Research Journal of Mathematical and Statistical Sciences
Vol. **1(1),** 19-22, February **(2013)** *Res. J. Mathematical & Statistical Sci.*

Unsteady Flow of Non-Newtonian [Oldroyd (1958) Model] Fluid with Transient Pressure Gradient through Porous Media in A Rectangular Channel

Kumar Ravindra[1], Mishra Gaurav[1] and Sharma A.K.[2]
[1]Department of Mathematics, Agra College, Agra, UP, INDIA
[2]Department of Mathematics, K.K.P.G. College, Etawah, UP, INDIA

Available online at: www.isca.in
Received 13th December 2012, revised 28th December 2012, accepted 24th January 2013

Abstract

The aim of the present paper is to investigate the unsteady flow of non-Newtonian Oldroyd model fluid of second order with transient pressure gradient through porous media in a long rectangular channel. The expression for velocity of the fluid is obtained in elegant form. The various deductions have also been discussed in detail.

Introduction

The study of physics of flow through porous media has become the basis of many scientific and engineering applications. This type of flow is of great importance in the petroleum engineering concerned with the movement of oil, gas and water through the reservoir of an oil or gas field to the hydrologist in the study of the migration of underground water and to the chemical engineer in the filtration process.

Many research workers have paid their attention towards the application of non- Newtonian fluid flow through porous media in various types of channel, such as Oldroyd[1], Gupta and Sharma[2], Kapur, Bhatt and Sachetti[3] , Singh, Shankar and Singh[4], Gupta and Gupta[5], Singh and Kumar[6], Hayat, Asghar and Siddiqui[7], Sharma and Pareek[8], Kundu and Sengupta[9], Hassianien[10], Sengupta and Basak[11], Pundhir and Pundhir[12], Rehman and Alam Sarkar[13], Agarwal and Agarwal[14], Singh, Kumar and Sharma[15], Kumar, Sharma and Singh[16] and Kumar, Mishra and Singh[17] etc.

In the present paper, the unsteady laminar flow of non-Newtonian Oldroyd fluid of second order with transient pressure gradient through porous media in a long rectangular channel. The various deductions have also been discussed in detail.

Basic theory and Equations of Motions

For slow motion, the Rheological equations for second order non-Newtonian [Oldroyd[1] model] fluid are:

$$\tau_{ij} = -p\delta_{ij} + \tau_{ij}^{'} \tag{1}$$

$$\left(1+\lambda_1 \frac{\partial}{\partial t}+\lambda_2 \frac{\partial^2}{\partial t^2}\right)\tau_{ij}^{'} = 2\mu\left(1+\mu_1 \frac{\partial}{\partial t}+\mu_2 \frac{\partial^2}{\partial t^2}\right)e_{ij} \tag{2}$$

$$e_{ij} = \frac{1}{2}\left(v_{i,j}+v_{j,i}\right) \tag{3}$$

where $\tau_{ij} =$ The stress tensor , $\tau_{ij}^{'} =$The deviatoric stress tensor, $e_{ij} =$The rate of strain tensor, $p =$The pressure, $\lambda_1 =$The stress relaxation time parameter, $\lambda_2 =$The additional material constant, $\mu_1 =$The strain rate retardation time parameter, $\mu_2 =$ The additional material constant, $\delta_{ij} =$The metric tensor, $\mu =$The coefficient of viscosity, $v_i =$The velocity components

Formulation of the Problem

Let us consider the walls of rectangular channel to be the $x = \pm a$ and $y = \pm b$. *z*-axis is taken towards the direction of motion of the fluid. Again let $0, 0, w(x, y, t)$ be the velocity components along *x, y, z*-directions respectively. A transient pressure gradient $-pe^{-\omega t}$ varying with time is applied to the non-Newtonian fluid.

