model for exponentially decaying inventory. Thereafter, several interesting papers for controlling the deteriorating items appeared in different journals and they were summarized by so many researchers. S.K. Goyal and B.C. Giri[2] Invited Review paper on recent trends in modelling of deteriorating inventory. Chandra K. Jaggi, Satish K. Goel and Mandeep Mittal[3] developed economic order quantity model for deteriorating items with imperfect quality and permissible delay on payment.O Fujiwara[4] given the model EOQ models for continuously deteriorating products using linear and exponential penalty costs. K.S Chaudhuri[5] developed an EOQ model for deteriorating items with a linear trend in demand and shortages in all cycles. H.M Wee[6] introduced A deterministic lot-size inventory model for deteriorating with shortages and a declining market. Further K. Chung and P.A. Ting[7] suggested heuristic for replenishment of deteriorating items with a linear trend in demand. Inventory model for ameliorating items for price dependent demand rate was proposed by Mondal et.al[8] and inventory model with price and time dependent demand was developed by you[9]. In general holding cost is assumed to be known and constant. But in realistic condition holding cost may not Goh[10] considered various functions to describe holding cost.

Since then, many research papers have appeared in different journals, A.K Jalan and K.S Chaudhuri[11] introduced Structural properties of an inventory system with deterioration and trended demand.

In this paper we have consider the demand of the item is time dependent and linear. In permissible delay in payment supplier does offer a certain fixed period to the retailer for settling the account. There are two cases are discussed first is permissible credit period is less than to Inventory cycle length or second is permissible credit period is greater than and equal to Inventory cycle length. The planned model optimizes retailer's order quantity by maximizing his total profits.

Notations and Assumptions

The fundamental Notations are used to develop the model.

The demand rate D (t) is dependent on time t, as follow $D(t) = a + bt,$ the annual demand as a function of time, where$a > 0, b > 0$. C is the unit purchase cost. I (t) is the inventory level at time t. $0 \le t \le T$. T is the Inventory cycle length. A_0 is the ordering cost. k is the unit selling price with $k > C$. h is the inventory holding cost per unit item per unit time. The deterioration cost per unit item, per unit time is C_2. and the deterioration rate is proportional to time. C_3 is the shortage cost per unit item per unit time. I(t) is the maximum inventory level for the ordering cycle . Q is the ordering quantity. N is permissible credit or permissible delay in settling the accounts. Ie is Interest earned per unit per unit time. Ip is Interest paid per unit item per unit time. The deterioration of time as follows by Weibull parameter (two) distribution θ (t) = $\alpha\beta t^{(\beta-1)}$ where 0<α< 1 is the scale parameter and β >0 is the shape parameter.

Assumptions: Demand rate is time dependent and linear. Shortages are not allowed and lead time is zero. Shortages are not allowed and lead time is zero. Shortages are not allowed and lead time is zero. P_1 (T) total profit for case first, $t_1 \le N \le T$. P_2 (T) total profit for case first , $t_1 \le T \le N$.

Formulation and Solution

The length of the cycle is T. Let I(t) be the inventory level at time t ($0 \le t < T$).
The demand rate $D(t) = a + bt,$ is time dependent,
The differential equation can be defined when the instantaneous state of I(t) over (O,T) are given by

$$\frac{dI(t)}{dt} + \propto\beta\, (t)^{\beta-1}\, I(t) = -(a + bt) \qquad 0 \le t \le T \tag{1}$$

Subject to boundary condition t=0, I(t) =Q, I(0) =Q

From equation (1) we get $I(t) = (1-\propto t^{\beta})\left\{Q - \left[a\left(t + \frac{\alpha t^{(\beta+1)}}{(\beta+1)}\right) + b\left(\frac{t^2}{2} + \frac{\alpha t^{(\beta+2)}}{(\beta+2)}\right)\right]\right\} \qquad O < t \le T$ (2)

