Research Journal of Mathematical and Statistical Sciences ______________________________
Vol. **1(1),** 7-13, February **(2013)** *Res. J. Mathematical & Statistical Sci.*

Occasionally Weakly Compatible Maps in Fuzzy Metric Spaces

Rangamma M., Mallikarjun Reddy G. and Srikanth Rao P.
Department of Mathematics, O.U., Hyderabad-500 007, INDIA

Available online at: www.isca.in
Received 8th September 2012, revised 28th December 2012, accepted 29th December 2012

Abstract

The aim of this paper is to establish some common fixed point theorems under occasionally weakly compatible maps for six self mappings. Our results generalize several fixed point theorems in fuzzy metric spaces.
Mathematics subject classification: 47H10, 54H25.

Keywords: Compatible maps, R-weakly commuting maps, weakly compatible maps, occasionally weakly compatible maps.

Introduction

In 1965, the concept of fuzzy sets was introduced by Zadeh[1].Following the concept of fuzzy sets, fuzzy metric spaces have been introduced by Kramosil and Michalek[2] and George and Veeramani[3] modified the notion of fuzzy metric spaces with the help of continuous t-norms. Recently many authors have proved fixed point theorems involving fuzzy sets. Vasuki[4] investigated some fixed point theorems in fuzzy metric spaces for R-weakly commuting maps and Pant[5] introduced the notion of reciprocal continuity of mappings in metric spaces. Pant and Jah[6] proved an analogue of result given by Balasubramanyam[7]. S.Kutukcu[8] extended the result of Pant and Jha[6]. Recently Al-Thagafi and N. Shahzad[9] weakened the concept of compatibility by giving a new notion of occasionally weakly compatible (owc) maps which is most general among all the commutative concepts. The main purpose of our paper is to prove common fixed point theorems for six self mappings under occasionally weakly compatibility condition.

Preliminaries

Definition 1.1: A binary operation $*$: $[0, 1] \times [0, 1] \rightarrow [0, 1]$ is called a continuous t-norm[10] if $*$ is satisfying the following conditions: i. $*$ is commutative and associative, ii. $*$ is continuous, iii. $a*1 = a$ for all $a \in [0, 1]$, iv. $a*b \leq c*d$ whenever $a\leq c$ and $b\leq d$ for all $a, b, c, d \in [0, 1]$.

Definition 1.2: A 3-tuple $(X, M, *)$ is said to be a fuzzy metric space[3] if X is an arbitrary set, $*$ is a continuous t-norm and M is a fuzzy set on $X^2 \times (0, \infty)$ satisfying the following conditions for all $x, y, z \in X$, $s, t > 0$.
(fm1) $M(x, y, t) > 0$, (fm2) $M(x, y, t) = 1$ if and only if $x=y$, (fm3) $M(x, y, t) = M(y, x, t)$, (fm4) $M(x, y, t) * M(y, z, s) \leq M(x, z, t+s)$, (fm5) $M(x, y, \bullet)$: $(0, \infty) \rightarrow [0, 1]$ is left continuous.

Then M is called a fuzzy metric on X. The function $M(x, y, t)$ denote the degree of nearness between x and y with respect to 't'. We identify $x=y$ with $M(x, y, t)=1$ for all $t>0$ and $M(x, y, t)=0$ with ∞ and we can find some topological properties and examples of fuzzy metric spaces in paper of George and Veeramani[3].

Example 1.3 (Induced fuzzy metric[3]) : Let (X, d) be a metric space.Define $a*b= ab$ for all $a,b\in[0,1]$ and let M_d fuzzy sets on $X^2 \times(o, \infty)$ defined as follows, $M_d(x, y, t)= \frac{t}{t+d(x,y)}$ then $(X, M_d, *)$ is a fuzzy metric space. We call this fuzzy metric induced by the metric d, the standard fuzzy metric. On the other hand note that there exists no metric on X satisfying the above $M_d(x, y, t)$.

Remark 1.4: Since $*$ is continuous, it follows from (fm4) that the limit of the sequence in fuzzy metric space is uniquely determined. Let $(X, M, *)$ is a fuzzy metric space with the following condition (fm6) $\lim_{t\to\infty} M(x, y, t)=1$ for all $x, y \in X$.

