Research Journal of Mathematical and Statistical Sciences ISSN 2320–6047
Vol. **1(2),** 7-15, March **(2013)** *Res. J. Mathematical & Statistical Sci.*

CPM Analysis of Rolai-Rinjlai Road Construction

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Available online at: www.isca.in
Received 04th November 2012, revised 28th December 2012, accepted 23rd January 2013

Abstract

The present work is based on empirical data of a part of the Rolai-Rinjlai road construction project, in which raw material is available at different quarries providing different options to the contractor. Considering the project as a network, we used CPM technique in an attempt to obtain the critical path of the network and suggested the best approach for acquiring material and construction of road under the stated constraints. We used crashing to further reduce the project completion time. The solution suggested by us provide a much shorter completion time as compared to the actual time taken by the project. To this end, we use crashing not only to minimise time economically, but also to identify viable means that were earlier ignored by the contractor which resulted in a schedule that required time as well as cost less than that taken by for the initial CPM solution.

Keywords: Network analysis, CPM, crashing, cost slope, road construction.

Introduction

There is a conscious effort at improving the roads in most cities of India with an aim to provide better travel and transport facilities. Such efforts may also attract more investment in the states. However, in public interest, it is imperative that the disruption of traffic caused by the construction process should be minimised. This would entail the contractor to subdivide the project into smaller sub-projects and minimise time of completion of each sub-project, that too at a relatively low cost. This requires study of the impact of time and cost of various stages of the project and the different constraints involved in the construction process as key variables. As a prototype of such an endeavour, the present study was initiated to explore the costs and chart a schedule that will result in minimum time required to complete the project.

Consequently, empirical data was taken from a part of a completed road construction project, namely, the Rolai -Borsi section of the Rolai- Rinjlai road construction in the industrial town Pithampur near Indore (State M.P., India), which actually took 97 days to complete, partly, because of disruption due to rain for some days. This study was therefore initiated to explore the cost and minimum expected time that will be required to complete the project. We note that had the project used the scheduling suggested by us, it may have been completed before the rainy season began. The data on the cost and duration of activities involved were obtained from the contractor of the road construction company. Critical path method (CPM) was used for the analysis, which is based on the computation of latest starting times of activities using the knowledge of the earliest ending time of the project. The analysis which involved crashing of both time and cost, paved the way for the determination of critical path and revealed that using CPM, the analyzed construction project may be completed in 45 days instead of the actual duration of 97 days, i.e. the expected completion time would be reduced by 42 days. Using the concept of 'crashing', it is shown that the duration can be further reduced to 27 days. The additional cost associated with the reduction in time is Rs.56755, which increases the initial expected project completion cost from Rs.54, 85,240.00 to Rs.5541995.00. Therefore, strict adherence to the schedule and taking resort to crashing will trim down the time required to complete a specific part of project. Such improvements, when incorporated in the sub-projects will make a significant difference in the completion time of the entire project.

The critical path method(CPM) is one of the commonly used network techniques developed to facilitate planning, scheduling and controlling of projects in an integrated manner with the aim to complete them within the constraints of given time. This provides a managerial device which acts as a tool to cater to a variety of needs such as system design, planning and control.CPM technique has been used by Yakhchali[5] and Sathish and Ganesan[6]. Haga and O'Keefe[1] used PERT(Project evaluation and Review Technique) which is another network technique that takes into consideration the randomness involved in the time to completion of a task, and is at times considered to be better than CPM, However, Stockhausen[3] made an attempt to combine the PERT/CPM and state after investigations, that there are no differences. This significant research will assist engineers and managers in making more realistic project completion and cost projections. Taking a cue from their conclusion, in this study, we have undertaken to use only CPM.

Reducing the project duration can be achieved by assigning more skilled labour into the project activities often in the form of overtime and by assigning more resources. This can be achieved through deployment of more resources in terms of monetary value to the activities to be crashed using a procedure called project crashing and time cost analysis. Adebowale and Oluboyede[4] used crashing while considering the challenges in scheduling of project, where the project manager is frequently faced with the problem of having to reduce the scheduled completion time of a project to meet the deadlines. Mario Vanhoucke[2] proposed an exact search procedure for scheduling vertical (or discrete) repetitive projects with work continuity constraint, which involves a trade-off between total project duration and the resource idle time. The process of expediting project schedule by compressing the total project duration is also helpful when managers want to avoid incoming bad weather - which is one of the aspects that caused delay in the actual road construction project - data obtained from which is used in the paper, and we also endeavour to address this aspect.