Following the stress-strain relations (1)-(3), the equation for unsteady motion through porous media is given by

$$\left(1+\lambda_1\frac{\partial}{\partial t}+\lambda_2\frac{\partial^2}{\partial t^2}\right)\frac{\partial w}{\partial t}=-\frac{1}{\rho}\left(1+\lambda_1\frac{\partial}{\partial t}+\lambda_2\frac{\partial^2}{\partial t^2}\right)\frac{\partial p}{\partial z}+\upsilon\left(1+\mu_1\frac{\partial}{\partial t}+\mu_2\frac{\partial^2}{\partial t^2}\right)\nabla^2 w-\ \frac{\upsilon}{K}\left(1+\lambda_1\frac{\partial}{\partial t}+\lambda_2\frac{\partial^2}{\partial t^2}\right)w \tag{4}$$

For the present problem, the boundary conditions are

$$\left.\begin{aligned} &w=0 \quad when\ \ x=\pm a, \quad -b\le y\le b\\ &w=0 \quad when\ \ y=\pm b, \quad -a\le x\le a\end{aligned}\right\} \tag{5}$$

Introducing the non-dimensional quantities

$$w^*=\frac{a}{\upsilon}w,\ \ p^*=\frac{a^2}{\rho\upsilon^2}p,\ \ t^*=\frac{\upsilon}{a^2}t,\ \ \left(x^*,y^*,z^*\right)=\frac{1}{a}(x,y,z),\ \ \lambda_1^*=\frac{\upsilon}{a^2}\lambda_1,$$

$$\mu_1^*=\frac{\upsilon}{a^2}\mu_1,\ \ \lambda_2^*=\frac{\upsilon^2}{a^4}\lambda_2,\qquad \mu_2^*=\frac{\upsilon^2}{a^4}\mu_2,\ \ K^*=\frac{1}{a^2}K$$

In equation (4) and dropping the stars, we get

$$\left(1+\lambda_1\frac{\partial}{\partial t}+\lambda_2\frac{\partial^2}{\partial t^2}\right)\frac{\partial w}{\partial t}=-\left(1+\lambda_1\frac{\partial}{\partial t}+\lambda_2\frac{\partial^2}{\partial t^2}\right)\frac{\partial p}{\partial z}+\left(1+\mu_1\frac{\partial}{\partial t}+\mu_2\frac{\partial^2}{\partial t^2}\right)\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}\right)-\ \frac{1}{K}\left(1+\lambda_1\frac{\partial}{\partial t}+\lambda_2\frac{\partial^2}{\partial t^2}\right)w \tag{6}$$

The boundary conditions (5) reduces to

$$w=0 \qquad when\ \ x=\pm 1,\quad -\frac{b}{a}\le y\le\frac{b}{a} \tag{7}$$

$$w=0 \qquad when\ \ y=\pm\frac{b}{a},\quad -1\le x\le 1 \tag{8}$$

Now we have to consider those type of situations of the flow which is transient in nature with respect to time and periodic in nature with respect to *y*. Subject to the nature of the boundary conditions (7) and (8) we choose the solution of (6) as

$$w=W(x)\,e^{-\omega t}\cos my \tag{9}$$

The boundary conditions (7) and (8) corresponding to (9) are

$$W=0 \qquad when\ \ x=\pm 1,\quad -\frac{b}{a}\le y\le\frac{b}{a} \tag{10}$$

$$W=0 \qquad when\ \ y=\pm\frac{b}{a},\quad -1\le x\le 1 \tag{11}$$

The boundary condition (11) will be satisfied if

$$\cos m\frac{b}{a}=0$$

or $$m\frac{b}{a}=(2n+1)\frac{\pi}{2}$$

or $$m=(2n+1)\frac{\pi a}{2b}\ ;\ \ n=0,\ 1,\ 2,3,\ldots\ldots\ldots\ldots \tag{12}$$

We may construct the solution as the sum of all possible solutions for each value of *n* of the form

$$w=\sum_{n=0}^{\infty}W(x)e^{-\omega t}\cos my \tag{13}$$

Putting the value of $\frac{\partial p}{\partial z}=-Pe^{-\omega t}$, $\omega>0$ in (6) we get

$$\sum_{n=o}^{\infty}\left\{\frac{d^2W(x)}{dx^2}-m^2W(x)\right\}\cos my+\sum_{n=o}^{\infty}\frac{\left(1-\lambda_1\omega+\lambda_2\omega^2\right)\left(\omega-\frac{1}{K}\right)}{\left(1-\mu_1\omega+\mu_2\omega^2\right)}W(x)\cos my+\frac{P\left(1-\lambda_1\omega+\lambda_2\omega^2\right)}{\left(1-\mu_1\omega+\mu_2\omega^2\right)}=0$$