Lemma 1.5: For all $x, y \in X$, $M(x, y, \bullet)$ is non decreasing.

Proposition 1.6: In a fuzzy metric space $(X, M, *)$, if $a* a \geq a$ for all $a\in [0, 1]$ then[13] $a*b= \min\{a, b\}$ for all $a, b \in [0, 1]$ i.e. the only t-norm satisfying $a*a\geq a$ for all $a \in [0, 1]$ is the norm $a*b=\min\{a, b\}$.

Definition 1.7: Two self maps A and S of a fuzzy metric space (X, M, *) are called compatible[11] if $\lim_{n\to\infty} M(ASx_n, SAx_n, t)=1$ whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} Ax_n= \lim_{n\to\infty} Sx_n=x$ for some x in X.

Definition 1.8: Two self maps A and S of a fuzzy metric space (X, M, *) are called weakly commuting[4] if $M(ASx, SAx, t)\geq M(Ax, Sx, t)$ for all x in X and t>0.

Definition 1.9: Two self maps A and S of a fuzzy metric space (X, M, *) are called R-weakly commuting[4] if there exists R> 0 such that $M(ASx, SAx, t)\geq M(Ax, Sx, \frac{t}{R})$ for all x in X and t>0.

Remark 1.10: Clearly, point wise R-weakly commuting implies weak commuting only when R≤1.

Remark 1.11: Compatible mappings are point wise R-weakly commuting but not conversely. The following theorem was proved by S.Kutukcu[8]

Theorem 1.12: Let (X, M, *) be complete fuzzy metric space with a*a≥a for all a∈ [0, 1] and the condition (fm6). Let (A, S) and (B, T) be point wise R-weakly commuting pairs of self mappings of X such that i. $A(X) \subset T(X)$, $B(X) \subset S(X)$, ii. There exists k ∈ (0, 1) such that $M(Ax, By, kt) \geq N(x, y, t)$ where $N(x, y, t)= M(Sx, Ax, t)*M(Ty, By, t)*M(Sx, Ty, t)*M(Ty, Ax, \alpha t)*M(Sx, By, (2-\alpha)t)$ for all x, y∈X, α∈ (0, 2) and t>0,

If one of the mappings in compatible pair (A, S) or (B, T) is continuous then A, B, S and T have a unique common fixed point.

Definition 1.13: Two self mappings A and S of a fuzzy metric space (X, M, *) are said to be weakly compatible[2] if they commute at their coincidence points that is the pair (A, S) is weakly compatible pair if Ax=Sx implies ASx=SAx for all x∈X.

Definition 1.14: Let X be a non empty set and S, T be two self maps on X. A point x in X is said to be a coincidence point[11] of S and T if Sx=Tx. A point W in X is said to be a point of coincidence of S and T if there exists a coincidence point x in X of S and T such that W=Sx=Tx. Recently, Al-Thagafi and N.Shahzad[9], introduced the concept of occasionally weakly compatible maps (OWC) as follows.

Definition 1.15: Two self maps are said to be Occasionally Weakly Compatible[9] if there exists at least one x in X for which Sx=Tx implies STx=TSx.

Example 1.16: Let $X=[1,\infty)$ endowed with Euclidean metric d, we define $M(x,y,t)=\dfrac{t}{t+d(x,y)}$ $\forall x,y\in X, t>0$. Let * be any continuous t-norm then (X, M, *) is a Fuzzy Metric space. Let S, T be self maps on X defined by $S(x)=4x-3, x\in X$ and $T(x)=x^2, x\in X$ then 1and 3 are the only points of coincidence of S and T. Also ST(1)=1=TS(1) but ST(3)=33 ≠ 81=TS(3), Clearly S and T are OWC but not Weakly compatible.

Remarks 1.17: OWC maps need not be weakly compatible.

Lemma 1.18: If $\forall x, y\in X$ and $t>0, k\in(0,1)$ such that[5] $M(x,y,kt)\geq M(x,y,t)$ then $x=y$.

Lemma 1.19: Let x be a set, S and T be OWC self maps on X. If f and g have a unique point of coincidence[11] w=Sx=Tx then w is the unique common fixed point of S and T.

Main Results

In this section we extend the **theorem 1.12** by using OWC which generalizes the results of Balasubramaniam[7] and S.Kutukcu[8].