Methodology

Project management consists of planning, designing, and implementing a set of activities in order to accomplish a particular goal or task. For many years, two of the most popular approaches to project management have been the Critical Path Method (CPM) and the Project Evaluation and Review Technique (PERT). In CPM network, the whole project consists of a number of clearly recognisable jobs and operations called activities. Activities are usually operations which takes time to carry out, and on which resources are expended. Nodes between activities are termed as events.

It is often necessary or desirable to shorten the duration of a project. Crashing refers to reducing the duration of an activity by devoting additional resources, typically at a higher cost. In crashing, the activity on the critical path with the minimum cost per unit time (as obtained from the time/cost slope) is selected to crash, the activity time is reduced, and the network is reanalyzed. The process is continued until a schedule is arrived at, which will enable the project completion within the desired time frame, the cost of further crashing outweighs the benefit, or all activities have been crashed to their shortest possible duration.

In this paper, we use the standard CPM to make a comparative study of the minimum time that has been arrived at by the actual project and the schedule and selection of options suggested by us. Next we crash the costs using the existing options as well as new options not earlier considered by the practitioner.

Project Outline: The contractor company started the work in Rolai-Borya-Borsi sub-project whose length is 5.5 Kms. The material required in the construction of the road up to Grade II (GII) can be described with the respective utility of the material as follows: i. Murram is used for earthwork, ii. GSB (Granular sub base) work material is a mixture of stones up to size of 80-100mm. iii. G I (Grade I) work material is a mixture of stones up to size of 65-80mm. iv. G II (Grade II) work material is a mixture of stones up to size of 40-50mm. The natural constraints of a road construction process as informed by the contractor is that layering must be done strictly in the following order: i. Murram, ii. GSB, iii. GI iv. GII.

This material was available at three nearby quarries Kalasura (quarry A), Rolai (quarry B) and Borya (quarry C) (Figure:1).Material available in quarry 'A' was murram and GSB, the distance of this quarry from the initial node is 5.6km and that from the end node is 11.1km. In quarry B, the only material available is murram and the distances of the quarry from the initial and end nodes are 1.4Kms and 6.9Kms respectively. All the material required for the construction (Murram, GSB, GI, GII) is available in the quarry C, the distance of which from initial node is 4.5kms and that from the end node is 4.0kms. A rough diagram of the road also showing positions of the quarries at which material is available is given in figure1.

Options currently available: In order to apply network analysis in this project we have considered two options: i. Option I: Murram is taken for the first three Kms from quarry B and remaining Kms from Quarry C. All other materials (GSB, GI, and GII) are taken from quarry C., ii. Option II: Murram is taken for the first three Kms from quarry B and remaining from Quarry C, GSB has to be taken in first three Kms from quarry A and for the remaining length of the road from quarry C, and GI and GII are taken for the entire construction from quarry C.

The actual road construction work has been done using Option I. By considering Option II as per our assumption, if GSB in the first three Kms is taken from Quarry A we intuitively feel that are there is a possibility of reducing the duration of the project as by implementing the project by Option-I. After incorporating these options, network can be analysed by CPM to determine earliest start times, latest start times, the critical path and to investigate time–cost trade-offs.

Our aim is to focus among all the jobs in the project, at those where management should concentrate its efforts in order to achieve the minimal latest starting times of all activities in networks with relevant precedence relations and time lag durations. The main purpose of this research is to study the application of project scheduling in routine road construction project. After networking; we analyze the critical and non critical activities of the project.

