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MHD Flow of Viscous Liquid Moving Steadily Under Pressure between Two Flat Plates with Constant Velocity

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Abstract

The aim of the present paper is to study the flow of viscous liquid moving steadily under the pressure between two flat plates with constant velocity under the influence of uniform magnetic field applied perpendicularly to the flow of viscous liquid. The expression for velocity of the viscous liquid is obtained in elegant form.

Keywords: Flow, viscous liquid, flat plates, magnetic field, elegant form.

Introduction

The study of viscous fluid flow through channels of various cross-sections has been investigated by several research workers using various techniques in different circumstances. In classical viscous fluid, we know that the fluid exert viscosity effect when there is a tendency of shear flow of the fluid. Various types of basic problems of diversified nature have been solved in this branch.

The study of flow through porous media is of considerable interest in the field of petroleum engineering concerned with the movement of oil and gas, ground water hydrology, heat transfer in cooling systems and chemical engineering for filtration process etc.

In hydromagnetic flow, we study of the flow of electrically conducting fluid in presence of Maxwell electromagnetic field. The flow of conducting fluid is effectively changed by the presence of the magnetic field and the magnetic field is also perturbed due to the motion of conducting fluid. This phenomena is called Magnetohydrodynamics and in short written as MHD. There are wider applications in engineering technology, astrophysics and other applied sciences. The magnetohydrodynamic flow of viscous problems has been given in the standard such as Alfven[1], Cowling[2], Chandrasekhar[3], Ferraro and Plumpton[4], Jefferey[5], Cabannes[6] and others.

Ahmadi and Manvi[7] derived a general equation of viscous flow through porous medium and applied them to some basic problems. On account of varied practical applications of the magnetohydrodynamic flow problems in pipes of various cross-sections through porous medium several authors such as Ram and Mishra[8], Gupta[9], Kuiry[10], Sengupta and Kumar[11], Avinash and Rao[12], Kumari and Varshney[13], Shakya and Johri[14], Kumar and Singh[15], Raveendranath and Prasada Rao[16], Reddy and Verma[17] have paid their attention in this direction.

The aim of the present paper is to study the flow of viscous liquid moving steadily under the pressure between two flat plates with constant velocity under the influence of uniform magnetic field applied perpendicularly to the flow of viscous liquid. The expression for velocity of the viscous liquid is obtained in elegant form.

Formulation of the Problem and Solution

Incompressible viscous liquid is moving steadily under pressure between flat plates $y=0$, $y=h$ the plate $y=0$ has constant velocity U in the direction of the axis of x and the plane $y=h$ is fixed. The plates are porous and the liquid is sucked uniformly over one and eject uniformly over the other obtain a solution under the influence of uniform transverse magnetic field.

Since the plate $y=0$ and $y=h$ are taken infinitely large the component of velocity (u,v) is independent from x, y *and* $w=0$. Let y be the direction perpendicular to the flow and width of the plates parallel to *z*- direction.

The equation of continuity

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0$$

Reduces as

$$\frac{\partial v}{\partial y}=0$$

This show v is independent of y so it a function of x or constant. But

$$v=\frac{\upsilon}{a} \tag{1}$$

where $\upsilon =$ coefficient of viscosity
$a =$ constant

Also Navier Stoke's equation in the absence of body force in two directions under the influence of uniform magnetic field.

$$v\frac{du}{dy}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\upsilon\frac{d^2u}{dy^2}-\frac{\sigma B_0^2 u}{\rho} \tag{2}$$

$$0=-\frac{1}{\rho}\frac{\partial p}{\partial x} \tag{3}$$

From equation (1) and (2), we get

$$\frac{\upsilon}{a}\frac{du}{dy}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\upsilon\frac{d^2u}{dy^2}-\frac{\sigma B_0^2 u}{\rho} \tag{4}$$

Introducing the following non-dimensional quantities:

$$u^*=\frac{a}{\upsilon}u,\quad x^*=\frac{1}{a}x,\quad y^*=\frac{1}{a}y,\quad p^*=\frac{a^2}{\rho\upsilon^2}p \tag{5}$$

in equation (4), we get (after dropping the stars)

$$\frac{du}{dy}=-\frac{\partial p}{\partial x}+\frac{d^2u}{dy^2}-H^2u \tag{6}$$

where $H=aB_0\sqrt{\frac{\sigma}{\mu}}$ is the Hartmann number.

P is independent of y from (3)

$$\therefore \frac{\partial p}{\partial x}=\frac{dp}{dx}=-P \text{ (say)}$$

$$\Rightarrow \quad \frac{d^2u}{dy^2}-\frac{du}{dy}-H^2u=-P \tag{7}$$

The boundary conditions are

$$\left.\begin{aligned} u=U \quad at \quad y=o \\ u=o \quad at \quad y=h \end{aligned}\right\} \tag{8}$$

Solution of equation (7) is

$$u(y)=A\left\{\cosh\frac{1}{2}y+\sinh\frac{1}{2}y+\cosh\frac{\sqrt{1+4H^2}}{2}y\right\}+B\sinh\frac{\sqrt{1+4H^2}}{2}y-\frac{P}{H^2} \tag{9}$$

Applying boundary conditions (8) in (9), we get

$$\begin{aligned} u(y)=\frac{UH^2+P}{2H^2}\left\{\cosh\frac{1}{2}y+\sinh\frac{1}{2}y+\cosh\frac{\sqrt{1+4H^2}}{2}y\right\}+ \\ \frac{\sinh\frac{\sqrt{1+4H^2}}{2}y}{\sinh\frac{\sqrt{1+4H^2}}{2}h}\left[\frac{P}{2H^2}-\frac{U}{2}\left\{\cosh\frac{1}{2}h+\sinh\frac{1}{2}h+\cosh\frac{\sqrt{1+4H^2}}{2}h\right\}\right]-\frac{P}{H^2} \end{aligned} \tag{10}$$

where

$$A = \frac{UH^2 + P}{2H^2} \quad \text{and} \quad B = \frac{1}{\sinh \frac{\sqrt{1+4H^2}}{2} h}\left[\frac{P}{H^2} - \frac{UH^2 + P}{2H^2}\left\{\cosh \frac{1}{2} h + \sinh \frac{1}{2} h + \cosh \frac{\sqrt{1+4H^2}}{2} h\right\}\right]$$

Conclusion

The nature of the Magneto-hydrodynamics is to reduce the velocity of the fluid therefore the presence of the uniform magnetic field between two flat plates will definitely reduced the velocity of non-Newtonian fluid.

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