Theorem: Let (X, M, *) be a fuzzy metric space with * being continuous t-norm such that a*a ≥ a for all a ∈ [0, 1]. Let S, T be self maps on X satisfying the following

(**2.**1.1) Pair (S, T) is occasionally weakly compatible

(**2.**1.2) There exists $q \in (0,1)$ such that

$M(Tx,Ty,qt) \geq \alpha M(Sx,Sy,t) + \beta[M(Tx,Sy,t) * M(Sy,Tx,k_1t) * M(Ty,Sx,k_2t)]$

$\forall x,y \in X$ with $k_1,k_2 \in (0,2)$ such that $k_1 + k_2 = 2, t > 0$ $\alpha,\beta > 0, \alpha+\beta > 1$

(**2**.1.3) One of S or T is injective then S and T have a unique coincidence point and a unique fixed point in X.

proof: Since the pair (S,T) is OWC there exists $x_0 \in X$ such that

(2.1.4) $Sx_0 = Tx_0$ implies $STx_0 = TSx_0$. We claim that x_0 is unique such that $Sx_0 = Tx_0$. If let $y_0 \in X$ such that

(2.1.5) $Sy_0 = Ty_0$

Using (2.1.2) $M(Tx_0,Ty_0,qt) \geq \alpha M(Sx_0,Sy_0,t) + \beta[M(Tx_0Sy_0,t) * M(Sy_0Tx_0,k_1t) * M(Ty_0Sx_0,k_2t)]$

Taking $k_1 = 1 = k_2$ and using (2.1.4) and (2.1.5)

$M(Tx_0,Ty_0,qt) \geq \alpha M(Tx_0,Ty_0,t) + \beta[M(Tx_0Ty_0,t) * M(Ty_0Tx_0,t) * M(Ty_0Tx_0,t)]$

$\geq \alpha M(Tx_0,Ty_0,t) + \beta[M(Tx_0Ty_0,t) * M(Ty_0Tx_0,t)]$

$> \alpha M(Tx_0,Ty_0,t) + \beta M(Tx_0Ty_0,t)$

$= (\alpha+\beta) M(Tx_0Ty_0,t)$

Implies $Tx_0 = Ty_0$ since $\alpha + \beta > 1$. Therefore $Sx_0 = Tx_0 = Ty_0 = Sy_0$

Now either S or T is injective in either case $x_0 = y_0$, hence x_0 is the unique coincidence point of S and T.

Let $Sx_0 = Tx_0 = k$ (say) then k is the unique point of coincidence. Then using lemma1.19 k is the unique common fixed point of S and T.

Theorem 2.2: Let (X, M, *) be a fuzzy metric space with * being continuous t-norm such that $a * a \geq a \quad \forall a \in [0,1]$. Let S, T be self maps on X satisfying (2.1.1) and (2.1.3) of above theorem along with the following condition

(2.2.1) there exists $q \in (0,1)$ such that

$M(Tx,Ty,qt) \geq \alpha M(Sx,Sy,t) + \beta[M(Sx,Ty,k_1t) * M(Sx,Tx,k_2t) * M(Tx,Sy,k_1t) * M(Ty,Sy,k_2t)]$ $\forall x,y \in X$ with $k_1,k_2 \in (0,2)$ such that $k_1 + k_2 = 2, t > 0$ $\alpha,\beta > 0, \alpha+\beta > 1$ then S and T have a unique coincidence point which is the unique common fixed point of S and T in X.

Proof: Since the pair (S, T) is OWC there exists $x_0 \in X$ such that

(2.2.2) $Sx_0 = Tx_0$ implies $STx_0 = TSx_0$

We claim that x_0 is unique such that $Sx_0 = Tx_0$**.** For if let $y_0 \in X$ such that

(2.2.3) $Sy_0 = Ty_0$

Using (2.2.1)

$M(Tx_0,Ty_0,qt) \geq \alpha M(Sx_0,Sy_0,t) + \beta[M(Sx_0,Ty_0,k_1t) * M(Sx_0,Sy_0,k_2t) * M(Tx_0,Sy_0,k_1t) * M(Tx_0,Sy_0,k_2t)]$

Taking $k_1 = 1 = k_2$ and using (2.2.2) and (2.2.3)