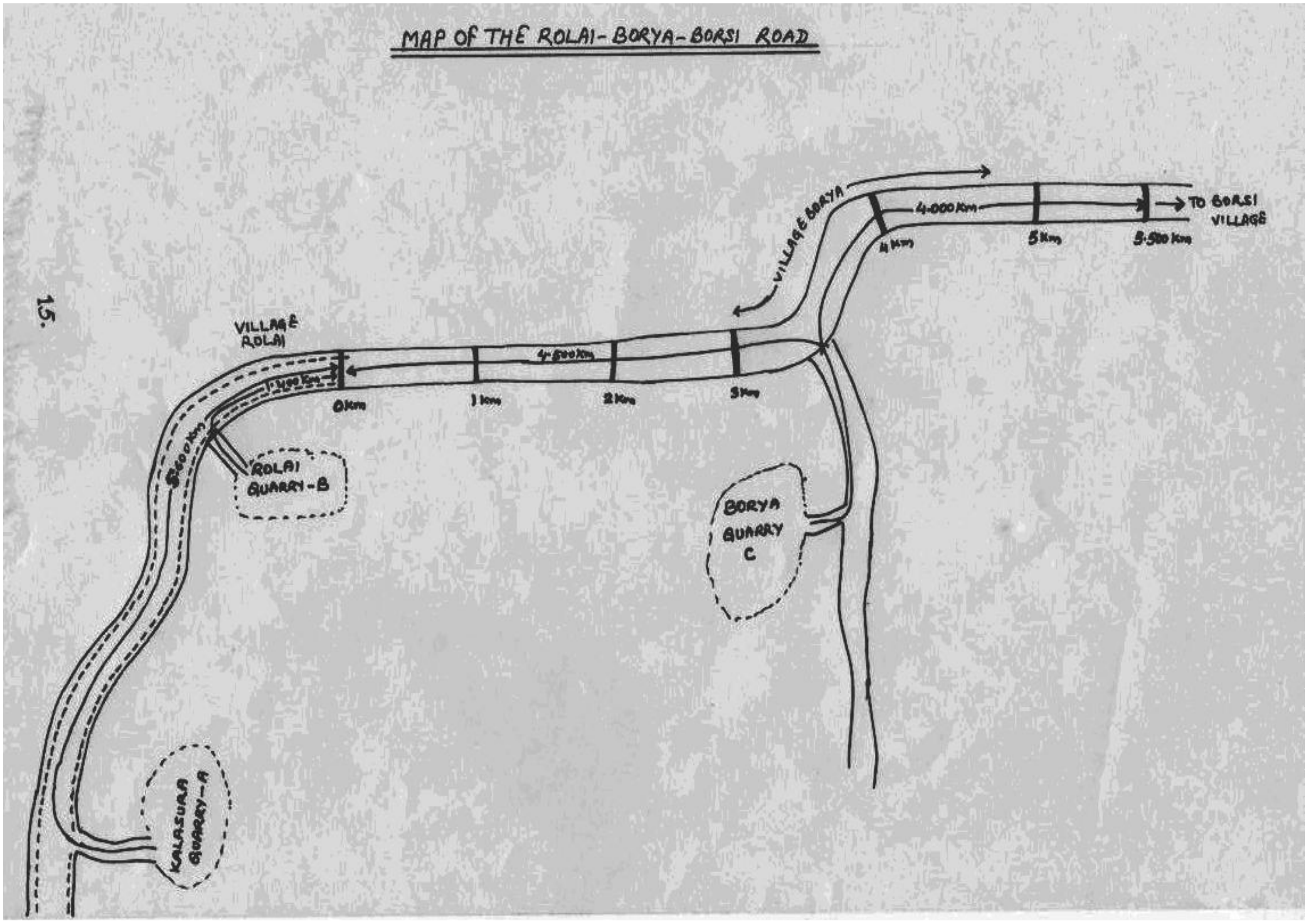

Figure-1
Map of Road diagram

For network analysis we will need to arrange construction activities using various available materials from different quarries. Table 1 shows the list of the activities along with the durations for these. In this table, first column shows the S.No., the second the description of the activity. The entries in the column labelled 'Activity' can be generally explained as follows:Y j(X) means that for the j^{th} Km of the road, raw material Y is to be taken from quarry X ; Y= M,GSB,GI,GII, (where M stands for Murram); j=1,..,6, X=A.B.C. For example, M1(B) means that Murram in the first Km is to be taken from quarry B(Rolai). The third column provides duration for completing each activity of the project.

In order to schedule the activities in the network we require an estimate of how much time each activity should take when it is done in the normal way. These estimates as provided by the site engineer are given in the column 4 of table 1. Given all the information in table: 1, we wish to develop answers to the following questions: i. How can the project be displayed in a graphical way to visualize the flow of the activities? (Network analysis), ii. What is the total time required to complete the project if no delays occur? (Critical path of the network), iii. When can the individual activities start and finish (at the earliest) if no delays occur and start and finish (at the latest) to meet this project completion time? (Forward and Backward pass computations), iv. Which are the critical activities where any delays must be avoided to prevent delaying project completion and for the other activities, how much delay can be tolerated without delaying project completion? (Critical and Non-Critical activities), v. If extra money is spent to expedite the project, what is the least expensive way of attempting to meet the target completion time of the project? (using crashing).

As table-1 indicates three types of information: Activity information (break down the project into its individual activities) precedence relationships (identify the immediate predecessor(s) for each activity) time information (estimate the duration of each activity).

The project network in figure-2 provides the network to visualize all activities of the project, the order in which certain activities must be performed and the *duration* of each activity. Each activity is represented by an *arc and* a node is used to separate an activity from each of its immediate predecessors. Dashes show the dummy activities, which are sometimes needed to represent the precedence constraint highlting the critical path computed by Forward and backward pass computations.