And the order quantity at t=T From equation (2) we get I(T) = 0, $Q = \left[a\left(T + \frac{\alpha T^{(\beta+1)}}{(\beta+1)}\right) + b\left(\frac{T^2}{2} + \frac{\alpha T^{(\beta+2)}}{(\beta+2)}\right)\right]$ (3)

The all costs are evaluated for the total profit as follows.Total sales revenue will be generated by the demand meet during the time period. =k.D.T (4)

Ordering cost = A_0 (5)

Purchase cost = CQ (6)

Deterioration cost = $C_2 \left[Q - \int_0^T D(t)dt\right], D_T = C2\left[a\frac{\alpha T^{(\beta+1)}}{(\beta+1)} + b\frac{\alpha T^{(\beta+2)}}{(\beta+2)}\right]$ (7)

The holding cost during the time period 0 to T, $H_T = h\int_0^T I(t)dt$

$$H = h\int_0^T (1-\propto t^{\beta})\left\{Q - \left[a\left(t + \frac{\alpha t^{(\beta+1)}}{(\beta+1)}\right) + b\left(\frac{t^2}{2} + \frac{\alpha t^{(\beta+2)}}{(\beta+2)}\right)\right]\right\}dt$$

Now holding cost will be

$$H = h\left\{Q\left[T - \frac{\alpha T^{(\beta+1)}}{(\beta+1)}\right] - a\left[\frac{T^2}{2} - \frac{\alpha T^{(\beta+2)}}{(\beta+2)}\left(\frac{\beta}{\beta+1}\right) - \frac{\alpha^2 T^{2(\beta+1)}}{2(\beta+1)^2}\right] + b\left[\frac{T^3}{6} - \frac{\alpha T^{(\beta+2)}}{2(\beta+2)} + \frac{\alpha T^{(\beta+3)}}{(\beta+3)} - \frac{\alpha^2 T^{2\beta+3}}{(\beta+2)(2\beta+3)}\right]\right\} \tag{8}$$

To determine the interest payable and earned, there will be two cases i.e. N<T and N≥T:

Case Ist: N<T

In this case, the retailer can earn interest on the revenue created from the sales the product up to N. Although he has to resolve the account at N, he has to arrange the money to get his rest stokes, for this retailer borrow the money at some certain rate of interest, the retailer Pay for the period N to T.

Interest earned on the revenue by the retailer.

$$IE_1 = kI_e\int_0^N D(t).tdt, = kI_e\int_0^N (a + bt).tdt, \; = kI_eN^2\left(\frac{a}{2} + \frac{bN}{3}\right) \tag{9}$$

The interest paid by the retailer during the time interval N to T.

$$I.P = CI_p\int_N^T I(t)dt \; = CI_P\int_N^T (1-\propto t\beta)\left\{Q - \left[a\left(t + \frac{\alpha t^{(\beta+1)}}{(\beta+1)}\right) + b\left(\frac{t^2}{2} + \frac{\alpha t^{(\beta+2)}}{(\beta+2)}\right)\right]\right\}dt$$

$$\mathrm{I.P.} = CI_P\left[Q\left\{(T-N) - \frac{\alpha}{(\beta+1)}\left(T^{(\beta+1)} - N^{(\beta+1)}\right)\right\} - a\left\{\frac{1}{2}(T^2-N^2) - \frac{\alpha\beta}{(\beta+1)(\beta+2)}\left(T^{(\beta+2)} - N^{(\beta+2)}\right) - \frac{\alpha^2}{2(\beta+1)^2}\left(T^{2(\beta+1)} - N^{2(\beta+1)}\right)\right\} + b\left\{\frac{1}{6}(T^3-N^3) - \frac{\alpha}{2(\beta+2)}\left(T^{(\beta+2)} - N^{(\beta+2)}\right) + \frac{\alpha}{(\beta+3)}\left(T^{(\beta+3)} - N^{(\beta+3)}\right) - \frac{\alpha^2}{(\beta+2)(2\beta+3)}\left(T^{(2\beta+3)} - N^{(2\beta+3)}\right)\right\}\right] \quad (10)$$

