Research Journal of Mathematical and Statistical Sciences ________________________________ISSN 2320–6047
Vol. 1(2), 19-21, March (2013) *Res. J. Mathematical & Statistical Sci.*

Review Paper

ABS Methods to Solve Optimization Problems: A Review

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Available online at: www.isca.in
Received 17th January 2013, revised 27th February 2013, accepted 4th March 2013

Abstract

The ABS methods have been studied a lot in last few years. The ABS methods have been used broadly for solving linear and nonlinear system of equations comprising large number of constraints and variables. This work gives a review of the work done by earlier researchers to apply ABS algorithm for different kind of optimization problems i.e.; for solving determined or undetermined linear systems, linear least squares optimization problem and integer equation and linear programming etc.
AMS Subject Classification: 65F30

Keywords: ABS algorithm, Non linear equations, Linear inequalities and Linear programming problem.

Introduction

ABS algorithms were introduced by Abaffy, Broyden and Spedicato to solve linear equations first in the form of the basic ABS class, later generalized as the scaled ABS class. These algorithms then applied to linear least squares, nonlinear equations and optimization problems. Earlier researchers developed a class of methods for solving system of nonlinear equations. But the convergence properties of these methods when applied to linear systems were not well known for the years. After that Gray proved the convergence of Broyden's methods for linear systems in at most 2n iterations (where n being the number of variables).

ABS methods have allowed to solve open problems in the literature. ABS methods provide a unification of the field of finitely terminating methods for linear systems and for feasible direction methods for linearly constrained optimization; due to the several alternative formulations of their algebra. For linear Diophantine equations, ABS methods have led to a new solution method that also provides the first generalization of the classical existence theorem of Euclides, Diophantus and Euler from a single n-dimensional equation to a general m-equations system in n variables.

ABS methods provide some new methods that are better than classical ones under several respects. The implicit LX method requires the same number of multiplications as Gaussian elimination (which is optimal under very general conditions) but requires less memory and does not need pivoting in general; moreover its application to the simplex method has not only a lower memory requirement, but is cheaper in multiplications up to one order. There are ABS methods for nonlinear problems that are better than Newton method in terms of convergence speed or in terms of required information.

Literature Survey

Elena[1] described numerical experiments with Gauss-ABS algorithms on tridiagonal systems of linear equations. The author presented a Gauss-ABS algorithm to solve tridiagonal system of linear equation on vector and parallel computers. The algorithm is based on a partitioning method which uses the properties of the projection matrix of the ABS implicit LU algorithm. Zun – Quan Xia[2] reformulated the simplex method for variables with upper bounds, the dual simplex method and the dual – primal simplex method in terms of the implicit LX algorithm of the ABS class.

Feng Enmin et. al.[3] discussed the application of the ABS algorithm to simplex method , the dual simplex method , the complementary problem. They considered the ABS formulation of the stopping criterion, the search direction, the minimal rule to determine the vectors entering and leaving the basis matrix and updating of the Abaffian matrix after a basis vector change. Gabriela Kalnova[4] dealt with a pivoting modification of the algorithm in the class of ABS methods.A new algorithm known as hybrid algorithm for the solution of the linear system with the Hankel matrix had been introduced.

Zunquan Xia[5] considered the classical primal affine scaled algorithm via the basic ABS algorithm. ABS methods are direct iteration methods for solving a linear system where the i-th iterate satisfies the first i equations, so that a system of m equations is solved in m steps. Majid Adib et.al.[6] developed a class of ABS type methods whose i-th iterate solves the first 2i equations,so that termination is achieved in at most m/2 steps. Hamid Esmaeili et.al.[7] gave a representation of the solutions of a system of m linear integer inequalities in n variables, $m \leq n$, with full rank coefficient marix. They applied this result to solve linear integer programming problems with $m \leq n$ inequalities.