or

$$\sum_{n=0}^{\infty}\left[\frac{d^2W(x)}{dx^2}-\left\{\frac{\left(1-\lambda_1\omega+\lambda_2\omega^2\right)}{\left(1-\mu_1\omega+\mu_2\omega^2\right)K}+m^2-\frac{\left(1-\lambda_1\omega+\lambda_2\omega^2\right)\omega}{\left(1-\mu_1\omega+\mu_2\omega^2\right)}\right\}W(x)+\frac{(-1)^n}{(2n+1)}\cdot\frac{4P}{\pi}\frac{\left(1-\lambda_1\omega+\lambda_2\omega^2\right)}{\left(1-\mu_1\omega+\mu_2\omega^2\right)}\right]\cos my=0 \quad \text{… (14)}$$

$\because 1=\frac{4}{\pi}\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)}\cos my$ (by Fourier cosine series in the interval of $-\frac{b}{a}\le y\le\frac{b}{a}$)

Equating the coefficient of *cos my* equal to zero, the value of $W(x)$ can be determined from

$$\frac{d^2W(x)}{dx^2}-\frac{Q^2}{a^2}W(x)+A_n=0 \tag{15}$$

where

$$Q^2=\left\{\frac{\left(1-\lambda_1\omega+\lambda_2\omega^2\right)}{\left(1-\mu_1\omega+\mu_2\omega^2\right)K}+m^2-\frac{\left(1-\lambda_1\omega+\lambda_2\omega^2\right)\omega}{\left(1-\mu_1\omega+\mu_2\omega^2\right)}\right\}a^2$$

$$A_n=\frac{(-1)^n}{(2n+1)}\cdot\frac{4P}{\pi}\cdot\frac{\left(1-\lambda_1\omega+\lambda_2\omega^2\right)}{\left(1-\mu_1\omega+\mu_2\omega^2\right)}$$

The general solution of (15) is

$$W(x)=C_1\cosh\frac{Q}{a}x+C_2\sinh\frac{Q}{a}x+\frac{a^2A_n}{Q^2} \tag{16}$$

Using boundary conditions (10), we get

$$W(x)=\frac{(-1)^n4P\left(1-\lambda_1\omega+\lambda_2\omega^2\right)}{(2n+1)\pi\left(1-\mu_1\omega+\mu_2\omega^2\right)Q^2}\left\{1-\frac{\cosh\frac{Q}{a}x}{\cosh\frac{Q}{a}}\right\} \tag{17}$$

Hence the velocity of Oldroyd fluid is given by

$$w(x,y,t)=\sum_{n=0}^{\infty}\frac{(-1)^n4P\left(1-\lambda_1\omega+\lambda_2\omega^2\right)}{(2n+1)\pi\left(1-\mu_1\omega+\mu_2\omega^2\right)Q^2}\left\{1-\frac{\cosh\frac{Q}{a}x}{\cosh\frac{Q}{a}}\right\}e^{-\omega t}\cos\frac{(2n+1)\pi a}{2b}y \tag{18}$$

Deductions

Case I: If we put $\lambda_2=0,\ \mu_2=0$ in equation (18) then we obtain all expressions for first order Oldroyd fluid of Singh, Kumar and Sharma[15].

Case II: If we take $\lambda_2=0,\ \mu_1=0\ and\ \ \mu_2=0$ in equation (18) then we get velocity expression for Maxwell visco-elastic fluid.

Case III: If we take $\lambda_1=0,\ \lambda_2=0,\ \mu_1=0\ and\ \ \mu_2=0$ in equation (18) then we get the velocity expression for purely viscous fluid.

Case IV: If porous media is withdrawn i.e. $K\to\infty$ in equation (18), we obtain all expressions of Kundu and Sengupta[9].

Conclusion

The nature of the porous medium is to reduce the velocity of the fluid therefore the presence of the porous medium in the rectangular channel will definitely reduced the velocity of non-Newtonian fluid and evidently velocity of the fluid in deductions case I, case II, case III and case IV will be slower than the velocity of the fluid obtained Kundu and Senguta[9].

Acknowledgement

The authors are thankful to Dr. K. K. Singh, Ex. Reader, Department of Mathematics, Agra College, Agra (U.P.) for his valuable suggestions and moral support.

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