The total profit per unit time $P_1(T)$ will be

$$P_1(T) = \frac{1}{T}\{\text{Sales revenue} + \text{Interest earned} - \text{Ordering cost} - \text{Purchasing cost} - \text{Deterioration cost} - \text{Holding cost} - \text{interest paid}\} \quad (11)$$

$$P_1(T) = \frac{1}{T}\left\{\left[k.D.T + kI_eN^2\left(\frac{a}{2} + \frac{bN}{3}\right)\right] - \left[A0 + CQ + C_2\left\{a\frac{\alpha T^{(\beta+1)}}{(\beta+1)} + b\frac{\alpha T^{(\beta+2)}}{(\beta+2)}\right\} + h\left\{Q\left[T - \frac{\alpha T^{(\beta+1)}}{(\beta+1)}\right] - a\left[\frac{T^2}{2} - \frac{\alpha T^{(\beta+2)}}{(\beta+2)}\left(\frac{\beta}{\beta+1}\right) - \frac{\alpha^2 T^{2(\beta+1)}}{2(\beta+1)^2}\right] + b\left[\frac{T^3}{6} - \frac{\alpha T^{(\beta+2)}}{2(\beta+2)} + \frac{\alpha T^{(\beta+3)}}{(\beta+3)} - \frac{\alpha^2 T^{2\beta+3}}{(\beta+2)(2\beta+3)}\right]\right\} + CI_P\left[Q\left\{(T-N) - \frac{\alpha}{(\beta+1)}\left(T^{(\beta+1)} - N^{(\beta+1)}\right)\right\} - a\left\{\frac{1}{2}(T^2-N^2) - \frac{\alpha\beta}{(\beta+1)(\beta+2)}\left(T^{(\beta+2)} - N^{(\beta+2)}\right) - \frac{\alpha^2}{2(\beta+1)^2}\left(T^{2(\beta+1)} - N^{2(\beta+1)}\right)\right\} + b\left\{\frac{1}{6}(T^3-N^3) - \frac{\alpha}{2(\beta+2)}\left(T^{(\beta+2)} - N^{(\beta+2)}\right) + \frac{\alpha}{(\beta+3)}\left(T^{(\beta+3)} - N^{(\beta+3)}\right) - \frac{\alpha^2}{(\beta+2)(2\beta+3)}\left(T^{(2\beta+3)} - N^{(2\beta+3)}\right)\right\}\right]\right]\right\} \quad (12)$$

The optimal value of T which maximizes the total profit $P_1(T)$ can be obtained by solving the equation. $\frac{dP_1(T)}{dT} = 0$