$M(Tx_0,Ty_0,qt) \geq \alpha M(Tx_0,Ty_0,t) + \beta[M(Tx_0,Ty_0,t) * M(Tx_0,Tx_0,t) * M(Tx_0,Ty_0,t) * M(Ty_0,Ty_0,t)]$

$= \alpha M(Tx_0, Ty_0, t) + \beta[M(Tx_0, Ty_0, t) * 1 * M(Tx_0, Ty_0, t) * 1]$
$= \alpha M(Tx_0, Ty_0, t) + \beta[M(Tx_0, Ty_0, t) * M(Tx_0, Ty_0, t)]$
$= \alpha M(Tx_0, Ty_0, t) + \beta M(Tx_0, Ty_0, t)$
$= (\alpha + \beta) M(Tx_0, Ty_0, t)$

Implies since $\alpha + \beta > 1$. Therefore $Sx_0 = Tx_0 = Ty_0 = Sy_0$. Since either S or T is injective, it follows in both the cases that $x_0 = y_0$. Hence x_0 is unique coincidence point of S and T. Let $Sx_0 = Tx_0 = k$ (say) using lemma 1.19, k is unique fixed point of S and T further it follows $x_0 = k$.

Theorem 2.3: Let (X, M, *) be a fuzzy metric space with * being continuous t-norm such that $a * a \geq a \quad \forall a \in [0,1]$. Let A, B, S, T, P and Q be self maps on X such that

(2.3.1) the Pairs (P, ST), (Q, AB) are OWC

(2.3.2) there exists $q \in (0,1)$ such that
$M(Px, Qy, qt) \geq M(STx, ABy, t) * M(STx, Px, t) * M(ABy, Qy, t) * M(STx, Qy, \alpha t) * M(ABy, Px, \beta t)$
$x, y \in X, t > 0$ with $\alpha, \beta \in (0,2), \alpha + \beta = 2$
(2.3.3) the pairs $(A, B), (T, S), (P, S), (Q, A), (P, T), (Q, B)$ are commuting. Then the maps A, S, P, T, Q and B have a unique common fixed point in X.
Proof: Since (P, ST), (Q, AB) are OWC there exists $x_0, y_0 \in X$ such that

(2.3.4) $STx_0 = Px_0$ implies $P(ST)x_0 = (ST)Px_0$ and

(2.3.5) $ABy_0 = Qy_0$ implies $Q(AB)y_0 = (AB)Qy_0$
We claim that $Px_0 = Qy_0$. Using (2.3.2)
$M(Px_0, Qy_0, qt) \geq M(STx_0, ABy_0, t) * M(STx_0, Px_0, t) * M(ABy_0, Qy_0, t) * M(STx_0, Qy_0, \alpha t) * M(ABy_0, Px_0, \beta t)$
Taking $\alpha = 1 = \beta$ and using (2.3.4) and (2.3.5)
$M(Px_0, Qy_0, qt) \geq M(Px_0, Qy_0, t) * M(Px_0, Px_0, t) * M(Qy_0, Qy_0, t) * M(Px_0, Qy_0, t) * M(Qy_0, Px_0, t)$
$\geq M(Px_0, Qy_0, t) * 1 * 1 * M(Px_0, Qy_0, t) * M(Qy_0, Px_0, t)$
$\geq M(Px_0, Qy_0, t)$

Now using lemma 1.18, $Px_0 = Qy_0$ and therefore

(2.3.6) $STx_0 = Px_0 = Qy_0 = ABy_0 = k$ (say)
Then from (2.3.4) and (2.3.5) implies
(2.3.7) $STk = Pk$
(2.3.8) $ABk = Qk$
We claim $Pk = Qk$. Using (2.3.5)
$M(Pk, Qk, qt) \geq M(STk, ABk, t) * M(STk, Pk, t) * M(ABk, Qk, t) * M(STk, Qk, \alpha t) * M(ABk, Pk, \beta t)$
$M(Pk, Qk, qt) \geq M(STk, ABk, t) * M(STk, Pk, t) * M(ABk, Qk, t) * M(STk, Qk, \alpha t) * M(ABk, Pk, \beta t)$
Taking $\alpha = 1 = \beta$ using (2.3.7) and (2.3.8)
$M(Pk, Qk, qt) \geq M(Pk, Qk, t) * 1 * 1 * M(Pk, Qk, t) * M(Pk, Qk, t)$
$M(Pk, Qk, t)$

Now using lemma1.18, $Pk = Qk$ and therefore

(2.3.9) $STk = Pk = Qk = ABk$

We prove $Pk = k$.