Elena Bodon et.al.[8] presented numerical experiments with conjugate ABS type algorithms for solving linear systems. These algorithms form a subclass of the scaled ABS algorithms. Hamid Esmaeili et. al.[9] described an approach based upon the ABS methods, to solve a general system of linear Diophantine equations. This approach determines if the system has a solution, generalizing the classical fundamental theorem of the single linear Diophantine equation. Elena Bodon[10] gave the listing of two codes implementing the ABS algorithms for solving linear determined and undetermined banded system of n variables and m equations.

Emilio Spedicato et.al.[11] described some numerical results on the Huang, Modified Huang, implicit LU and implicit QR algorithm in the ABS class with iterative refinement. These algorithms are tested on over determined linear system of equations to find the minimal norm solution in the least squares sense. Elena Bodon[12] gave the listing of the codes implementing the ABS algorithms for solving linear least square problems. Emilio Spedicato et.al.[13] presented a method, called the IABS-MPVT algorithm, for solving a system of linear equations and linear inequalities. It is characterized by solving the system of linear equations first via the ABS algorithms and then solving the unconstrained minimization problem obtained by substituting the ABS general form of solutions into the system of linear inequalities and using a parallel algorithm for a minimization stage.

Emilio Spedicato et.al.[14] presented some methods for solving nonlinear integer optimization via the (basic) integet ABS algorithm. They described the application of this algorithm (BI-ABS algorithm) to solve a special class of nonlinear integer optimization, namely quadratic programming and nonlinear programming with integer linear equality constraints.

H. Esmaeili et.al.[15] had used the ABS algorithms for linear real systems to solve full rank linear inequalities and Linear programming problems where the number of inequalities is less than or equal to the number of variables. They obtained the conditions of both optimality and unboundedness in the context of ABS algorithms. Zun-Quan Xia et.al.[16] presented a special ABS algorithm "ABS Algorithms For Diophantine Linear Equations and Integer LP Problems" for solving such equations which is effective in computation and storage,not requiring the computation of the greatest common divisor. They discussed the ILP problem with upper and lower bounds on the variables using this result.

Kevyan Amini[17] presented that how to update the general solution of a linear system obtained by use of the scaled ABS method when the matrix coefficient is subjected to a rank one change. Hamid Esmaeili[18] gave an ABS representation theorem for polyhedral sets and its applications. He attempted to find explicitly all solutions of a system of m linear inequalities in n variables, $m \leq n$, with full rank matrix. With the help of this theorem one can easily be seen that the problem of finding the least squares points in the polyhedral set is equivalent to a nonnegative least squares problem.

Qiang Guo et.al.[18] developed a method, called the Multi-Stage ABS algorithm, for solving the over-determined system of linear inequalities and the system combined with the over-determined linear inequalities and the equations. The multi-stage ABS algorithm is characterized by translating the system of inequalities to an equation system with slack variables. By the Multi-Stage ABS method one can determine whether the solution exists or not.

A.Galantai et.al.[19] described ABS methods for nonlinear systems of algebraic equations. They gave a survey of the theory and practice of nonlinear ABS methods including various types of generalizations and computer testing. Emilio Spedicato et.al.[20] considered the application of the ABS procedure to the linear system arising in the primal-dual interior point method where Newton method is used to compute the path to the solution. When approaching the solution the linear system becomes more and more ill conditioned.

Cheng-Zhi Gao et.al.[21] developed ABS algorithms for solving a class of linear Diophantine inequalities and integer linear programming problems where the number of inequalities is less than or equal to the number of variables. Spedicato Emilio et.al.[22] discussed ABS methods for continuous and integer linear equations and optimization. Mostafa Khorramizadeh[23] obtained solution of Newton systems of Primal-Dual infeasible interior point methods by ABS algorithm. He proved that when activity matrix is square, the parameters of the ABS algorithm for solving system of linear equations can always be chosen so that the search vectors do not change in every iteration of the infeasible interior point algorithm.

Conclusion

Since ABS methods have their own limitations that's why these are not well known. We are trying our best to apply these methods to different types of programming problems i.e. Fractional Programming Problem, Quadratic Programming Problem, Separable Programming problem and linear plus linear fractional programming problems etc. All these types of programming problems shall be discussed in subsequent research papers one by one.