$$\frac{dP_1(T)}{dT} = \left\{\left[kI_eN^2\left(-\frac{a}{2T^2} - \frac{bN}{3T^2}\right)\right] - \left[-\frac{A_0}{T^2} + \frac{C}{T}Q' - \frac{C}{T^2}Q + C_2\left\{a\frac{\alpha\beta T^{(\beta-1)}}{(\beta+1)} + b\frac{\alpha(\beta+1)T^{\beta}}{(\beta+2)}\right\} + h\left\{Q'\left[1 - \frac{\alpha T^{\beta}}{(\beta+1)}\right] - Q\left[\frac{\alpha\beta T^{(\beta-1)}}{(\beta+1)}\right] - a\left[\frac{1}{2} - \frac{\alpha\beta T^{\beta}}{(\beta+2)} - \frac{\alpha^2(2\beta+1)T^{2\beta}}{2(\beta+1)^2}\right] + b\left[\frac{T}{3} - \frac{\alpha(\beta+1)T^{\beta}}{2(\beta+2)} + \frac{\alpha(\beta+2)T^{((\beta+1))}}{(\beta+3)} - \frac{\alpha^2(2\beta+2)T^{(2\beta+1)}}{(\beta+2)(2\beta+3)}\right]\right\} + CI_P\left[Q'\left\{\left(1 - \frac{N}{T}\right) - \frac{\alpha}{(\beta+1)}\left(T^{\beta} - \frac{N^{(\beta+1)}}{T}\right)\right\} + Q\left\{\frac{N}{T^2} - \frac{\alpha}{(\beta+1)}\left(\beta T^{(\beta-1)} + \frac{N^{(\beta+1)}}{T^2}\right)\right\} - a\left\{\frac{1}{2}\left(1 - \frac{N^2}{T^2}\right) - \frac{\alpha\beta}{(\beta+1)(\beta+2)}\left((\beta+1)T^{\beta} + \frac{N^{(\beta+2)}}{T^2}\right) - \frac{\alpha^2}{2(\beta+1)^2}\left((2\beta+1)T^{2\beta} + \frac{N^{2(\beta+1)}}{T^2}\right)\right\} + b\left\{\frac{1}{6}\left(2T + \frac{N^3}{T^2}\right) - \frac{\alpha}{2(\beta+2)}\left((\beta+1)T^{\beta} + \frac{N^{(\beta+2)}}{T^2}\right) + \frac{\alpha}{(\beta+3)}\left((\beta+2)T^{(\beta+1)} + \frac{N^{(\beta+3)}}{T^2}\right) - \frac{\alpha^2}{(\beta+2)(2\beta+3)}\left((2\beta+2)T^{(2\beta+1)} + \frac{N^{(2\beta+3)}}{T^2}\right)\right\}\right]\right]\right\} = 0 \quad (13)$$

Using the software Mathematica 5.1, we can calculate the optimal value of $T = T_1$, which maximizes $P_1(T)$ can be obtained by the equation (13). $\frac{dP_1(T)}{dT} = 0$

Provided $\frac{d^2P_1(T)}{dT^2} < 0$ (14)

Where $Q = \left[a\left(T + \frac{\alpha T^{(\beta+1)}}{(\beta+1)}\right) + b\left(\frac{T^2}{2} + \frac{\alpha T^{(\beta+2)}}{(\beta+2)}\right)\right]$, $Q' = \frac{dQ}{dT} = \left[a\left(1 - \alpha T^{\beta}\right) + b\left(T + \alpha T^{(\beta+1)}\right)\right]$

Case II^nd: - N≥T

In this case, the retailer can make interest on the total revenue, which is generated from trading the product during the time interval 0 to T. The interest paid by the retailer during the time interval o to T is zero. Interest earned per cycle is defined as below.

(i)Interest earned on the revenue by the retailer.

$$IE_2 = kI_e\left[\int_0^T D(t).\,tdt + DT.(N-T)\right]$$

$$IE_2 = kI_e\left[\frac{aT^2}{2} + \frac{bT^3}{3} + DT.(N-T)\right] \quad (15)$$

The total profit per unit time $P_2(T)$ will be

$$P_2(T) = \frac{1}{T}\{\text{Sales revenue} + \text{Interest earned} - \text{Ordering cost} - \text{Purchasing cost} - \text{Deterioration cost} - \text{Holding cost}\} \quad (16)$$

$$P_2(T) = \frac{1}{T}\left\{\left[k.D.T + kI_e\left[\frac{aT^2}{2} + \frac{bT^3}{3} + DT.(N-T)\right]\right] - \left[A_0 + CQ + C_2\left\{a\frac{\alpha T^{(\beta+1)}}{(\beta+1)} + b\frac{\alpha T^{(\beta+2)}}{(\beta+2)}\right\} + h\left\{Q\left[T - \frac{\alpha T^{(\beta+1)}}{(\beta+1)}\right] - a\left[\frac{T^2}{2} - \frac{\alpha T^{(\beta+2)}}{(\beta+2)}\left(\frac{\beta}{\beta+1}\right) - \frac{\alpha^2 T^{2(\beta+1)}}{2(\beta+1)^2}\right] + b\left[\frac{T^3}{6} - \frac{\alpha T^{(\beta+2)}}{2(\beta+2)} + \frac{\alpha T^{(\beta+3)}}{(\beta+3)} - \frac{\alpha^2 T^{2\beta+3}}{(\beta+2)(2\beta+3)}\right]\right\}\right]\right\} \quad (17)$$