From (2.3.3) $M(Pk, k, qt) = M(Pk, Qy_0, qt)$

$\geq M(STk, ABy_0, t) * M(STk, Pk, t) * M(ABy_0, Qy_0, t) * M(STk, Qy_0, \alpha t) * M(ABy_0, Pk, \beta t)$

Taking $\alpha = 1 = \beta$ using (2.3.7), (2.3.8) and (2.3.9)

$M(Pk, k, qt) \geq M(Pk, k, t) * 1 * 1 * M(Pk, k, t) * M(Pk, k, t)$

$\geq M(Pk, k, t)$

Using lemma 1.18, $Pk = k$

Therefore $STk = Pk = Qk = ABk = k$

Put $x = Tk$ and $y = k$ with $\alpha = 1 = \beta$ in (2.3.2)

$M(Px, Qy, qt) = M(PTk, Qk, qt)$

$\geq M(STTk, ABk, t) * M(STTk, PTk, t) * M(ABk, Qk, t)$

$*M(STTk, Qk, t) * M(ABk, PTk, t)$

$\geq M(Tk, k, t) * M(Tk, Tk, t) * M(k, k, t) * M(Tk, k, t) * M(k, Tk, t)$

$M(Tk, k, qt) \geq M(Tk, k, t)$

Which implies $Tk = k$.

Since $STk = k$ implies $Sk = k$

To show $Bk = k$ put $x = k, y = Bk$ with $\alpha = 1 = \beta$ in (2.3.2)

$M(Pk, QBk, qt) \geq M(STk, ABBk, t) * M(STk, Pk, t) * M(ABBk, QBk, t) * M(STk, QBk, t) * M(ABBk, Pk, t)$

$M(Pk, Bk, qt) \geq M(Pk, Bk, t) * M(Pk, Pk, t) * M(Bk, Bk, t) * M(k, Bk, t) * M(Bk, Pk, t)$

$M(k, Bk, qt) \geq M(k, Bk, t) * 1 * 1 * M(k, Bk, t) * M(Bk, k, t)$

$M(k, Bk, qt) \geq M(Bk, k, t)$

Which implies $Bk = k$, since $ABk = k$ implies $Ak = k$.

Therefore $Ak = Bk = Sk = Tk = Pk = Qk = k$.

i.e. k is the common fixed point of A,B,S,T,P and Q.

for uniqueness, let $\omega(\omega \neq z)$ be another common fixed point of A,B,S,T,P and Q with $\alpha = 1 = \beta$ in (2.3.2) we write

M(Pk,Qω,qt) ≥M(STk, ABω, t)*M(STk,Pz, t)*M(ABω,Qω, t) *M(STk,Qω,t)*M(ABω,Pk, t)

M(k,ω,qt) ≥M(k,ω, t)*M(k, k, t)*M(ω,ω, t)*M(k,ω, t)*M(ω,k, t)

M(k,ω,qt) ≥M(k,ω, t)

Which shows k =ω '

If we put B = T =I_x (the identity map on X) in the theorem 2.3 we have the following.

Corollary (2.4): Let (X, M, *) be a fuzzy metric space with * being continuous t-norm such that $a * a \geq a \quad \forall a \in [0,1]$. Let A, S, P and Q be self maps on X such that

(2.4.1) Pairs (P, S), (Q, A) are OWC

(2.4.2) there exists $q \in (0,1)$ such that

$M(Px, Qy, qt) \geq M(Sx, Ay, t) * M(Sx, Px, t) * M(Ay, Qy, t) * M(Sx, Qy, \alpha t) * M(Ay, Px, \beta t)$

$x, y \in X, t > 0$ with $\alpha, \beta \in (0,2), \alpha + \beta = 2$

Then the maps A, S, P, and Q have a unique common fixed point in X.

If we put P= Q, B = T =I_x in theorem 2.3 we have the following.