Table-1
Activity List for the road Construction Project (Option no.1)

S. NO.	Activity	Description	Duration
1.	M1(B)	Muram from quarry B in 1st km	4 days
2.	M2(B)	Muram from quarry B in 2nd km	5 days
3.	M3(B)	Muram from quarry B in 3rd km	6days
4.	M4(C)	Muram from quarry C in 4th km	5days
5.	M5(C)	Muram from quarry C in 5th km	7days
6.	M6(C)	Muram from quarry C in 6th km	4days
7.	GSB1(C)	GSB from quarry C in 1st km	6days
8.	GSB2(C)	GSB from quarry C in 2nd km	5days
9.	GSB3(C)	GSB from quarry C in 3rd km	3days
10.	GSB4(C)	GSB from quarry C in 4th km	3days
11.	GSB5(C)	GSB from quarry C in 5th km	5days
12.	GSB6(C)	GSB from quarry C in 6th km	3days
13.	GI 1(C)	Grade I from quarry C in 1st km	6days
14.	GI 2(C)	Grade I from quarry C in 2nd km	5days
15.	GI 3(C)	Grade I from quarry C in 3rd km	3days
16.	GI 4(C)	Grade I from quarry C in 4th km	3days
17.	GI 5(C)	Grade I from quarry C in 5th km	5days
18.	GI 6(C)	Grade I from quarry C in 6th km	3days
19.	GII 1(C)	Grade II from quarry C in1st km	5days
20.	GII 2(C)	Grade II from quarry C in 2d km	5days
21.	GII 3(C)	Grade II from quarry C in 3rd km	3days
22.	GII 4(C)	Grade II from quarry C in 4th km	3days
23.	GII 5(C)	Grade II from quarry C in 5th km	5days
24.	GII 6(C)	Grade II from quarry C in 6th km	3days

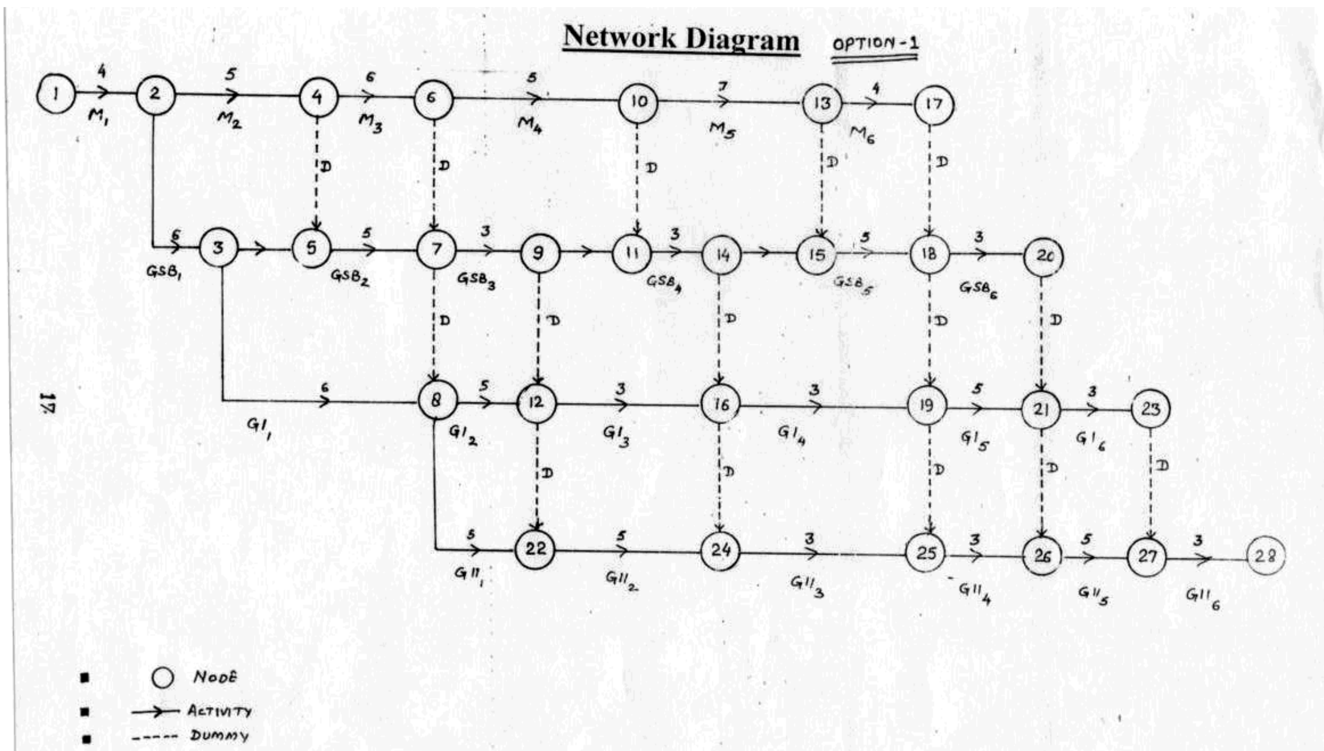

Figure-2
Network Diagram of option-1

Scheduling the Project with CPM: CPM procedure is very efficient scheduling procedure for larger projects. The starting and finishing times of each activity if no delays occur anywhere in the project are called the earliest start time and the earliest finish time of the activity. We use the usual forward and backward pass methods:

Initial event is supposed to occur as time equal to zero, that is E_s^1=T_E^1 =0. We would first start the activity #1.Earliest start time of activities emanating from event 2 is the earliest occurrence time of event 2. Since duration for this activity is 4 days, the E_s^2 = $T_E^2 = E_s^1 + t_E^{1-2}$would be 0 + 4 = 4. Same would be carried out for activities #2, #3 and #4 and E_c^{ij} worked out accordingly. When we next go to activity #5, we see two activities (#3 and #4) are merging. Here we would take maximum of E_c^{ij} of the both as both must be complete before #5 can be started i.e. E_s^5 = T_E^5 =Max.(T_E^3 +0,T_E^4 +0)=10. In the same way, we can complete rest of the diagram. We find that the project would take 45 days to be completed.

The project completion time is 45 days. So its Early Finish or Late Finish must be the same. In other words, call it early or late, it must be finished by 45 days. We start from last activity #28, and take T_L^{28}=45. This activity has duration of 3 days. Calculating backward, we find that $L_s$should be T_L^{27}= T_L^{28}-3=45-3=42. In other words, if activity #28 was started as late as on 42nd day and it takes 3 days, it could still meet the deadline of 45 days. We go back further and trace tails of arrows touching #28 activities. We can continue likewise till we hit a "Burst Event" like #23 and #26 T_L^{21}=Min.(T_L^{23}-3, T_L^{26}-0)=37. In such a situation, we shall take minimum of the $L_s$of the two activities. Critical path is highlighted in Figure: 2 by the darker arrows. It is the activities on this path that we must monitor with special care to keep the project on schedule.

Critical Path of option-1: Activities 1-2,2-4,4-6,6-10,10-13,13-15,15-18,18-19,19-21,21-26,26-27,27-28 have 0 float hence are critical. The path 1-2-4-6-10-13-15-18-19-21-26-27-28 is the critical path with the project duration of 45 days. In other words, delay in some activity would not increase the project duration.

Next we have analysed network for option-2 by CPM method. We have drawn the network in figure-3 and the critical path found by CPM analysis is highlighted by the darker arrows. Activities on this critical path must be monitored with special care to keep the project on schedule.