Acknowledgement

One of the author Dr Adarsh Mangal expresses heartiest thanks to his Academic Mentor Dr Sanjay Jain for valuable suggestions while working.

References

1. Bodon Elena, Numerical experiments with Gauss-ABS algorithms on tridiagonal systems of linear equations, QDMSIA, 31 **(1992)**

2. Xia Zunquan, ABS reformulation of some versions of the simplex method for linear programming Q MSIA, 10 **(1995)**

3. Feng E., Wang X. and Wang X.L, On the application of the ABS algorithm to linear programming and linear complementarity, OMS, **8,** 133-142 **(1997)**

4. Gabriela Kalnova, Pivoting algorithm in class of ABS methods Archivum Mathematicum (BRNO), Tomus **32,** 167-180 **(1996)**

5. Xia Zunquan, ABD derivation of affine-type projection for interior point methods via the ABS algorithm, QDMSIA 35 **(1996)**

6. Adib Majid, Mahdavi-Amiri Nezam and Spedicato Emilio, ABS type methods for solving m linear equations in m/2 steps, QDMSIA, 8 **(2000)**

7. Esmaeili Hamid, Mahdavi-Amiri and Spedicato Emilio, ABS solution of a class of linear integer inequalities and integer LP problems QDMSIA, 3 **(2001)**

8. Bodon Elena, Luskan Ladislav and Spedicato Emilio, Computational experiments with conjugate type ABS algorithms QDMSIA, 26 **(2001)**

9. Esmaeili Hamid, Mahdavi-Amiri Nezam and Spedicato Emilio, A class of ABS algorithm for Diophantine linear systems, Numer. Math., **90,** 101-115 **(2001)**

10. Bodon Elena, ABS codes for banded linear determined and undetermined systems QDMSIA, 9 **(2001)**

11. Spedicato Emilio and Bodon Elena, Numerical experiments of the ABS algorithm with iterative refinement on linear least squares problem QDMSIA, 7 **(2002)**

12. Bodon Elena, ABS codes for linear least squares, QDMSIA, 3 **(2002)**

13. Spedicato Emilio, Li-Ping Pang, Xia Zun- Quan and Wang Wei ,A method for solving linear inequality system QDMSIA, 19 **(2004)**

14. Spedicato Emilio, Xia Zun-Quan and Zhang Li- Wei , Application of the integer ABS algorithms to integer nonlinear optimization, QDMSIA, 18 **(2004)**

15. Esmaeili Hamid, Mahdavi-Amiri ,Nezam and Spedicato Emilio ,Explicit ABS solution of a class of linear inequality system and LP problems, Bulletin of the Iranian Mathematical Society, **30(2),** 21-38 **(2004)**

16. Xia Zun-Quan and Zou Mei-Feng , ABS algorithm for Diophantine linear equations and integer LP problems, QDMSIA, 3 **(2004)**

17. Amini Keyvan, Solving rank one revised linear system by the scaled ABS method, ANZIAM J., **46,** 225-236 **(2004)**

18. Guo Qiang and Liu Jian-Guo, An two phase ABS method for solving over determined system of linear inequalities, J. Appl. Math. & Computing, **21(1-2),** 259-267 **(2006)**

19. Galantai A. and Spedicato E., ABS methods for nonlinear systems of algebraic equations **(2007)**

20. Emilio Spedicato and Marco Bonomi, Application of ABS methods to the primal-dual interior point method for linear programming, QDMSIA, 6 **(2007)**

21. Gao Cheng-Zhi and Dong Yu-Lin, ABS algorithm for solving a class of linear Diophantine inequalities and integer LP problems, J. Appl. Math. & Informatics, **26(1-2),** 349-353 **(2008)**

22. Spedicato Emilio, Bodon Elena, Xia Zunquan and Mahdavi-Amiri Nezam, ABS methods for continuous and integer linear equations and optimization, CEJOR, **18,** 73-95 **(2010)**

23. Khorramizadeh Mostafa, On solving newton systems of primal-dual infeasible interior point methods using ABS methods, Applied Mathematical Sciences, **6(106),** 5263-5272 **(2012)**