Corollary (2.5): Let (X, M, *) be a fuzzy metric space with * being continuous t-norm such that $a * a \geq a \quad \forall a \in [0,1]$. Let A, S, P be self maps on X such that
(2.5.1) Pairs (P, S), (P, A) are OWC
(2.5.2) there exists $q \in (0,1)$ such that

$$M(Px,Py,qt) \geq M(Sx,Ay,t) * M(Sx,Px,t) * M(Ay,Py,t) * M(Sx,Py,\alpha t) * M(Ay,Px,\beta t)$$

$x, y \in X, t > 0$ with $\alpha, \beta \in (0,2), \alpha + \beta = 2$
Then the maps A, S, P have a unique common fixed point in X.

If we put P =Q, A = S and B = T =I_x in theorem 2.3 we have the following.

Corollary (2.6): Let (X, M, *) be a fuzzy metric space with * being continuous t-norm such that $a * a \geq a \quad \forall a \in [0,1]$. Let A, S, P be self maps on X such that

(2.6.1) the pair (P, S) is OWC
(2.6.2) there exists $q \in (0,1)$ such that

$$M(Px,Py,qt) \geq M(Sx,Sy,t) * M(Sx,Px,t) * M(Sy,Py,t) * M(Sx,Py,\alpha t) * M(Sy,Px,\beta t)$$

$x, y \in X, t > 0$ with $\alpha, \beta \in (0,2), \alpha + \beta = 2$

Then the maps P, S have a unique common fixed point in X. If we put A= S and B = T = I_x in theorem 2.3 we have the following.

Corollary (2.7): Let (X, M, *) be a fuzzy metric space with * being continuous t-norm such that $a * a \geq a \quad \forall a \in [0,1]$. Let A, S, P and Q be self maps on X such that
(2.7.1) Pairs (P.S), (Q, S) are OWC
(2.7.2) there exists $q \in (0,1)$ such that

$$M(Px,Qy,qt) \geq M(Sx,Sy,t) * M(Sx,Px,t) * M(Sy,Qy,t) * M(Sx,Qy,\alpha t) * M(Sy,Px,\beta t)$$

$x, y \in X, t > 0$ with $\alpha, \beta \in (0,2), \alpha + \beta = 2$

Then the maps P, S and Q have a unique common fixed point in X.

Remark (2.8): Since a*b = min{a, b} then the condition (2.3.4) in the theorem 2.3 becomes

$$M(Px,Qy,qt) \geq \min\{M(STx,ABy,t), M(STx,Px,t), M(ABy,Qy,t), M(STx,Qy,\alpha t), M(ABy,Px,\beta t)\}$$

$x, y \in X, t > 0$ with $\alpha, \beta \in (0,2), \alpha + \beta = 2$.

Now, we prove the theorem 2.3 from the compatible fuzzy metric space to complete metric space.

Theorem 2.9: Let A, B, S, T, P and Q be self mappings of a complete metric(X, d). Suppose that the pairs (P, ST) and (Q, AB) are OWC and also(A, B), (S, T), (P, S), (A, Q), (T, P), (Q, B) are commuting mappings with one of P, Q, AB, ST are continuous. If there exists a constant k $\in$ (0, 1) such that for all x, y $\in$ X d(Px, Qy) $\leq$ k max{d(STx, Px), d(ABy, Qy), d(STx, ABy), $\frac{1}{2}$[d(ABy, Px)+d(STx, Qy)]}Then A, B, S, T, P and Q have a unique common fixed point in X.

Proof: The proof follows from theorem 2.3, by considering the induced fuzzy metric space (X, M, *) where a*b = min{a, b} and M(x, y, t) = $\frac{t}{t+d(x,y)}$.

This result also generalizes the results of Pant and Jha[6], Balasubramaniam[7] and S.Kutukcu[8] for complete metric space in the aforesaid sense.

Conclusion

This article investigates common fixed point theorems for six self mappings. The concept of occasionally weakly compatible maps in Fuzzy metric spaces has also been used. Several Fixed point theorems in Fuzzy metric spaces such as fixed point theorems for four, three and two self mappings have been derived in the present study as particular cases.