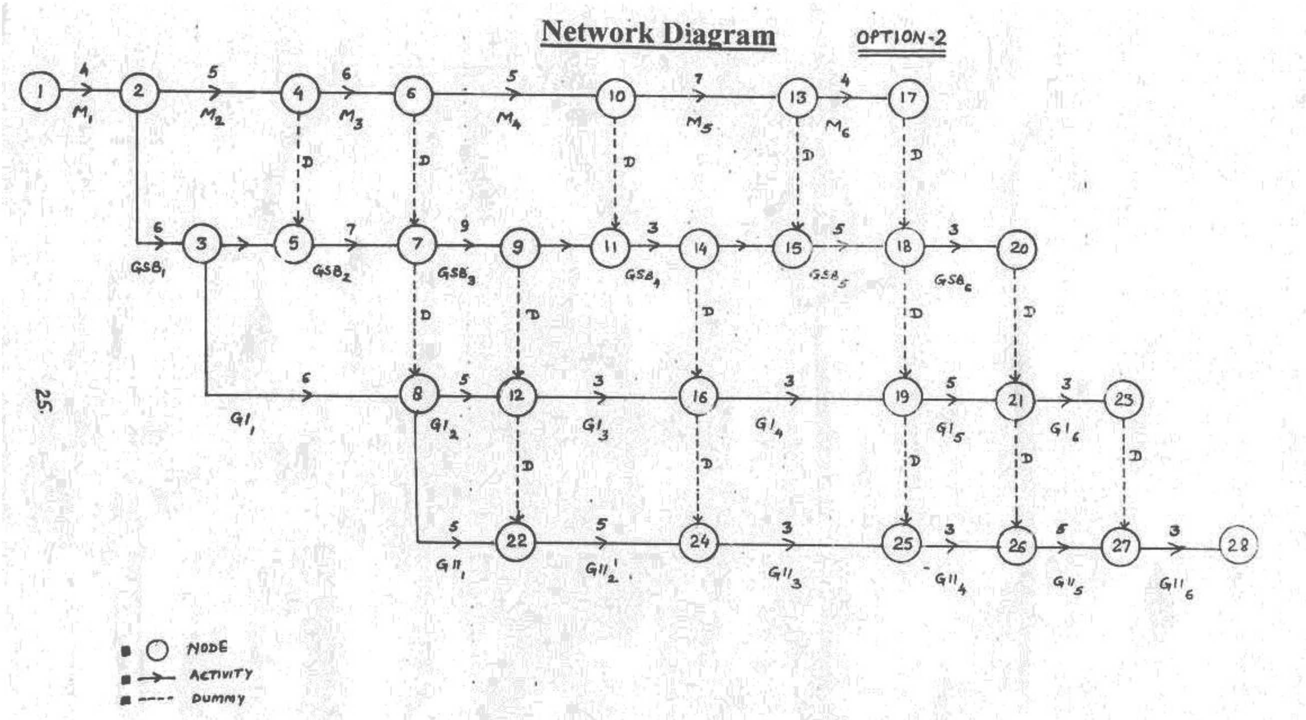

Figure 3
Network Diagram of option-2

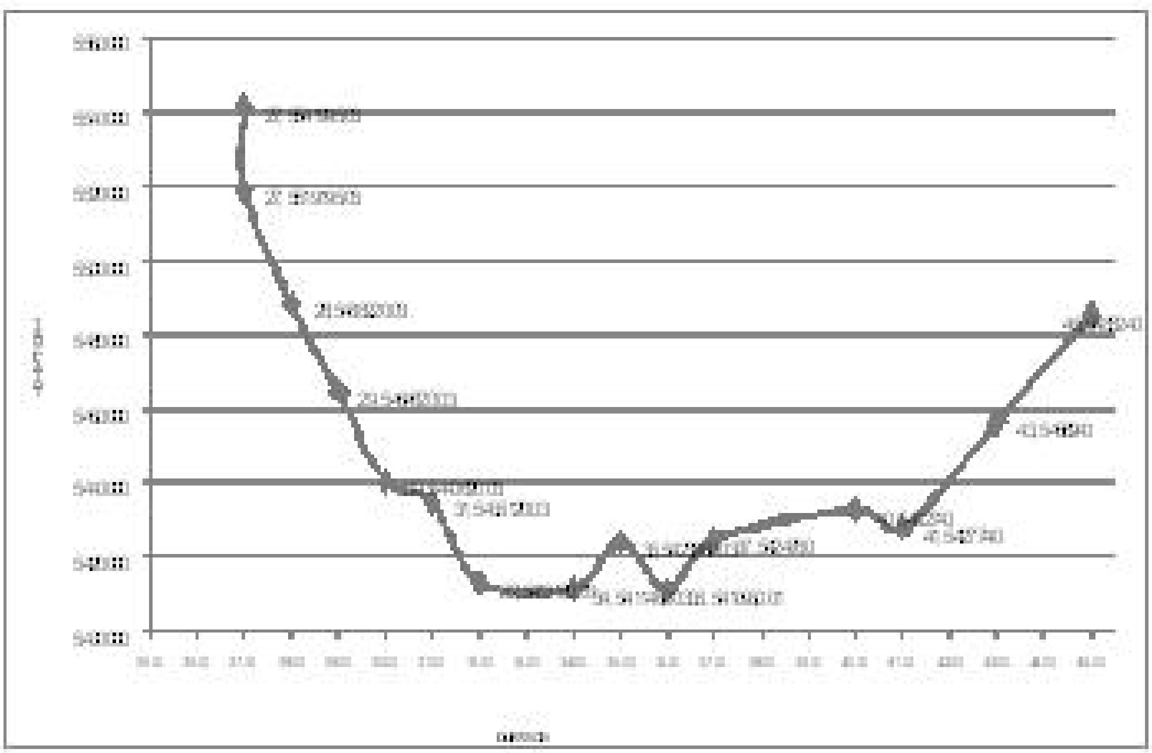

Figure-4
Project duration Vs cost analysis

Critical Path of option-2: Activities 1-2,2-3,3-5,5-7,7-9,9-11,11-14,14-15,15-18,18-19,19-21,21-26,26-27,27-28 have 0 float hence the path is critical. The path 1-2-3-5-7-9-11-14-15-18-19-21-26-27-28 is the critical path with the project duration of 47 days.

Result: From the value of the total float, we can draw the following conclusions: When the float for an activity is positive, it may indicate that the resources for the activity are more than adequate.