The optimal value of T which maximizes the total profit $P_2(T)$ can be obtained by solving the equation $\frac{dP_2(T)}{dT} = 0$

$$\frac{dP_2(T)}{dT} = \Big\{\Big[\ kI_e\left(\frac{a}{2} - \frac{2bT}{3} - D\right)\Big] - \Big[-\frac{A_0}{T^2} + \frac{C}{T}Q' - \frac{C}{T^2}Q + C_2\left\{a\frac{\alpha\beta T^{(\beta-1)}}{(\beta+1)} + b\frac{\alpha(\beta+1)T^{\beta}}{(\beta+2)}\right\} + h\Big\{Q'\left[1 - \frac{\alpha T^{\beta}}{(\beta+1)}\right] - Q\left[\frac{\alpha\beta T^{(\beta-1)}}{(\beta+1)}\right] - a\left[\frac{1}{2} - \frac{\alpha\beta T^{\beta}}{(\beta+2)} - \frac{\alpha^2(2\beta+1)T^{2\beta}}{2(\beta+1)^2}\right] + b\left[\frac{T}{3} - \frac{\alpha(\beta+1)T^{\beta}}{2(\beta+2)} + \frac{\alpha(\beta+2)T^{((\beta+1))}}{(\beta+3)} - \frac{\alpha^2(2\beta+2)T^{(2\beta+1)}}{(\beta+2)(2\beta+3)}\right]\Big\}\Big]\Big\} = 0 \quad (18)$$

Using the software Mathematica 5.1, we can calculate the optimal value of $T = T_2$, which maximizes $P_2(T)$ can be obtained by the equation (13). $\frac{dP_2(T)}{dT} = 0$

Provided $\frac{d^2P_2(T)}{dT^2} < 0$ (19)

Where $Q = \left[a\left(T + \frac{\alpha T^{(\beta+1)}}{(\beta+1)}\right) + b\left(\frac{T^2}{2} + \frac{\alpha T^{(\beta+2)}}{(\beta+2)}\right)\right]$, $Q' = \frac{dQ}{dT} = \left[a\left(1 - \alpha T^{\beta}\right) + b\left(T + \alpha T^{(\beta+1)}\right)\right]$

Numerical Example

Example:- 1. Let us consider D = 24000 units/year, A_0 = 50/cycle, h = 2.5, C = 12, C_2 =.03, C_3 =2.1 □ = 25, θ (t) =.02, N = .015 year, Ie = 5/year, Ip = 7/year

Case I[st]:- N<T
Based on above input data and Using the software mathematica-5.1, we calculate the optimal value of T* and Q* simultaneously by equation no. (13) and equation no. (3).
T* = 0.0154 year, Q* = 218 units approximate.

Case II[nd]: - N≥T
Based on above input data and Using the software mathematica-5.1, we calculate the optimal value of T* and Q* simultaneously by equation no. (18) and equation no. (3).
T* = 0.0102 year, Q* = 238 units approximate.

Conclusion

This paper presented a profit maximising inventory model with Weibull deteriorating item. When the credit period increase the retailer wants more order frequently to take the maximum gain on the total revenue. In Case Ist (N<T), the retailer can make interest on the revenue produced from the sales the product up to N. Although he has to resolve the account at N, he has to concord the money to get his rest stokes, for this he took the money at some certain rate of interest, the retailer Pay for the period N to T. But in the Case IInd (N≥T) the retailer can make interest on the total revenue, which is generated from trading the product during the time interval 0 to T. The interest paid by the retailer during the time interval o to T is zero. For the more study this paper can be applied in several ways for example we may add exponential deterioration rate, stock dependent demand rate and price dependent demand rate.

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