References

1. Zadeh L.A., Fuzzy Sets, Information and Control, **89,** 338-353 **(1965)**

2. Kramosil I. and Michalek J., Fuzzy metric and statistical metric spaces, Kybernetica, **11,** 336-344 **(1975)**

3. George A., Veeramani P., On some results in fuzzy metric spaces, Fuzzy Sets and Systems, **64,** 395-399 **(1994)**

4. Vasuki R., Common fixed points for R-weakly commuting maps in fuzzy metric spaces, *Indian J.Pure.Appl.Math.,* **30,** 419-423 **(1999)**

5. Pant R.P., Common fixed points of four mappings, *Bull.Cal.Math.Soc.,* **90,** 281-286 **(1998)**

6. Pant R.P., Jha K., Aremarkon, Common fixed points of four mappings in a fuzzy metric space, *J.Fuzzy Math.,* **12(2),** 433-437 **(2004)**

7. Balasubramaniam P., MuraliSankar S. and Pant R.P., Common fixed points of four mappings in a fuzzy metric space, *J.Fuzzy Math.,* **10(2),** 379-384 **(2002)**

8. Kutukcu S., Sharma S. and Tokgoz H., A fixed point theorem in fuzzy metric spaces, *Int.J.Math.Analysis,* **1(18),** 861-872 **(2007)**

9. Al-Thagafi M.A., Shahzad N., A note on occasionally weakly compatible maps, *Int. J. Math. Anal.,* **3(2),** 55-58 **(2009)**

10. Klement E.P., Mesiar R. and Pap E., Triangular Norms in Trends in Logic, Vol.8, Kluwer Academic Publishers, Dordrecht, **(2000)**

11. Jungck G. and Rhodes B.E, Fixed point for Occasionally weakly compatible mappings, *Fixed point theory*, **7(2)**, 287-296 **(2006)**

12. Grabiec M., Fixed points in fuzzy metric spaces, *Fuzzy Sets and Systems*, **27,** 385-389 **(1989)**

13. Mishra S.N., Sharma N. and Singh S.L., Common fixed Points of maps on fuzzy metric spaces, *Internet.J.Math & Math.Sci*., **17**, 253-258 **(1994)**

14. Jungck G., Compatible mappings and common fixed points (2), *Internat.J.Math. Math. Sci*., 285-288 **(1988)**

15. Cho Y.J., Pathak H.K., Kang S.M. and Jung J.S., Common fixed points of compatible maps of type (β) on fuzzy metric spaces, *Fuzzy Sets and Systems*, **93,** 99-111 **(1998)**

16. Kutukcu S., A fixed point theorem in manger spaces, *Int.Math.F*., **1(32)**, 1543-1554 **(2006)**

17. Kutukcu S., Yildiz C. and Juna A., On common fixed points in Menger Probabilistic metric spaces, *Int.J.Contemp.Math.Sci.,* **2(8),** 383-391 **(2007)**

18. Kutukcu S., A fixed point theorem for contraction type mappings in Menger Spaces, *Am.J.Appl.Sci*., **4(6)**, 371-373 **(2007)**

19. Mishra S.N., Common fixed points of Compatible mappings in PM-Spaces, *Math.Japon*., **36,** 283-289 **(1991)**

20. Hadzic O., Common fixed point theorems for families of mapping in complete metric space, *Math.Japon*., **29,** 127-134 **(1984)**

21. Pant R.P., Common fixed point theorems for contractive maps, *J.Mematica ath.Anal.Appl.*, **226**, 251-258 **(1998)**

22. Rhoades B.E., Contractive definitions and continuity, *Contemparary Math*., **72,** 233-245 **(1988)**

23. Sharma S., Common fixed point theorems in fuzzy metric spaces, *Fuzzy Sets and Systems*, **127**, 345-352 **(2002)**

24. Sharma S. and Deshpande B., Discontinuity and weak compatibility in fixed point consideration on non-complete fuzzy metric spaces, *J.Fuzzy Math.,* **11(2),** 671-686 **(2003)**

25. Sharma S. and Deshpande B., Compatible multivalued mappings satisfying an implicit relation, *Southeast Asian. Bull.Math*., **30**, 535-540 **(2006)**

26. Al-Thagafi A. and Naseer Shahzad, Generalized I-Non expansive self maps and invariant approximation Acta Mathematica sinica, English Series, **24(5)**, 867-876 **(2008)**