In option-I, the CPM network analysis shows(in figure-2) that there is no slack for activities 1-2,2-4,4-6,6-10,10-13,13-15,15-18,18-19,19-21,21-26,26-27,27-28, and hence no delay on the path 1-2-4-6-10-13-15-18-19-21-26-27-28 can be allowed. In the execution of the project the critical path (1-2-4-6-10-13-15-18-19-21-26-27-28) with duration of 45 days needs utmost care. Any delay in any activity on this path will upset the whole project and will delay its completion.

In Option-II by network analysis using CPM method as shows in figure-3, there is no slack for activities 1-2,2-3,3-5,5-7,7-9,9-11,11-14,14-15,15-18,18-19,19-21,21-26,26-27,27-28 and hence no delay on the critical path1-2-3-5-7-9-11-14-15-18-19-21-26-27-28 can be allowed. In the execution of the project, (1-2-3-5-7-9-11-14-15-18-19-21-26-27-28) is the critical path with duration of 47 days, which needs optimum care.

Crashing considerations: As per the discussion of CPM in the preceding section, we recall that there is a considerable variation in fixed activity time and one needs to determine how much time will be actually needed for each activity and hence, for the overall project. The project duration will turn out to be 45 days under option I and 47 days under option II. We consider only the first option as it results in shorter time duration. Incidentally, this was the option used in the actual project.

We would next like to investigate the possibilities of reduction of project duration further down from 45 days. For example, this may be required if the deadline of the project is less than 45 days for the construction company and the company is able to increase the resources of the project. The question arises that if extra money is spent to expedite the project, what is the least expensive way of attempting to meet the target of the completion time (45 days)? CPM provides an excellent procedure for investigating time–cost trade-offs, so we will use this approach, that is the process of *crashing*. These special measures might include using overtime, hiring additional resources, using special time-saving materials, special equipment, and so forth.

We next explain the considerations for addition of resources and find the direct and indirect costs of the project. For direct costs we consider the cost of Earthwork, GSB, GI and GII for each Km and then find the total cost of all the activities. The indirect costs considered are: Salary of the staff, Surveying and Levelling expenses at the site, expenses on food for the staff and workers at the site, expenditure on transportation of staff up to the site, Stationery, Insurance, Entertainment and other miscellaneous expenses, Security expenses at site.We proposed increase in some resources (machineries) for reducing activity time and then computed the corresponding cost slope. For example in a critical path, normal time duration of the first activity M1(B) is 4 days and normal cost is Rs.206230. But after increasing the resources, crash cost will increase to Rs. 238950 due to which we are able to crash the time by 2 days. Thus the cost slope turns o ut to be Rs.16360 per day. Similarly, we can find the cost slopes of each activity in the critical path.

Crashing of Project Network for Option-1: Step 1: After knowing the Earliest time estimates and the Latest time estimates for the various events in the network, the critical path activities can be identified as: 1-2-4-6-10-13-15-18-19-21-26-27-28 with the project duration of 45 days.
Step 2: Find the project cost by the formula
Project cost = (Direct cost + (Indirect cost*project duration))
The project cost is calculated to be: Rs. 5485240

Step 3: Find the minimum cost slope by the formula
Cost slope = (Crash cost - Normal cost)/ (Normal time - Crash time)
For this we will crash the least expensive critical activity in the critical path. In the present project, 19-21 is the least expensive amongst the critical activities, with a crash cost of Rs.1000 per day. This can be crashed by 2 days.

Step 4: Identify the activity with the minimum cost slope and crash that activity by 2 days. Identify the new critical path and find the cost of the project by the formula
Project Cost= ((Project Direct Cost +Crashing cost of crashed activity) + (Indirect Cost*project duration)). The project cost will be as Rs. Rs.5456240 and the new critical path will be: CP = 1-2-4-6-10-13-15-18-20-21-26-27-28 with project duration 43 days.

Step 5: In the new critical path, select the activity with the next minimum cost slope, and crash again by 2 days.

The activity 26-27 has minimum cost slope Rs.1250 and is crashed by its duration, i.e. by 2 days. Similarly in the resulting critical path, select the activity with the next minimum cost slope, and crash it as shown in Table:2, and repeat this step until all the activities along the critical path are crashed up to maximum possible time and find optimal duration of the project.

From the curve in figure-4 (from table: 2) we find that the minimum cost corresponding to the project duration of 36 days is Rs.5410960. An unusual phenomenon that is visible in this graph is that for some part of crashing in time the cost also reduces. This has been enabled by taking resort to some new, relatively inexpensive options that the contractor was not ready to accept earlier, such as using more vehicles to carry the material simultaneously from different quarries.

