



# The Relationship between Least Square and Linear Programming

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Available online at: [www.isca.in](http://www.isca.in)

Received 22<sup>nd</sup> November 2012, revised 19<sup>th</sup> January 2013, accepted 20<sup>th</sup> February 2013

## Abstract

The predication is an important tool for planning where the aim of any statistician is to predicate the values of dependent variable which minimize the errors (the different between actual and predicated value). The least square method is classical method which is used to achieve this purpose. The predication by using least square method depends on minimizing the sum square of error. This paper introduces the restrictions of least square method, while the predication by using linear programming method depends on the assumption of minimizing the sum of absolute errors.

**Keywords:** Least square method, linear programming method, absolute errors.

## Introduction

The purpose of least square and linear programming methods are minimize the errors for any linear mode<sup>1</sup>. so to compare these methods we introduce the constraints of least square method<sup>2</sup>, and we put assumption of minimizing of the sum of absolute errors for linear programming method.

## Methodology

**Least Square Method:** The estimation of parameter model<sup>3</sup>.  $B_j, j = 1, 2, \dots, k$  of least square method is defined as follows<sup>4</sup>:-

$$\text{Min } z = e_1^2 + e_2^2 + \dots + e_n^2$$

s.t:

$$B_0 + B_1x_{11} + \dots + B_kx_{ik} + e_i = y_i, i=1,2,\dots,n \quad B_j, e_i \text{ unrestricted in sign}$$

**Linear Programming Method:** The predication formula by using this method depends on the assumption:

$$\text{Min } z = |e_1| + |e_2| + \dots + |e_n|$$

So, to estimate the parameter model  $B_j$  suppose<sup>5</sup>  $e_i = e_i^+ - e_i^-$

Because  $e_i$  unrestricted in sign also<sup>6</sup>. the restriction of linear programming method is nonnegative variables ( $B_j$ )<sup>7</sup>.

So, the model becomes as follows:-

$$\text{Min } z = e_1^+ + e_1^- + \dots + e_n^+ + e_n^-$$

s.t:

$$B_0 + B_1x_{1i} + \dots + B_kx_{ik} + e_i^+ - e_i^- = y_i \quad B_j \text{ unrestricted in sign } e_i^+, e_i^- \geq 0$$

**Example:** Consider, the linear model<sup>8</sup>  $y_i = B_0 + B_1x_i + e_i$

Where:  $y_i$  represent the monthly average income per capita.  $x_i$  represent the monthly average expenditure per capita.

So, to estimate the parameter model  $B_j$  by using least square method is defined as follows<sup>9</sup>  $\text{Min } z = e_1^2 + \dots + e_{16}^2$

s.t:

$$B_0 + 10.2B_1 + e_1 = 6.10$$

$$B_0 + 10.32B_1 + e_2 = 5.48$$

$$B_0, B_1, e_1, \dots, e_{16} \text{ unrestricted in sign}$$

**Table-1**  
 Show the values of parameter, errors and objective functions for two methods

Variables	Least square	Linear programming (simplex)*
$b_0^+$	3.568073	3.0018506
$b_0^-$	0	0
$b_1^+$	0.280519	0.31850788
$b_1^-$	0	0
$e_1^+$	-0.3293668	0
$e_1^-$	0	0.15063103
$e_2^+$	-0.98302908	0
$e_2^-$	0	0.80885172
$e_3^+$	-0.4235225	0
$e_3^-$	0	0.25618345
$e_4^+$	-0.2165086	0
$e_4^-$	0	0.12134842
$e_5^+$	-0.04045596	0
$e_5^-$	0	0
$e_6^+$	-0.13255939	0
$e_6^-$	0	0.12895234
$e_7^+$	0.13289566	0.09661418
$e_7^-$	0	0
$e_8^+$	0.29562419	0.16627026
$e_8^-$	0	0
$e_9^+$	0.40074107	0.26530853
$e_9^-$	0	0
$e_{10}^+$	0.3992064	0.18096179
$e_{10}^-$	0	0
$e_{11}^+$	0.3912076	0.08520817
$e_{11}^-$	0	0
$e_{12}^+$	0.50671882	0.07687327
$e_{12}^-$	0	0
$e_{13}^+$	0.35105728	0
$e_{13}^-$	0	0.14184997
$e_{14}^+$	0.73965561	0.17342989
$e_{14}^-$	0	0
$e_{15}^+$	0.75389175	0
$e_{15}^-$	0	0
$e_{16}^+$	-1.5151926	0
$e_{16}^-$	0	2.7876320
$R_1$	-	0
$R_2$	-	0
$R_3$	-	0
$R_4$	-	0
$R_5$	-	0
$R_6$	-	0
$R_7$	-	0
$R_8$	-	0
$R_9$	-	0
$R_{10}$	-	0
$R_{11}$	-	0
$R_{12}$	-	0
$R_{13}$	-	0
$R_{14}$	-	0
$R_{15}$	-	0
$R_{16}$	-	0
Z	5.689599	5.440116

\*: treatioNo = 26

And the estimation parameter model Bj by using linear programming method (simplex)<sup>10</sup>, is defined as follows:

$$\text{Min } z = e_1^+ + e_1^- + \dots + e_{16}^+ + e_{16}^-$$

s.t:

$$B_0^+ - B_0^- + 10.20(B_1^+ - B_1^-) + e_1^+ - e_1^- + R_1 = 6.10$$

$$B_0^+ - B_0^- + 10.32(B_1^+ - B_1^-) + e_2^+ - e_2^- + R_2 = 5.48$$

$$B_0^+ - B_0^- + 48.40(B_1^+ - B_1^-) + e_{16}^+ - e_{16}^- + R_{16} = 15.63$$

$$B_0^+, B_0^-, B_1^+, B_1^-, e_1^+, e_1^-, \dots, e_{16}^+, e_{16}^-, R_1, \dots, R_{16} \geq 0$$

## Result and Discussion

The table (1) shows the values of parameters, errors and the objective function values for two methods.

## Conclusion

The researcher reach the following conclusions: i. The estimation of parameter of both methods is approximately equal. ii. The value of objective function of both methods is approximately equal, as well. iii. There is strong relationship between both methods. iv. The aim of both methods is to minimize the errors.

**Table-2**  
 Shows the monthly average income per capita (y) and the monthly average expenditure per capita(x)

Y	X
6.10	10.20
5.48	10.32
6.09	10.50
6.83	12.40
7.41	13.84
7.59	14.81
8.15	15.86
9	18.31
9.15	18.47
9.76	20.65
10.40	22.96
11.43	26.22
11.74	27.88
12.67	29.81
14.07	34.75
15.63	48.40

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