Table-2
Result of Calculations based on Crashing

	Activity Crashed	Days Saved	Direct Cost	Indirect Cost @ Rs.15500/day	Total Project Cost
45	Nil	0	Rs.4787740.00	Rs.697500	Rs.5485240.00
43	19-21	2	Rs.4789740.00	Rs.666500	Rs.5456240.00
41	26-27	2	Rs.4792240.00	Rs.635500	Rs.5427740.00
40	15-18,25-26	1	Rs.4812740.00	Rs.620000	Rs.5432740.00
37	4-6	3	Rs.4851260.00	Rs.573500	Rs.5424760.00
36	22-24	1	Rs.4852960.01	Rs.558000	Rs.5410960.01
35	6-10,3-8	1	Rs.4881460.03	Rs.542500	Rs.5423960.03
34	8-12	1	Rs.4884460.03	Rs.527000	Rs.5411460.03
32	1-2	2	Rs.4917180.03	Rs.496000	Rs.5413180.03
31	2-4	1	Rs.4954620.03	Rs.480500	Rs.5435120.03
30	16-19,5-7	1	Rs.4975620.03	Rs.465000	Rs.5440620.03
29	10-13	1	Rs.5015020.03	Rs.449500	Rs.5464520.03
28	2-3,24-25	1	Rs.5054220.03	Rs.434000	Rs.5488220.03
27	13-17	1	Rs.5100695.03	Rs.418500	Rs.5519195.03
27	18-20	0	Rs.5123495.03	Rs.418500	Rs.5541995.03

Interpretation: The actual execution of the project had taken 97 days for completion. Using either option, through CPM technique, we gain at least 40 days.

As per the assumption of Option 1, using CPM it will take 45 days to complete the project, while by using Option 2, using CPM it will take 47 days to complete the project.

In the same project, we have applied crashing, and since all the activities of the critical path have been crashed, , no further crashing of the project of the network is possible. It may be seen that cost is minimum when the project duration is 36 days. The result of crashing is as follows:
Normal duration of the project = 45 days, Maximum Crash duration of the project = 27 days, Optimal duration of the project = 36 days, Minimum cost of the project after crashing= Rs.5410960.0.

Conclusion

A major challenge in planning a project is to determine how the resources should be allocated to the activities. CPM assumes that each activity has available - all the resources needed to perform the activity in the normal way (or on a crashed basis). In this study, a small amount of overlapping i.e. simultaneously laying the same layer at different spots has provided the slack/float needed to compensate for the “unexpected” delays that inevitably seem to slip into a schedule.
Using CPM, our road project as per the better of the two options, i.e., using option-1, will take 45 days to complete the project, whereas the actual project took 97 days for completion. Part of the delay was due to heavy rains, which resulted in inaccessibility of the quarries and interrupted the progress of the project.

Next keeping a sharp eye on the bottom line of the project; we have applied the plan for crashing the project in an attempt to reduce the project duration further from 45 days. This resulted in a plan corresponding to the minimum cost of Rs.5410960 which would enable project completion in 36 day. This cost is even less than the total cost of the project using only CPM schedule without crashing. That is, at the above optimum duration (36days) the project can be completed with 20% decrease in time as well as1.35%

decrease in cost, which is quite appreciable. In fact, using crashing, we have also arrived at a schedule with time duration reduced by 40% i.e., to 27 days but for achieving this, 1.03% additional cost has to be incurred. We thus conclude that the schedule proposed by us provides much shorter completion time as compared to the actual time taken by the project and paves the way for use of CPM scheduling for road construction projects to be a lucrative and viable option.

Acknowledgement

The authors would like to convey thanks to the contractor company for providing the data of Rolai-Rinjlai Road project and permission to use it for this study. We are also thankful to the site engineer for providing the relevant information and for their cooperation.

References

1. Haga W.A. and O'Keefe T., Paper presented at the 4th International conference of the Academy of Business and Administrative Sciences Conference Quebec City, Canada, 1 (**2001**).
2. Vanhoucke M., *Faculteit Economy Hooveniersberg 24 B-9000 GENT ,1* (**2004**)
3. Stockhausen M., Proceedings of The National Conference On Undergraduate Research (NCUR) University of Montana, Mi1/17/2013 11:34:04 AMssoula, 180 (**2010**)
4. Adebowale, S. A. and Oluboyede, E. D., Network analysis and building construction: Implications for timing and costing of activities, *Journal of Civil Engineering and Construction Technology,* **2,** *90,* (**2011**)
5. Yakhchali H. S., *Proceedings of the World Congress on Engineering,* **1,** (**2011**)
6. Sathish S. and Ganesan K., *Mathematical Theory and Modeling,* **2**, 53, (**2012**).